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Cohomology, superderivations and Abelian extensions of 3-Lie superalgebras

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Abstract. The main object of the study of this paper is the notion of 3-Lie superalgebras with superderivations. We consider a representation $(\Phi, \mathcal{P})$ of a 3-Lie superalgebra $\mathcal{Q}$ on $\mathcal{P}$ and construct first-order cohomologies by using superderivations of $\mathcal{P}$ and $\mathcal{Q}$ which induce a Lie superalgebra $\mathcal{T}_\Phi$ and its representation Ψ. Then, we consider an abelian extension of 3-Lie superalgebras of the form $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} \mathcal{Q} \to 0$ and construct an obstruction class to the extensibility of a compatible pair of superderivations. Moreover, we prove that a pair of superderivations is extensible if and only if its obstruction class is trivial under some suitable conditions.

1. Introduction

Filippov introduced n-Lie algebras in 1985 [5]. n-Lie algebras, in particular 3-Lie algebras are important in mathematical physics. Lie superalgebras are the $\mathbb{Z}_2$-graded Lie algebras which was introduced by Kac [6]. These are too interesting from a purely mathematical point of view. The notion of 3-Lie superalgebras are generalization of 3-Lie algebras extending to a $\mathbb{Z}_2$-graded case. n-Lie superalgebras are more general structures that include n-Lie algebras and Lie superalgebras whose definition was introduced by Cantarini *et al*. [1].

Derivation algebra is an important topic in Lie algebras, which has widespread applications in physics and geometry. A superderivation of a Lie superalgebra is a certain generalization of the derivation of a Lie algebra. The structure of the superderivation of Lie superalgebras was studied in [10, 13]. Cohomology is an important tool in modern mathematics and theoretical physics; its range of applications includes algebra and topology, as well as the theory of smooth manifolds and holomorphic functions. The cohomology of Lie algebras was defined by Chevalley *et al*. [2]. Leites introduced the cohomology of Lie superalgebras and extended some of the basic structures and results of classical theories to Lie superalgebras [7]. Further, cohomology for n-Lie superalgebras was discussed in [9].

Recently, Tang *et al*. studied a Lie algebra with a derivation from the cohomological point of view and constructed a cohomology theory that controls, among other things, simultaneous deformations of a

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Lie algebra with a derivation [12]. These results have been extended to associative algebras [3], Leibniz algebras [4], 3-Lie colour algebras [16], 3-Lie algebras [15], Lie triple systems [14], and n-Lie algebras [11]. Generalized representations of 3-Lie algebras and 3-Lie superalgebras were introduced in [8, 18]. Zhao *et al*. studied a representation of a Lie superalgebra with a superderivation pair and its corresponding cohomologies [17].

The aim of this paper is to generalize the results of Xu [15] to the 3-Lie superalgebra case. First, we take a representation $(\Phi, \mathcal{P})$ of a 3-Lie superalgebra Q on $\mathcal{P}$ and construct 2-cocycles by using superderivations of $\mathcal{P}$ and Q and hence first-order cohomologies. This construction develops a Lie superalgebra $\mathcal{T}_\Phi$ by the representation Φ and the space $\mathcal{H}^1(Q;\mathcal{P})$ of the first-order cohomology class gives a representation Ψ of the Lie superalgebra $\mathcal{T}_\Phi$. Furthermore, we consider the representation of 3-Lie superalgebras given by abelian extensions of 3-Lie superalgebras of the form $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} Q \to 0$ and construct an obstruction class to the extensibility of a compatible pair of superderivations of $\mathcal{P}$ and Q to those of $\mathcal{L}$.

2. Preliminaries

In this section, we recall representations and cohomologies of 3-Lie superalgebras and their relations to abelian extension of 3-Lie superalgebras of the form $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} Q \to 0$ with $[\mathcal{P}, \mathcal{P}, \mathcal{L}] = 0$. We show that $\mathcal{H}^1(Q;\mathcal{P}) = 0$, then the extension splits.

Let $\mathbb{Z}_2 = \{\bar{0}, \bar{1}\}$ be the field of two elements. Throughout the paper, we denote $\mathbb{F}$ as a field of characteristic zero. A superspace is a $\mathbb{Z}_2$-graded vector space $\mathcal{V} = \mathcal{V}_{\bar{0}} \oplus \mathcal{V}_{\bar{1}}$. A *subsuperspace* is a $\mathbb{Z}_2$-graded vector space which is closed under bracket operation. The nonzero elements of $\mathcal{V}_{\bar{0}} \cup \mathcal{V}_{\bar{1}}$ are said to be *homogeneous* and whenever the degree function occurs in a formula, the corresponding elements are supposed to be homogeneous. A *superalgebra* is a superspace $\mathcal{L} = \mathcal{L}_{\bar{0}} \oplus \mathcal{L}_{\bar{1}}$ endowed with an algebra structure such that $\mathcal{L}_\alpha \mathcal{L}_\beta \subseteq \mathcal{L}_{\alpha+\beta}$ for $\alpha, \beta \in \mathbb{Z}_2$.

Definition 2.1. *A* 3*-Lie superalgebra is a $\mathbb{Z}_2$-graded vector space $\mathcal{L} = \mathcal{L}_{\bar{0}} \oplus \mathcal{L}_{\bar{1}}$ equipped with a trilinear map $[.,.,.] : \wedge^3 \mathcal{L} \to \mathcal{L}$ satisfying:*

1. $|[x_1, x_2, x_3]| = |x_1| + |x_2| + |x_3|$,
2. $[x_1, x_2, x_3] = -(-1)^{|x_1||x_2|}[x_2, x_1, x_3] = -(-1)^{|x_2||x_3|}[x_1, x_3, x_2]$,
3. $[x_1, x_2, [x_3, x_4, x_5]] = [[x_1, x_2, x_3], x_4, x_5] + (-1)^{|x_3|(|x_1|+|x_2|)}[x_3, [x_1, x_2, x_4], x_5] + (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}[x_3, x_4, [x_1, x_2, x_5]]$,

for $x_1, x_2, x_3, x_4, x_5 \in \mathcal{L}$ and $|x_i|$ is the degree of homogeneous element x_i, where $|x_i| \in \mathbb{Z}_2$.

A subsuperspace $\mathcal{N}$ of a 3-Lie superalgebra $\mathcal{L}$ is said to be 3 -*Lie subsuperalgebra* if it is closed under the superbracket. If $\mathcal{L}$ and $\mathcal{M}$ are 3-Lie superalgebras, then a 3-*Lie superalgebra homomorphism* $\theta : \mathcal{L} \to \mathcal{M}$ is an even linear map satisfying $\theta([x, y, z]) = [\theta(x), \theta(y), \theta(z)]$ for $x, y, z \in \mathcal{L}$.

Let $\mathcal{V}$ be a $\mathbb{Z}_2$-graded vector space and $End(\mathcal{V})$ be the set of all $\mathbb{F}$-linear mappings of $\mathcal{V}$ into itself. In $End(\mathcal{V})$, define a $\mathbb{Z}_2$-gradation by $End_\beta(\mathcal{V}) = \{\phi \in End(\mathcal{V}) | \phi(\mathcal{V}_\alpha) \subset \mathcal{V}_{\beta+\alpha}$ for $\alpha, \beta \in \mathbb{Z}_2\}$. Then $End(\mathcal{V})$ becomes an associative superalgebra. For each $\beta \in \mathbb{Z}_2$, $End_\beta(\mathcal{V})$ consists of linear mappings of a $\mathbb{Z}_2$-graded vector space $\mathcal{V}$ into itself which are homogeneous of degree β. If $\mathcal{L}$ is an associative superalgebra and we define a commutator on $\mathcal{L}$ by $[l, l'] = ll' - (-1)^{\alpha\beta} l'l$, for $l \in \mathcal{L}_\alpha$, $l' \in \mathcal{L}_\beta$, α, $\beta \in \mathbb{Z}_2$. Then, $\mathcal{L}$ is a Lie superalgebra. The Lie superalgebra connected with the associative superalgebra $End(\mathcal{V})$ is called general linear Lie superalgebra of $\mathcal{V}$ which is denoted by $gl(\mathcal{V})$.

Definition 2.2. *A superderivation of a* 3*-Lie superalgebra $\mathcal{L}$ is a linear map $\mathcal{D} : \mathcal{L} \to \mathcal{L}$ of degree β satisfying:*

$$\mathcal{D}([x, y, z]) = [\mathcal{D}(x), y, z] + (-1)^{\beta|x|}[x, \mathcal{D}(y), z] + (-1)^{\beta(|x|+|y|)}[x, y, \mathcal{D}(z)],$$

for $x, y, z \in \mathcal{L}$ and $\beta \in \mathbb{Z}_2$.

We denote $Der(\mathcal{L})$ as the space of superderivations of $\mathcal{L}$. Define an even skew-supersymmetric bilinear map $ad : \wedge^2\mathcal{L} \to gl(\mathcal{L})$ by

$$ad(x_1, x_2)x_3 = [x_1, x_2, x_3],$$

for $x_1, x_2, x_3 \in \mathcal{L}$.

Definition 2.3. *A representation of a* 3*-Lie superalgebra* $(\mathcal{L}, [.,.,.])$ *on a superspace* $\mathcal{V}$ *is a bilinear map* $\Phi : \wedge^2\mathcal{L} \to gl(\mathcal{V})$ *such that the following equalities hold:*

1. $|\Phi(x_1, x_2)| = |x_1| + |x_2|$,
2. $\Phi(x_1, x_2) = -(-1)^{|x_1||x_2|}\Phi(x_2, x_1)$,
3. $\Phi(x_1, x_2)\Phi(x_3, x_4) = \Phi([x_1, x_2, x_3], x_4)+(-1)^{|x_3|(|x_1|+|x_2|)}\Phi(x_3, [x_1, x_2, x_4])+(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3, x_4)\Phi(x_1, x_2)$,
4. $\Phi(x_1, [x_2, x_3, x_4]) = (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3, x_4)\Phi(x_1, x_2) - (-1)^{|x_1|(|x_2|+|x_4|)+|x_3||x_4|}\Phi(x_2, x_4)\Phi(x_1, x_3) + (-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2, x_3)\Phi(x_1, x_4)$,

for $x_1, x_2, x_3, x_4 \in \mathcal{L}$.

We denote a representation of $\mathcal{L}$ on a superspace $\mathcal{V}$ by $(\Phi, \mathcal{V})$.

Now onwards, we always assume that $(\mathcal{L}, [.,.,.])$ is a 3-Lie superalgebra and we shall write, for any $X = x_1 \wedge x_2 \in \wedge^2\mathcal{L}$, $x_3 \in \mathcal{L}$,

$$[X, x_3] := [x_1, x_2, x_3] \in \mathcal{L}. \tag{1}$$

We shall use the following bilinear operation $[.,.,.]_{\mathbb{F}}$ on $\wedge^2\mathcal{L}$ given by

$$[X, Y]_{\mathbb{F}} = [X, y_1] \wedge y_2 + (-1)^{|y_1||X|}y_1 \wedge [X, y_2] \in \mathcal{L}, \tag{2}$$

for $X = x_1 \wedge x_2$, $Y = y_1 \wedge y_2$, and $|X| = |x_1| + |x_2|$. One can see that $\wedge^2\mathcal{L}$ is a Leibniz superalgebra with respect to $[.,.]_{\mathbb{F}}$.

Let $(\Phi, \mathcal{V})$ be a representation of $\mathcal{L}$. Cohomology groups of $\mathcal{L}$ with coefficients in $\mathcal{V}$ are defined as in [9]. At first, the space $C^{p-1}(\mathcal{L}; \mathcal{V})$ of p-cochains is the set of multilinear maps of the form

$$f : \underbrace{\wedge^2\mathcal{L} \otimes \wedge^2\mathcal{L} \otimes \cdots \otimes \wedge^2\mathcal{L}}_{p-1} \otimes \mathcal{L} \to \mathcal{V}, \tag{3}$$

while the coboundary operator $\delta_\Phi : C^{p-1}(\mathcal{L}; \mathcal{V}) \to C^p(\mathcal{L}; \mathcal{V})$ is given by

$$\begin{aligned}
&(\delta_\Phi f)(X_1, X_2, \ldots, X_p, z)\\
&= \sum_{1\le j<k\le p} (-1)^j(-1)^{|X_j|(|X_{j+1}|+\cdots+|X_{k-1}|)} f(X_1, \ldots, \hat{X}_j, \ldots, X_{k-1}, [x_j^1, x_j^2, x_k^1] \wedge x_k^2, X_{k+1}, \ldots, X_p, x)\\
&\quad + \sum_{1\le j<k\le p} (-1)^j(-1)^{|X_j|(|X_{j+1}|+\cdots+|X_{k-1}|)+|x_k^1||X_j|} f(X_1, \ldots, \hat{X}_j, \ldots, X_{k-1}, x_k^1 \wedge [x_j^1, x_j^2, x_k^2], X_{k+1}, \ldots, X_p, x)\\
&\quad + \sum_{j=1}^{p}(-1)^j(-1)^{|X_j|(|X_{j+1}|+\cdots+|X_p|)} f(X_1, \ldots, \hat{X}_j, \ldots, X_p, [X_j, x])\\
&\quad + \sum_{j=1}^{p}(-1)^{j+1}(-1)^{|X_j|(|f|+|X_1|+\cdots+|X_{j-1}|)}\Phi(X_j)f(X_1, \ldots, \hat{X}_j, \ldots, X_p, x)\\
&\quad + (-1)^{p+1}(-1)^{(|x_p^2|+|x|)(|f|+|X_1|+\cdots+|X_{p-1}|+|x_p^1|)}\Phi(x_p^2, x)f(X_1, \ldots, X_{p-1}, x_p^1)\\
&\quad + (-1)^{p+1}(-1)^{(|x_p^1|+|x|)(|f|+|X_1|+\cdots+|X_{p-1}|)+|X_p||x|}\Phi(x, x_p^1)f(X_1, \ldots, X_{p-1}, x_p^2),
\end{aligned} \tag{4}$$

for $X_i = x_i \wedge y_i \in \wedge^2\mathcal{L}$ and $z \in \mathcal{L}$. The p^{th} cohomology group is $\mathcal{H}^p(\mathcal{L}; \mathcal{V}) = \mathcal{Z}^p(\mathcal{L}; \mathcal{V})/\mathcal{B}^p(\mathcal{L}; \mathcal{V})$, where $\mathcal{Z}^p(\mathcal{L}; \mathcal{V})$ (respectively, $\mathcal{B}^p(\mathcal{L}; \mathcal{V})$) is the space of $(p + 1)$-cocycles (respectively, $(p + 1)$-coboundaries). We

denote $(p+1)$-cocycles of even degree as $(\mathcal{Z}^p(\mathcal{L};\mathcal{V}))_{\bar{0}}$.
By using Eq. (4), for $f \in C^0(\mathcal{L};\mathcal{V})$, $X_1 = x_1 \wedge x_2 \in \wedge^2\mathcal{L}$ and $x_3 \in \mathcal{L}$, we have

$$\begin{aligned}(\delta_\Phi f)(X_1, x_3) &= -f([X_1, x_3]) + (-1)^{|f|(|x_1|+|x_2|)}\Phi(X_1)f(x_3) + (-1)^{(|f|+|x_1|)(|x_2|+|x_3|)}\Phi(x_2, x_3)f(x_1)\\ &\quad + (-1)^{|f|(|x_1|+|x_3|)+|x_3|(|x_1|+|x_2|)}\Phi(x_3, x_1)f(x_2)\\ &= -f([x_1, x_2, x_3]) + (-1)^{|f|(|x_1|+|x_2|)}\Phi(x_1, x_2)f(x_3) + (-1)^{(|f|+|x_1|)(|x_2|+|x_3|)}\Phi(x_2, x_3)f(x_1)\\ &\quad + (-1)^{|f|(|x_1|+|x_3|)+|x_3|(|x_1|+|x_2|)}\Phi(x_3, x_1)f(x_2),\end{aligned} \tag{5}$$

and for $f \in C^1(\mathcal{L};\mathcal{V})$, $X_1 = x_1 \wedge x_2$, $X_2 = x_3 \wedge x_4$ and $x_5 \in \mathcal{L}$,

$$\begin{aligned}(\delta_\Phi f)(X_1, X_2, x_5) &= -f([X_1, X_2]_{\mathbb{F}}, x_5) - (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}f(X_2, [X_1, x_5]) + f(X_1, [X_2, x_5])\\ &\quad + (-1)^{|f|(|x_1|+|x_2|)}\Phi(X_1)f(X_2, x_5) - (-1)^{(|f|+|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(X_2)f(X_1, x_5)\\ &\quad - (-1)^{(|f|+|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4, x_5)f(X_1, x_3)\\ &\quad - (-1)^{(|f|+|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\Phi(x_5, x_3)f(X_1, x_4)\\ &= -f([x_1, x_2, x_3], x_4, x_5) - (-1)^{|x_3|(|x_1|+|x_2|)}f(x_3, [x_1, x_2, x_4], x_5)\\ &\quad - (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}f(x_3, x_4, [x_1, x_2, x_5]) + f(x_1, x_2, [x_3, x_4, x_5])\\ &\quad + (-1)^{|f|(|x_1|+|x_2|)}\Phi(x_1, x_2)f(x_3, x_4, x_5) - (-1)^{(|f|+|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3, x_4)f(x_1, x_2, x_5)\\ &\quad - (-1)^{(|f|+|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4, x_5)f(x_1, x_2, x_3)\\ &\quad - (-1)^{(|f|+|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\Phi(x_5, x_3)f(x_1, x_2, x_4),\end{aligned} \tag{6}$$

where $[.,.,.]_{\mathbb{F}}$ is given by Eq. (2).

Suppose $\mathcal{L}$, $\mathcal{P}$ and Q are 3-Lie superalgebras. If $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} Q \to 0$ is an exact sequence of 3-Lie superalgebras and $[\mathcal{P},\mathcal{P},\mathcal{L}] = 0$, then we call $\mathcal{L}$ an abelian extension of Q by $\mathcal{P}$. An even linear map $s : Q \to \mathcal{L}$ is called a section if it satisfies $\pi s = Id_Q$. If there exists a section s of π, which is a 3-Lie superalgebra homomorphism, then we say that the abelian extension splits.

Now we construct a representation of Q on $\mathcal{P}$ and a cohomology class. Fix any section $s : Q \to \mathcal{L}$ of π and define $\Phi : \wedge^2 Q \to gl(\mathcal{P})$ by

$$\Phi(x, y)(v) = [s(x), s(y), v]_{\mathcal{L}}, \tag{7}$$

for $x, y \in Q$ and $v \in \mathcal{P}$. It is easy to check that Φ is independent of the choice of s. Moreover, since

$$[s(x), s(y), s(z)]_{\mathcal{L}} - s([x, y, z]_Q) \in \mathcal{P}, \tag{8}$$

for $x, y, z \in Q$, we have a map $\Omega : \wedge^3 Q \to gl(\mathcal{P})$ given by

$$\Omega(x, y, z) = [s(x), s(y), s(z)]_{\mathcal{L}} - s([x, y, z]_Q) \in \mathcal{P}, \tag{9}$$

for $x, y, z \in Q$.

Lemma 2.4. *Let $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} Q \to 0$ be an abelian extension of 3-Lie superalgebras. Then*

1. *Φ as given in Eq. (7) is a representation of Q on $\mathcal{P}$.*
2. *Ω as given in Eq. (9) is a 2-cocycle associated to $(\Phi, \mathcal{P})$.*

Proof. 1. By the equality

$$\begin{aligned}[s(x_1), u, [s(y_1), s(y_2), s(y_3)]_{\mathcal{L}}]_{\mathcal{L}} &= [[s(x_1), u, s(y_1)]_{\mathcal{L}}, s(y_2), s(y_3)]_{\mathcal{L}}\\ &\quad + (-1)^{|y_1|(|x_1|+|u|)}[s(y_1), [s(x_1), u, s(y_2)]_{\mathcal{L}}, s(y_3)]_{\mathcal{L}}\\ &\quad + (-1)^{(|x_1|+|u|)(|y_1|+|y_2|)}[s(y_1), s(y_2), [s(x_1), u, s(y_3)]_{\mathcal{L}}]_{\mathcal{L}}\\ \Longrightarrow \Phi(x_1, [y_1, y_2, y_3]_Q) &= (-1)^{(|x_1|+|y_1|)(|y_2|+|y_3|)}\Phi(y_2, y_3)\Phi(x_1, y_1)\\ &\quad - (-1)^{|x_1|(|y_1|+|y_3|)+|y_2||y_3|}\Phi(y_1, y_3)\Phi(x_1, y_2)\\ &\quad + (-1)^{|x_1|(|y_1|+|y_2|))}\Phi(y_1, y_2)\Phi(x_1, y_3).\end{aligned} \tag{10}$$

Therefore, Φ is a representation of $\mathcal{Q}$ on $\mathcal{P}$.

2. By the equality

$$\begin{aligned}[s(x_1), s(x_2), [s(y_1), s(y_2), s(y_3)]_{\mathcal{L}}]_{\mathcal{L}} = & [[s(x_1), s(x_2), s(y_1)]_{\mathcal{L}}, s(y_2), s(y_3)]_{\mathcal{L}} \\ & + (-1)^{|y_1|(|x_1|+|x_2|)}[s(y_1), [s(x_1), s(x_2), s(y_2)]_{\mathcal{L}}, s(y_3)]_{\mathcal{L}} \\ & + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}[s(y_1), s(y_2), [s(x_1), s(x_2), s(y_3)]_{\mathcal{L}}]_{\mathcal{L}},\end{aligned} \tag{11}$$

we have

$$\begin{aligned}& [s(x_1), s(x_2), \Omega(y_1, y_2, y_3)]_{\mathcal{L}} + [s(x_1), s(x_2), s([y_1, y_2, y_3]_{\mathcal{Q}})]_{\mathcal{L}} \\ & = [\Omega(x_1, x_2, y_1), s(y_2), s(y_3)]_{\mathcal{L}} + [s([x_1, x_2, y_1]_{\mathcal{Q}}), s(y_2), s(y_3)]_{\mathcal{L}} \\ & \quad + (-1)^{|y_1|(|x_1|+|x_2|)}[s(y_1), \Omega(x_1, x_2, y_2), s(y_3)]_{\mathcal{L}} \\ & \quad + (-1)^{|y_1|(|x_1|+|x_2|)}[s(y_1), s([x_1, x_2, y_2]_{\mathcal{Q}}), s(y_3)]_{\mathcal{L}} \\ & \quad + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}[s(y_1), s(y_2), \Omega(x_1, x_2, y_3)]_{\mathcal{L}} \\ & \quad + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}[s(y_1), s(y_2), s([x_1, x_2, y_3]_{\mathcal{Q}})]_{\mathcal{L}} \\ & \implies \Phi(x_1, x_2)\Omega(y_1, y_2, y_3) + \Omega(x_1, x_2, [y_1, y_2, y_3]_{\mathcal{Q}}) + s([x_1, x_2, [y_1, y_2, y_3]_{\mathcal{Q}}]_{\mathcal{Q}}) \\ & = (-1)^{(|x_1|+|x_2|+|y_1|)(|y_2|+|y_3|)}\Phi(y_2, y_3)\Omega(x_1, x_2, y_1) + \Omega([x_1, x_2, y_1]_{\mathcal{Q}}, y_2, y_3) \\ & \quad + s([x_1, x_2, y_1]_{\mathcal{Q}}, y_2, y_3) + (-1)^{|y_1|(|x_1|+|x_2|)+|y_3|(|x_1|+|x_2|+|y_1|+|y_2|)}\Phi(y_3, y_1)\Omega(x_1, x_2, y_2) \\ & \quad + (-1)^{|y_1|(|x_1|+|x_2|)}\Omega(y_1, [x_1, x_2, y_2]_{\mathcal{Q}}, y_3) + (-1)^{|y_1|(|x_1|+|x_2|)}s([y_1, [x_1, x_2, y_2]_{\mathcal{Q}}, y_3]_{\mathcal{Q}}) \\ & \quad + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}\Phi(y_1, y_2)\Omega(x_1, x_2, y_3) \\ & \quad + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}\Omega(y_1, y_2, [x_1, x_2, y_3]_{\mathcal{Q}}) \\ & \quad + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}s([y_1, y_2, [x_1, x_2, y_3]_{\mathcal{Q}}]_{\mathcal{Q}}) \\ & \implies \Phi(x_1, x_2)\Omega(y_1, y_2, y_3) + \Omega(x_1, x_2, [y_1, y_2, y_3]_{\mathcal{Q}}) \\ & = (-1)^{(|x_1|+|x_2|+|y_1|)(|y_2|+|y_3|)}\Phi(y_2, y_3)\Omega(x_1, x_2, y_1) + \Omega([x_1, x_2, y_1]_{\mathcal{Q}}, y_2, y_3) \\ & \quad + (-1)^{|y_1|(|x_1|+|x_2|)+|y_3|(|x_1|+|x_2|+|y_1|+|y_2|)}\Phi(y_3, y_1)\Omega(x_1, x_2, y_2) \\ & \quad + (-1)^{|y_1|(|x_1|+|x_2|)}\Omega(y_1, [x_1, x_2, y_2]_{\mathcal{Q}}, y_3) + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}\Phi(y_1, y_2)\Omega(x_1, x_2, y_3) \\ & \quad + (-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}\Omega(y_1, y_2, [x_1, x_2, y_3]_{\mathcal{Q}}).\end{aligned}$$

Hence, Ω is a 2-cocycle associated to $(\Phi, \mathcal{P})$. □

Corollary 2.5. *Let $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} \mathcal{Q} \to 0$ be an extension of 3-Lie superalgebras with $[\mathcal{P}, \mathcal{P}, \mathcal{L}] = 0$. Then the cohomology class $[\Omega]$ does not depend on the choice of the section of π.*

Proof. Let s_1 and s_2 be sections of π and Ω_1, Ω_2 be defined by Eq. (9) which are corresponding to s_1, s_2, respectively. For any $x \in \mathcal{Q}$, set $\lambda(x) = s_1(x) - s_2(x)$.
Now, $(\pi\lambda)(x) = x - x = 0$, $\lambda(x) \in \mathcal{P}$, and $\lambda \in (C^0(\mathcal{L}; \mathcal{V}))_{\bar{0}}$. Then

$$\begin{aligned}& \Omega_1(x, y, z) - \Omega_2(x, y, z) \\ & = [s_1(x), s_1(y), s_1(z)]_{\mathcal{L}} - s_1([x, y, z]_{\mathcal{Q}}) - [s_2(x), s_2(y), s_2(z)]_{\mathcal{L}} + s_2([x, y, z]_{\mathcal{Q}}) \\ & = [s_2(x) + \lambda(x), s_2(y) + \lambda(y), s_2(z) + \lambda(z)]_{\mathcal{L}} - s_2([x, y, z]_{\mathcal{Q}}) + \lambda([x, y, z]_{\mathcal{Q}}) \\ & \quad - [s_2(x), s_2(y), s_2(z)]_{\mathcal{L}} + s_2([x, y, z]_{\mathcal{Q}}) \\ & = [s_2(x), s_2(y), \lambda(z)]_{\mathcal{L}} + [\lambda(x), s_2(y), s_2(z)]_{\mathcal{L}} + [s_2(x), \lambda(y), s_2(z)]_{\mathcal{L}} - \lambda([x, y, z]_{\mathcal{Q}}) \\ & = -\lambda([x, y, z]_{\mathcal{Q}}) + \Phi(x, y)\lambda(z) + (-1)^{|x|(|y|+|z|)}\Phi(y, z)\lambda(x) + (-1)^{|z|(|x|+|y|)}\Phi(z, x)\lambda(y) \\ & = (\delta_\Phi \lambda)(x, y, z),\end{aligned}$$

which completes the proof. □

Proposition 2.6. *If $(\Phi,\mathcal{P})$ is a representation of $\mathcal{Q}$ and Ω is a 2-cocycle given by the representation $(\Phi,\mathcal{P})$, then $\mathcal{L}_{\Phi,\Omega} := \mathcal{Q}\oplus\mathcal{P}$ is a 3-Lie superalgebra with the superbracket given by*

$$[x+u,y+v,z+w]_{\mathcal{L}_{\Phi,\Omega}} :=[x,y,z]_{\mathcal{Q}}+\Omega(x,y,z)+\Phi(x,y)(w)+(-1)^{|y||z|}\Phi(z,x)(v)+(-1)^{|x|(|y|+|z|)}\Phi(y,z)(u), \tag{12}$$

where $x,y,z\in\mathcal{Q}$ and $u,v,w\in\mathcal{P}$.

Proof. We have

$$\begin{aligned}
&[x_1+u_1,x_2+u_2,[y_1+v_1,y_2+v_2,y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}]_{\mathcal{L}_{\Phi,\Omega}}\\
&=[x_1,x_2,[y_1,y_2,y_3]_{\mathcal{Q}}]_{\mathcal{Q}}+\Omega(x_1,x_2,[y_1,y_2,y_3]_{\mathcal{Q}})+\Phi(x_1,x_2)(\Omega(y_1,y_2,y_3)\\
&\quad+\Phi(y_1,y_2)(v_3)+(-1)^{|y_2||y_3|}\Phi(y_3,y_1)(v_2)+(-1)^{|y_1|(|y_2|+|y_3|)}\Phi(y_2,y_3)(v_1))\\
&\quad+(-1)^{|x_2|(|y_1|+|y_2|+|y_3|)}\Phi([y_1,y_2,y_3]_{\mathcal{Q}},x_1)(u_2)\\
&\quad+(-1)^{|x_1|(|x_2|+|y_1|+|y_2|+|y_3|)}\Phi(x_2,[y_1,y_2,y_3]_{\mathcal{Q}})(u_1),
\end{aligned} \tag{13}$$

$$\begin{aligned}
&[[x_1+u_1,x_2+u_2,y_1+v_1]_{\mathcal{L}_{\Phi,\Omega}},y_2+v_2,y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}\\
&=[[x_1,x_2,y_1]_{\mathcal{Q}},y_2,y_3]_{\mathcal{Q}}+\Omega([x_1,x_2,y_1]_{\mathcal{Q}},y_2,y_3)\\
&\quad+(-1)^{(|x_1|+|x_2|+|y_1|)(|y_2|+|y_3|)}\Phi(y_2,y_3)(\Omega(x_1,x_2,y_1)\\
&\quad+\Phi(x_1,x_2)(v_1)+(-1)^{|x_2||y_1|}\Phi(y_1,x_1)(u_2)+(-1)^{|x_1|(|x_2|+|y_1|)}\Phi(x_2,y_1)(u_1))\\
&\quad+\Phi([x_1,x_2,y_1]_{\mathcal{Q}},y_2)(v_3)+(-1)^{|y_2||y_3|}\Phi(y_3,[x_1,x_2,y_1]_{\mathcal{Q}})(v_2),
\end{aligned} \tag{14}$$

$$\begin{aligned}
&(-1)^{|y_1|(|x_1|+|x_2|)}[y_1+v_1,[x_1+u_1,x_2+u_2,y_2+v_2]_{\mathcal{L}_{\Phi,\Omega}},y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}\\
&=(-1)^{|y_1|(|x_1|+|x_2|)}[y_1,[x_1,x_2,y_2]_{\mathcal{Q}},y_3]_{\mathcal{Q}}+(-1)^{|y_1|(|x_1|+|x_2|)}\Omega(y_1,[x_1,x_2,y_2]_{\mathcal{Q}},y_3)\\
&\quad+(-1)^{|y_3|(|x_1|+|x_2|+|y_2|)+|y_1|(|x_1|+|x_2|)}\Phi(y_3,y_1)(\Omega(x_1,x_2,y_2)+\Phi(x_1,x_2)(v_2)\\
&\quad+(-1)^{|x_2||y_2|}\Phi(y_2,x_1)(u_2)+(-1)^{|x_1|(|x_2|+|y_2|)}\Phi(x_2,y_2)(u_1))\\
&\quad+(-1)^{|y_1|(|x_1|+|x_2|)}\Phi(y_1,[x_1,x_2,y_2]_{\mathcal{Q}})(v_3)\\
&\quad+(-1)^{|y_1|(|y_2|+|y_3|)}\Phi([x_1,x_2,y_2]_{\mathcal{Q}},y_3)(v_1),
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
&(-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}[y_1+v_1,y_2+v_2,[x_1+u_1,x_2+u_2,y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}]_{\mathcal{L}_{\Phi,\Omega}}\\
&=(-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}[y_1,y_2,[x_1,x_2,y_3]_{\mathcal{Q}}]_{\mathcal{Q}}\\
&\quad+(-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}\Omega(y_1,y_2,[x_1,x_2,y_3]_{\mathcal{Q}})\\
&\quad+(-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}\Phi(y_1,y_2)(\Omega(x_1,x_2,y_3)+\Phi(x_1,x_2)(v_3)\\
&\quad+(-1)^{|x_2||y_3|}\Phi(y_3,x_1)(u_2)+(-1)^{|x_1|(|x_2|+|y_3|)}\Phi(x_2,y_3)(u_1))\\
&\quad+(-1)^{|y_1|(|x_1|+|x_2|)+|y_2||y_3|}\Phi([x_1,x_2,y_3]_{\mathcal{Q}},y_1)(v_2)\\
&\quad+(-1)^{|y_2|(|x_1|+|x_2|)+|y_1|(|y_2|+|y_3|)}\Phi(y_2,[x_1,x_2,y_3]_{\mathcal{Q}})(v_1).
\end{aligned} \tag{16}$$

We will see that Eq. (13)=Eqs. (14 + 15 + 16), if Φ is a representation and Ω is a 2-cocycle. Hence, we have

$$\begin{aligned}
&[x_1+u_1,x_2+u_2,[y_1+v_1,y_2+v_2,y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}]_{\mathcal{L}_{\Phi,\Omega}}\\
&=[[x_1+u_1,x_2+u_2,y_1+v_1]_{\mathcal{L}_{\Phi,\Omega}},y_2+v_2,y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}\\
&\quad+(-1)^{|y_1|(|x_1|+|x_2|)}[y_1+v_1,[x_1+u_1,x_2+u_2,y_2+v_2]_{\mathcal{L}_{\Phi,\Omega}},y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}\\
&\quad+(-1)^{(|x_1|+|x_2|)(|y_1|+|y_2|)}[y_1+v_1,y_2+v_2,[x_1+u_1,x_2+u_2,y_3+v_3]_{\mathcal{L}_{\Phi,\Omega}}]_{\mathcal{L}_{\Phi,\Omega}},
\end{aligned} \tag{17}$$

which implies $\mathcal{L}_{\Phi,\Omega}$ is a 3-Lie superalgebra. □

Proposition 2.7. *If $\mathcal{H}^1(\mathcal{Q};\mathcal{P}) = 0$, then the extension is split.*

Proof. It is sufficient to show that there is a section of π which is a 3-Lie superalgebra homomorphism. It is known that the representation $(\Phi,\mathcal{P})$ given by Eq. (7) is independent of the choice of the sections of π. Consider the 2-cocycle Ω given by Lemma 2.4. Since $\mathcal{H}^1(\mathcal{Q};\mathcal{P}) = 0$, there exists an $\xi \in (C^0(\mathcal{Q};\mathcal{P}))_{\bar{0}}$ such that $\Omega = \delta_\Phi \xi$. For any $x, y, z \in \mathcal{Q}$, it follows that

$$\Omega(x,y,z) = -\xi([x,y,z]_{\mathcal{Q}}) + \Phi(x,y)\xi(z) + (-1)^{|x|(|y|+|z|)}\Phi(y,z)\xi(x) + (-1)^{|z|(|x|+|y|)}\Phi(z,x)\xi(y). \tag{18}$$

Define an even linear map $s^{'} : \mathcal{Q} \to \mathcal{P}$ by $s^{'}(x) := s(x) - \xi(x)$. Note that $s^{'}$ is also a section of π. Then, for any $x, y, z \in \mathcal{Q}$, we have

$$\begin{aligned}
&[s^{'}(x), s^{'}(y), s^{'}(z)]_{\mathcal{L}}\\
&= [s(x) - \xi(x), s(y) - \xi(y), s(z) - \xi(z)]_{\mathcal{L}}\\
&= [s_1(x), s_1(y), s_1(z)]_{\mathcal{L}} - \Phi(x,y)\xi(z) - (-1)^{|x|(|y|+|z|)}\Phi(y,z)\xi(x) - (-1)^{|z|(|x|+|y|)}\Phi(z,x)\xi(y)\\
&= s([x,y,z]_{\mathcal{Q}}) + \Omega(x,y,z) - \Phi(x,y)\xi(z) - (-1)^{|x|(|y|+|z|)}\Phi(y,z)\xi(x) - (-1)^{|z|(|x|+|y|)}\Phi(z,x)\xi(y)\\
&= s([x,y,z]_{\mathcal{Q}}) - \xi([x,y,z]_{\mathcal{Q}})\\
&= s^{'}([x,y,z]_{\mathcal{Q}}).
\end{aligned}$$

Hence, $s^{'}$ is a 3-Lie superalgebra homomorphism. □

3. Cohomology Classes and a Lie superalgebra

Let $\mathcal{L}, \mathcal{P}$ and $\mathcal{Q}$ be 3-Lie superalgebras. Let $(\Phi,\mathcal{P})$ be a representation of a 3-Lie superalgebra $\mathcal{Q}$ on $\mathcal{P}$. In this section, we use superderivations of $\mathcal{P}$ and $\mathcal{Q}$ to construct first-order cohomology class. By using this, we construct a Lie superalgebra and its representation on $\mathcal{H}^1(\mathcal{Q};\mathcal{P})$.

Given a representation $(\Phi,\mathcal{P})$ of $\mathcal{Q}$. Suppose $\Omega \in (C^1(\mathcal{Q};\mathcal{P}))_{\bar{0}}$. For any pair $(\mathcal{D}_p, \mathcal{D}_q) \in Der(\mathcal{P}) \times Der(\mathcal{Q})$, define a 2-cochain $Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)} \in C^1(\mathcal{Q};\mathcal{P})$ as

$$Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)} = \mathcal{D}_p\Omega - \Omega(\mathcal{D}_q \otimes Id_{\mathcal{Q}} \otimes Id_{\mathcal{Q}}) - \Omega(Id_{\mathcal{Q}} \otimes \mathcal{D}_q \otimes Id_{\mathcal{Q}}) - \Omega(Id_{\mathcal{Q}} \otimes Id_{\mathcal{Q}} \otimes \mathcal{D}_q), \tag{19}$$

where "$Id_{\mathcal{Q}}''$ denotes the identity map and the degree of identity map is always even. This is equivalent to

$$Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x,y,z) := \mathcal{D}_p(\Omega(x,y,z)) - \Omega(\mathcal{D}_q(x),y,z) - (-1)^{\alpha|x|}\Omega(x,\mathcal{D}_q(y),z) - (-1)^{\alpha(|x|+|y|)}\Omega(x,y,\mathcal{D}_q(z)), \tag{20}$$

for $x, y, z \in \mathcal{Q}$ and $|Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}| = |\mathcal{D}_p| = |\mathcal{D}_q| = \alpha$, where $\alpha \in \mathbb{Z}_2$.

We begin with the following lemma.

Lemma 3.1. *Let $(\Phi,\mathcal{P})$ be a representation of $\mathcal{Q}$ and $\Omega \in (C^1(\mathcal{Q};\mathcal{P}))_{\bar{0}}$ associated to the representation $(\Phi,\mathcal{P})$. Assume that a pair $(\mathcal{D}_p, \mathcal{D}_q) \in Der(\mathcal{P}) \times Der(\mathcal{Q})$ satisfies that*

$$\mathcal{D}_p\Phi(x,y) - (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\mathcal{D}_p = \Phi(\mathcal{D}_q(x),y) + (-1)^{\alpha|x|}\Phi(x,\mathcal{D}_q(y)), \tag{21}$$

for $x, y \in \mathcal{Q}$ and $|\mathcal{D}_p| = |\mathcal{D}_q| = \alpha$. If Ω is a 2-cocycle, then $Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)} \in C^1(\mathcal{Q};\mathcal{P})$ given by Eq. (20) is also a 2-cocycle.

Proof. It is sufficent to show that $\delta_\Phi Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)} = 0$. Since, Ω is a 2-cocycle, so $\delta_\Phi\Omega = 0$. By Eq. (6) it follows that, for any $x_i \in \mathcal{Q}$,

$$\begin{aligned}
0 = &- \Omega([x_1,x_2,x_3]_{\mathcal{Q}}, x_4, x_5) - (-1)^{|x_3|(|x_1|+|x_2|)}\Omega(x_3,[x_1,x_2,x_4]_{\mathcal{Q}},x_5)\\
&- (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Omega(x_3,x_4,[x_1,x_2,x_5]_{\mathcal{Q}}) + \Omega(x_1,x_2,[x_3,x_4,x_5]_{\mathcal{Q}})\\
&+ \Phi(x_1,x_2)\Omega(x_3,x_4,x_5) - (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3,x_4)\Omega(x_1,x_2,x_5)\\
&- (-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4,x_5)\Omega(x_1,x_2,x_3)\\
&- (-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+(|x_3|+|x_4|)|x_5|}\Phi(x_5,x_3)\Omega(x_1,x_2,x_4).
\end{aligned} \tag{22}$$

Now $|Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}| = |\mathcal{D}_p| = |\mathcal{D}_q| = \alpha$, we have

$$\begin{aligned}
&\delta_{\Phi} Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, x_3, x_4, x_5)\\
&= -\underbrace{Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}([x_1, x_2, x_3]_Q, x_4, x_5)}_{(1)} - \underbrace{(-1)^{|x_3|(|x_1|+|x_2|)}Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_3, [x_1, x_2, x_4]_Q, x_5)}_{(2)}\\
&\quad - \underbrace{(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_3, x_4, [x_1, x_2, x_5]_Q)}_{(3)} + \underbrace{Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, [x_3, x_4, x_5]_Q)}_{(4)}\\
&\quad + \underbrace{(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1, x_2)Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_3, x_4, x_5)}_{(5)}\\
&\quad - \underbrace{(-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3, x_4)Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, x_5)}_{(6)}\\
&\quad - \underbrace{(-1)^{(\alpha+|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4, x_5)Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, x_3)}_{(7)}\\
&\quad - \underbrace{(-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_5|)+(|x_3|+|x_4|)|x_5|}\Phi(x_5, x_3)Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, x_4)}_{(8)}.
\end{aligned} \tag{23}$$

Applying the definition of $Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}$ as in Eq. (20), we get

$$\begin{aligned}
(1) = &-\mathcal{D}_p(\Omega([x_1, x_2, x_3]_Q, x_4, x_5)) + \Omega([\mathcal{D}_q(x_1), x_2, x_3]_Q, x_4, x_5)\\
&+ (-1)^{\alpha|x_1|}\Omega([x_1, \mathcal{D}_q(x_2), x_3]_Q, x_4, x_5)\\
&+ (-1)^{\alpha(|x_1|+|x_2|)}\Omega([x_1, x_2, \mathcal{D}_q(x_3)]_Q, x_4, x_5)\\
&+ (-1)^{\alpha(|x_1|+|x_2|+|x_3|)}\Omega([x_1, x_2, x_3]_Q, \mathcal{D}_q(x_4), x_5)\\
&+ (-1)^{\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega([x_1, x_2, x_3]_Q, x_4, \mathcal{D}_q(x_5)),
\end{aligned} \tag{24}$$

$$\begin{aligned}
(2) = &-(-1)^{|x_3|(|x_1|+|x_2|)}\mathcal{D}_p(\Omega(x_3, [x_1, x_2, x_4]_Q, x_5))\\
&+ (-1)^{|x_3|(|x_1|+|x_2|)}\Omega(\mathcal{D}_q(x_3), [x_1, x_2, x_4]_Q, x_5)\\
&+ (-1)^{|x_3|(\alpha+|x_1|+|x_2|)}\Omega(x_3, [\mathcal{D}_q(x_1), x_2, x_4]_Q, x_5)\\
&+ (-1)^{|x_3|(|x_1|+|x_2|)+\alpha(|x_1|+|x_3|)}\Omega(x_3, [x_1, \mathcal{D}_q(x_2), x_4]_Q, x_5)\\
&+ (-1)^{|x_3|(|x_1|+|x_2|)+\alpha(|x_1|+|x_2|+|x_3|)}\Omega(x_3, [x_1, x_2, \mathcal{D}_q(x_4)]_Q, x_5)\\
&+ (-1)^{|x_3|(|x_1|+|x_2|)+\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega(x_3, [x_1, x_2, x_4]_Q, \mathcal{D}_q(x_5)),
\end{aligned} \tag{25}$$

$$\begin{aligned}
(3) = &-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\mathcal{D}_p(\Omega(x_3, x_4, [x_1, x_2, x_5]_Q))\\
&+ (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Omega(\mathcal{D}_q(x_3), x_4, [x_1, x_2, x_5]_Q)\\
&+ (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha|x_3|}\Omega(x_3, \mathcal{D}_q(x_4), [x_1, x_2, x_5]_Q)\\
&+ (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha(|x_3|+|x_4|)}\Omega(x_3, x_4, [\mathcal{D}_q(x_1), x_2, x_5]_Q)\\
&+ (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha(|x_1|+|x_3|+|x_4|)}\Omega(x_3, x_4, [x_1, \mathcal{D}_q(x_2), x_5]_Q)\\
&+ (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega(x_3, x_4, [x_1, x_2, \mathcal{D}_q(x_5)]_Q),
\end{aligned} \tag{26}$$

$$\begin{aligned}(4) = {} & \mathcal{D}_p(\Omega(x_1, x_2, [x_3, x_4, x_5]_Q)) - \Omega(\mathcal{D}_q(x_1), x_2, [x_3, x_4, x_5]_Q) \\ & - (-1)^{\alpha|x_1|}\Omega(x_1, \mathcal{D}_q(x_2), [x_3, x_4, x_5]_Q) \\ & - (-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1, x_2, [\mathcal{D}_q(x_3), x_4, x_5]_Q) \\ & - (-1)^{\alpha(|x_1|+|x_2|+|x_3|)}\Omega(x_1, x_2, [x_3, \mathcal{D}_q(x_4), x_5]_Q) \\ & - (-1)^{\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega(x_1, x_2, [x_3, x_4, \mathcal{D}_q(x_5)]_Q),\end{aligned} \tag{27}$$

$$\begin{aligned}(5) = {} & (-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1, x_2)(\mathcal{D}_p(\Omega(x_3, x_4, x_5)) - \Omega(\mathcal{D}_q(x_3), x_4, x_5) \\ & - (-1)^{\alpha|x_3|}\Omega(x_3, \mathcal{D}_q(x_4), x_5) - (-1)^{\alpha(|x_3|+|x_4|)}\Omega(x_3, x_4, \mathcal{D}_q(x_5))),\end{aligned} \tag{28}$$

$$\begin{aligned}(6) = {} & -(-1)^{(|x_3|+|x_4|)(\alpha+|x_1|+|x_2|)}\Phi(x_3, x_4)(\mathcal{D}_p(\Omega(x_1, x_2, x_5)) - \Omega(\mathcal{D}_q(x_1), x_2, x_5) \\ & - (-1)^{\alpha|x_1|}\Omega(x_1, \mathcal{D}_q(x_2), x_5) - (-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1, x_2, \mathcal{D}_q(x_5))),\end{aligned} \tag{29}$$

$$\begin{aligned}(7) = {} & -(-1)^{(|x_4|+|x_5|)(\alpha+|x_1|+|x_2|+|x_3|)}\Phi(x_4, x_5)(\mathcal{D}_p(\Omega(x_1, x_2, x_3)) - \Omega(\mathcal{D}_q(x_1), x_2, x_3) \\ & - (-1)^{\alpha|x_1|}\Omega(x_1, \mathcal{D}_q(x_2), x_3) - (-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1, x_2, \mathcal{D}_q(x_3))),\end{aligned} \tag{30}$$

$$\begin{aligned}(8) = {} & -(-1)^{(|x_3|+|x_5|)(\alpha+|x_1|+|x_2|)+|x_5|(|x_3|+|x_4|)}\Phi(x_5, x_3)(\mathcal{D}_p(\Omega(x_1, x_2, x_4)) - \Omega(\mathcal{D}_q(x_1), x_2, x_4) \\ & - (-1)^{\alpha|x_1|}\Omega(x_1, \mathcal{D}_q(x_2), x_4) - (-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1, x_2, \mathcal{D}_q(x_4))).\end{aligned} \tag{31}$$

$$\begin{aligned}& - \mathcal{D}_p(\Omega([x_1, x_2, x_3]_Q, x_4, x_5)) - (-1)^{|x_3|(|x_1|+|x_2|)}\mathcal{D}_p(\Omega(x_3, [x_1, x_2, x_4]_Q, x_5)) \\ & - (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\mathcal{D}_p(\Omega(x_3, x_4, [x_1, x_2, x_5]_Q)) + \mathcal{D}_p(\Omega(x_1, x_2, [x_3, x_4, x_5]_Q)) \\ & = -\mathcal{D}_p(\Phi(x_1, x_2)\Omega(x_3, x_4, x_5)) + (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\mathcal{D}_p(\Phi(x_3, x_4)\Omega(x_1, x_2, x_5)) \\ & \quad + (-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\mathcal{D}_p(\Phi(x_4, x_5)\Omega(x_1, x_2, x_3)) \\ & \quad + (-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\mathcal{D}_p(\Phi(x_5, x_3)\Omega(x_1, x_2, x_4)).\end{aligned} \tag{32}$$

For Eqs. (24)-(32), by suitable combinations and with the aid of Eq. (22), we get

$$\begin{aligned}& \Omega([\mathcal{D}_q(x_1), x_2, x_3]_Q, x_4, x_5) + (-1)^{|x_3|(\alpha+|x_1|+|x_2|)}\Omega(x_3, [\mathcal{D}_q(x_1), x_2, x_4]_Q, x_5) \\ & \quad + (-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_4|)}\Omega(x_3, x_4, [\mathcal{D}_q(x_1), x_2, x_5]_Q) \\ & \quad - \Omega(\mathcal{D}_q(x_1), x_2, [x_3, x_4, x_5]_Q) + (-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3, x_4)\Omega(\mathcal{D}_q(x_1), x_2, x_5) \\ & \quad + (-1)^{(\alpha+|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4, x_5)\Omega(\mathcal{D}_q(x_1), x_2, x_3) \\ & \quad - (-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\Phi(x_5, x_3)\Omega(\mathcal{D}_q(x_1), x_2, x_4) \\ & = \Phi(\mathcal{D}_q(x_1), x_2)\Omega(x_3, x_4, x_5),\end{aligned} \tag{33}$$

$$\begin{aligned}& (-1)^{\alpha|x_1|}\Omega([x_1, \mathcal{D}_q(x_2), x_3]_Q, x_4, x_5) + (-1)^{(|x_3|+\alpha)(|x_1|+|x_2|)}\Omega(x_3, [x_1, \mathcal{D}_q(x_2), x_4]_Q, x_5) \\ & \quad + (-1)^{\alpha(|x_1|+|x_3|+|x_4|)+(|x_1|+|x_2|)(|x_3|+|x_4|)}\Omega(x_3, x_4, [x_1, \mathcal{D}_q(x_2), x_5]_Q) \\ & \quad - (-1)^{\alpha|x_1|}\Omega(x_1, \mathcal{D}_q(x_2), [x_3, x_4, x_5]_Q) \\ & \quad + (-1)^{\alpha(|x_1|+|x_3|+|x_4|)+(|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3, x_4)\Omega(x_1, \mathcal{D}_q(x_2), x_5) \\ & \quad + (-1)^{\alpha(|x_1|+|x_4|+|x_5|)+(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4, x_5)\Omega(x_1, \mathcal{D}_q(x_2), x_3) \\ & \quad + (-1)^{\alpha(|x_1|+|x_3|+|x_5|)+(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\Phi(x_5, x_3)\Omega(x_1, \mathcal{D}_q(x_2), x_4) \\ & = (-1)^{\alpha|x_1|}\Phi(x_1, \mathcal{D}_q(x_2))\Omega(x_3, x_4, x_5),\end{aligned} \tag{34}$$

$$\begin{aligned}
&(-1)^{\alpha(|x_1|+|x_2|)}\Omega([x_1,x_2,\mathcal{D}_q(x_3)]_Q,x_4,x_5)+(-1)^{|x_3|(|x_1|+|x_2|)}\Omega(\mathcal{D}_q(x_3),[x_1,x_2,x_4]_Q,x_5)\\
&\quad+(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Omega(\mathcal{D}_q(x_3),x_4,[x_1,x_2,x_5]_Q)\\
&\quad-(-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1,x_2,[\mathcal{D}_q(x_3),x_4,x_5]_Q)\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)\Omega(\mathcal{D}_q(x_3),x_4,x_5)\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|+|x_4|+|x_5|)+(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4,x_5)\Omega(x_1,x_2,\mathcal{D}_q(x_3))\\
&=-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(\mathcal{D}_q(x_3),x_4)\Omega(x_1,x_2,x_5)\\
&\quad-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(\alpha+|x_3|+|x_4|)}\Phi(x_5,\mathcal{D}_q(x_3))\Omega(x_1,x_2,x_4),
\end{aligned}\tag{35}$$

$$\begin{aligned}
&(-1)^{\alpha(|x_1|+|x_2|+|x_3|)}\Omega([x_1,x_2,x_3]_Q,\mathcal{D}_q(x_4),x_5)\\
&\quad+(-1)^{|x_3|(|x_1|+|x_2|)+\alpha(|x_1|+|x_2|+|x_3|)}\Omega(x_3,[x_1,x_2,\mathcal{D}_q(x_4)]_Q,x_5)\\
&\quad+(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha|x_3|}\Omega(x_3,\mathcal{D}_q(x_4),[x_1,x_2,x_5]_Q)\\
&\quad-(-1)^{\alpha(|x_1|+|x_2|+|x_3|)}\Omega(x_1,x_2,[x_3,\mathcal{D}_q(x_4),x_5]_Q)\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|+|x_3|)}\Phi(x_1,x_2)\Omega(x_3,\mathcal{D}_q(x_4),x_5)\\
&\quad+(-1)^{(|x_3|+|x_5|)(\alpha+|x_1|+|x_2|)+|x_5|(|x_3|+|x_4|)+\alpha(|x_1|+|x_2|)}\Phi(x_5,x_3)\Omega(x_1,x_2,\mathcal{D}_q(x_4))\\
&=-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha|x_3|}\Phi(x_3,\mathcal{D}_q(x_4))\Omega(x_1,x_2,x_5)\\
&\quad-(-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(\mathcal{D}_q(x_4),x_5)\Omega(x_1,x_2,x_3),
\end{aligned}\tag{36}$$

and

$$\begin{aligned}
&(-1)^{\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega([x_1,x_2,x_3]_Q,x_4,\mathcal{D}_q(x_5))\\
&\quad+(-1)^{|x_3|(|x_1|+|x_2|)+\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega(x_3,[x_1,x_2,x_4]_Q,\mathcal{D}_q(x_5))\\
&\quad+(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega(x_3,x_4,[x_1,x_2,\mathcal{D}_q(x_5)]_Q)\\
&\quad-(-1)^{\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Omega(x_1,x_2,[x_3,x_4,\mathcal{D}_q(x_5)]_Q)\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|+|x_3|+|x_4|)}\Phi(x_1,x_2)\Omega(x_3,x_4,\mathcal{D}_q(x_5))\\
&\quad+(-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha(|x_1|+|x_2|)}\Phi(x_3,x_4)\Omega(x_1,x_2,\mathcal{D}_q(x_5))\\
&=-(-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4,\mathcal{D}_q(x_5))\Omega(x_1,x_2,x_3)\\
&\quad-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+(|x_3|+|x_4|)(|x_5|+\alpha)}\Phi(\mathcal{D}_q(x_5),x_3)\Omega(x_1,x_2,x_4).
\end{aligned}\tag{37}$$

By inserting Eqs. (33)-(37) into Eq. (22), we get

$$\begin{aligned}
&(\delta_\Phi Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)})(x_1,x_2,x_3,x_4,x_5)\\
&=-\mathcal{D}_p(\Phi(x_1,x_2)\Omega(x_3,x_4,x_5))+(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\mathcal{D}_p(\Phi(x_3,x_4)\Omega(x_1,x_2,x_5))\\
&\quad+(-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\mathcal{D}_p(\Phi(x_4,x_5)\Omega(x_1,x_2,x_3))\\
&\quad+(-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\mathcal{D}_p(\Phi(x_5,x_3)\Omega(x_1,x_2,x_4))\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)(\mathcal{D}_p(\Omega(x_3,x_4,x_5)))\\
&\quad-(-1)^{(\alpha+|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(x_3,x_4)(\mathcal{D}_p(\Omega(x_1,x_2,x_5))\\
&\quad-(-1)^{(\alpha+|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(x_4,x_5)(\mathcal{D}_p(\Omega(x_1,x_2,x_3)))\\
&\quad-(-1)^{(\alpha+|x_1|+|x_2|)+|x_5|(|x_3|+|x_4|)(|x_3|+|x_5|)}\Phi(x_5,x_3)(\mathcal{D}_p(\Omega(x_1,x_2,x_4)))\\
&\quad+\Phi(\mathcal{D}_q(x_1),x_2)\Omega(x_3,x_4,x_5)+(-1)^{\alpha|x_1|}\Phi(x_1,\mathcal{D}_q(x_2))\Omega(x_3,x_4,x_5)\\
&\quad-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}\Phi(\mathcal{D}_q(x_3),x_4)\Omega(x_1,x_2,x_5)\\
&\quad-(-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(\alpha+|x_3|+|x_4|)}\Phi(x_5,\mathcal{D}_q(x_3))\Omega(x_1,x_2,x_4)
\end{aligned}$$

$$
\begin{aligned}
&- (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)+\alpha|x_3|}\Phi(x_3, \mathcal{D}_q(x_4))\Omega(x_1, x_2, x_5)\\
&- (-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}\Phi(\mathcal{D}_q(x_4), x_5)\Omega(x_1, x_2, x_3)\\
&- (-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)+\alpha|x_4|}\Phi(x_4, \mathcal{D}_q(x_5))\Omega(x_1, x_2, x_3)\\
&- (-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}\Phi(\mathcal{D}_q(x_5), x_3)\Omega(x_1, x_2, x_4)\\
= &-(\mathcal{D}_p(\Phi(x_1, x_2)) - (-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1, x_2)\mathcal{D}_p - \Phi(\mathcal{D}_q(x_1), x_2)\\
&- (-1)^{\alpha|x_1|}\Phi(x_1, \mathcal{D}_q(x_2)))\Omega(x_3, x_4, x_5) + (-1)^{(|x_1|+|x_2|)(|x_3|+|x_4|)}(\mathcal{D}_p(\Phi(x_3, x_4))\\
&- (-1)^{\alpha(|x_3|+|x_4|)}\Phi(x_3, x_4)\mathcal{D}_p - \Phi(\mathcal{D}_q(x_3), x_4) - (-1)^{\alpha|x_3|}\Phi(x_3, \mathcal{D}_q(x_4)))\Omega(x_1, x_2, x_5)\\
&+ (-1)^{(|x_1|+|x_2|+|x_3|)(|x_4|+|x_5|)}(\mathcal{D}_p(\Phi(x_4, x_5)) - (-1)^{\alpha(|x_4|+|x_5|)}\Phi(x_4, x_5)\mathcal{D}_p - \Phi(\mathcal{D}_q(x_4), x_5)\\
&- (-1)^{\alpha|x_4|}\Phi(x_4, \mathcal{D}_q(x_5)))\Omega(x_1, x_2, x_3) + (-1)^{(|x_1|+|x_2|)(|x_3|+|x_5|)+|x_5|(|x_3|+|x_4|)}(\mathcal{D}_p(\Phi(x_5, x_3))\\
&- (-1)^{\alpha(|x_3|+|x_5|)}\Phi(x_5, x_3)\mathcal{D}_p - \Phi(\mathcal{D}_q(x_5), x_3) - (-1)^{\alpha|x_5|}\Phi(x_5, \mathcal{D}_q(x_3)))\Omega(x_1, x_2, x_4)\\
= &\ 0.
\end{aligned}
$$

□

Definition 3.2. *Let $(\Phi, \mathcal{P})$ be a representation of $\mathcal{Q}$. A pair of superderivations $(\mathcal{D}_p, \mathcal{D}_q) \in Der(\mathcal{P}) \times Der(\mathcal{Q})$ is called compatible (with respect to Φ) if Eq. (21) holds.*

Now, we are ready to construct a Lie superalgebra and its representation on the first-order cohomology group. Set

$$\mathcal{T}_\Phi = \{(\mathcal{D}_p, \mathcal{D}_q) \in Der(\mathcal{P}) \times Der(\mathcal{Q}) | (\mathcal{D}_p, \mathcal{D}_q) \text{ is compatible with respect to } \Phi\}. \tag{38}$$

We have the following.

Lemma 3.3. *There is an even linear map $\Psi : \mathcal{T}_\Phi \to gl(\mathcal{H}^1(\mathcal{Q}; \mathcal{P}))$ given by*

$$\Psi(\mathcal{D}_p, \mathcal{D}_q)([\Phi]) = [Ob^\Omega_{(\mathcal{D}_p,\mathcal{D}_q)}] \quad for\ \Omega \in (\mathcal{Z}^1(\mathcal{Q}; \mathcal{P}))_{\bar{0}}, \tag{39}$$

where $[Ob^\Omega_{(\mathcal{D}_p,\mathcal{D}_q)}]$ is given by Eq. (20).

Proof. Since $(\mathcal{D}_p, \mathcal{D}_q)$ is compatible with respect to Φ, in Lemma 3.1 it is proved that $Ob^\Omega_{(\mathcal{D}_p,\mathcal{D}_q)}$ is a 2-cocycle whenever Ω is a 2-cocycle. So it is sufficient to show that if $\delta_\Phi\lambda$ is a 2-coboundary, then $\Phi(\mathcal{D}_p, \mathcal{D}_q)(\delta_\Phi\lambda) = 0$ which implies that Ψ is well-defined. Here $|\lambda| = 0$ and $|\mathcal{D}_p| = |\mathcal{D}_q| = \alpha$.

$$
\begin{aligned}
&(\Psi(\mathcal{D}_p, \mathcal{D}_q)(\delta_\Phi\lambda))(x, y, z)\\
= &\ \mathcal{D}_p(\delta_\Phi\lambda)(x, y, z) - (\delta_\Phi\lambda)(\mathcal{D}_q(x), y, z) - (-1)^{\alpha|x|}(\delta_\Phi\lambda)(x, \mathcal{D}_q(y), z)\\
&- (-1)^{\alpha(|x|+|y|)}(\delta_\Phi\lambda)(x, y, \mathcal{D}_q(z))\\
= &\ \mathcal{D}_p(-\lambda([x, y, z]_\mathcal{Q}) + \Phi(x, y)\lambda(z) + (-1)^{|x|(|y|+|z|)}\Phi(y, z)\lambda(x)\\
&+ (-1)^{|z|(|x|+|y|)}\Phi(z, x)\lambda(y)) - (-\lambda([\mathcal{D}_q(x), y, z]_\mathcal{Q}) + \Phi(\mathcal{D}_q(x), y)\lambda(z)\\
&+ (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y, z)\lambda(\mathcal{D}_q(x)) + (-1)^{|z|(|x|+|y|+\alpha)}\Phi(z, \mathcal{D}_q(x))\lambda(y))\\
&- (-1)^{\alpha|x|}(-\lambda([x, \mathcal{D}_q(y), z]_\mathcal{Q}) + \Phi(x, \mathcal{D}_q(y))\lambda(z) + (-1)^{|x|(\alpha+|y|+|z|)}\Phi(\mathcal{D}_q(y), z)\lambda(x)\\
&+ (-1)^{|z|(|x|+|y|+\alpha)}\Phi(z, x)\lambda(\mathcal{D}_q(y))) - (-1)^{\alpha(|x|+|y|)}(-\lambda([x, y, \mathcal{D}_q(z)]_\mathcal{Q}) + \Phi(x, y)\lambda(\mathcal{D}_q(z))\\
&+ (-1)^{|x|(\alpha+|y|+|z|)}\Phi(y, \mathcal{D}_q(z))\lambda(x) + (-1)^{(|z|+\alpha)(|x|+|y|)}\Phi(\mathcal{D}_q(z), x)\lambda(y)).
\end{aligned}
$$

Since, $\mathcal{D}_q$ is a superderivation,

$$
\begin{aligned}
&\lambda([\mathcal{D}_q(x), y, z]_\mathcal{Q}) + (-1)^{\alpha|x|}\lambda([x, \mathcal{D}_q(y), z]_\mathcal{Q}) + (-1)^{\alpha(|x|+|y|)}\lambda([x, y, \mathcal{D}_q(z)]_\mathcal{Q})\\
= &\ \lambda(\mathcal{D}_q([x, y, z]_\mathcal{Q})).
\end{aligned}
$$

Then, we have

$$\begin{aligned}
&(\Psi(\mathcal{D}_p,\mathcal{D}_q)(\delta_\Phi\lambda))(x,y,z)\\
&= (\mathcal{D}_p(\Phi(x,y))\lambda(z) - \Phi(\mathcal{D}_q(x),y)\lambda(z) - (-1)^{\alpha|x|}\Phi(x,\mathcal{D}_q(y))\lambda(z))\\
&\quad + (-1)^{|x|(|y|+|z|)}(\mathcal{D}_p(\Phi(y,z))\lambda(x) - \Phi(\mathcal{D}_q(y),z)\lambda(x) - (-1)^{\alpha|y|}\Phi(y,\mathcal{D}_q(z))\lambda(x))\\
&\quad + (-1)^{|z|(|x|+|y|)}(\mathcal{D}_p(\Phi(z,x))\lambda(y) - \Phi(\mathcal{D}_q(z),x)\lambda(y) - (-1)^{\alpha|z|}\Phi(z,\mathcal{D}_q(x))\lambda(y))\\
&\quad - (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y,z)\lambda(\mathcal{D}_q(x)) - (-1)^{\alpha|x|+|z|(|x|+|y|+\alpha)}\Phi(z,x)\lambda(\mathcal{D}_q(y))\\
&\quad - (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\lambda(\mathcal{D}_q(z)) - \mathcal{D}_p(\lambda([x,y,z]_{\mathcal{Q}})) + \lambda(\mathcal{D}_q([x,y,z]_{\mathcal{Q}})).
\end{aligned}$$

By using Eq. (21), we have

$$\begin{aligned}
&\mathcal{D}_p(\Phi(x,y))\lambda(z) - \Phi(\mathcal{D}_q(x),y)\lambda(z) - (-1)^{\alpha|x|}\Phi(x,\mathcal{D}_q(y))\lambda(z)\\
&= (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\mathcal{D}_p(\lambda(z)),
\end{aligned}$$

$$\begin{aligned}
&(-1)^{|x|(|y|+|z|)}(\mathcal{D}_p(\Phi(y,z))\lambda(x) - \Phi(\mathcal{D}_q(y),z)\lambda(x) - (-1)^{\alpha|y|}\Phi(y,\mathcal{D}_q(z))\lambda(x))\\
&= (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y,z)\mathcal{D}_p(\lambda(x)),
\end{aligned}$$

$$\begin{aligned}
&(-1)^{|z|(|x|+|y|)}(\mathcal{D}_p(\Phi(z,x))\lambda(y) - \Phi(\mathcal{D}_q(z),x)\lambda(y) - (-1)^{\alpha|z|}\Phi(z,\mathcal{D}_q(x))\lambda(y))\\
&= (-1)^{|z|(|x|+|y|)+\alpha(|z|+|x|)}\Phi(z,x)\mathcal{D}_p(\lambda(y)).
\end{aligned}$$

$$\begin{aligned}
&(\Psi(\mathcal{D}_p,\mathcal{D}_q)(\delta_\Phi\lambda))(x,y,z)\\
&= (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\mathcal{D}_p(\lambda(z)) + (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y,z)\mathcal{D}_p(\lambda(x))\\
&\quad + (-1)^{|z|(|x|+|y|)+\alpha(|z|+|x|)}\Phi(z,x)\mathcal{D}_p(\lambda(y)) - (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y,z)\lambda(\mathcal{D}_q(x))\\
&\quad - (-1)^{|z|(|x|+|y|)+\alpha(|z|+|x|)}\Phi(z,x)\mathcal{D}_q(\lambda(y)) - (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\mathcal{D}_q(\lambda(z))\\
&\quad - \mathcal{D}_p(\lambda([x,y,z]_{\mathcal{Q}})) + \lambda(\mathcal{D}_q([x,y,z]_{\mathcal{Q}}))\\
&= \delta_\Phi(\mathcal{D}_p\lambda - \lambda\mathcal{D}_q)(x,y,z),
\end{aligned} \tag{40}$$

which implies that $[Ob^{\Omega,1}_{(\mathcal{D}_p,\mathcal{D}_q)}] = [Ob^{\Omega,2}_{(\mathcal{D}_p,\mathcal{D}_q)}] \in \mathcal{H}^1(\mathcal{Q};\mathcal{P})$ as required. □

Below is the main result of this section.

Theorem 3.4. *Keep notations as above. For any representation $(\Phi,\mathcal{P})$ of $\mathcal{Q}$, $\mathcal{T}_\Phi$ is a Lie subsuperalgebra of $Der(\mathcal{P})\times Der(\mathcal{Q})$ and the map Ψ given by Eq. (39) is a Lie superalgebra homomorphism.*

Proof. Take $|\mathcal{D}_{p_1}| = \alpha_1$ and $|\mathcal{D}_{p_2}| = \alpha_2$.

$$\begin{aligned}
&(\mathcal{D}_{p_1}\mathcal{D}_{p_2} - (-1)^{\alpha_1\alpha_2}\mathcal{D}_{p_2}\mathcal{D}_{p_1})\Phi(x,y) - (-1)^{(\alpha_1+\alpha_2)(|x|+|y|)}\Phi(x,y)(\mathcal{D}_{p_1}\mathcal{D}_{p_2} - (-1)^{\alpha_1\alpha_2}\mathcal{D}_{p_2}\mathcal{D}_{p_1})\\
&= \underbrace{\mathcal{D}_{p_1}((-1)^{\alpha_2(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_2} + \Phi(\mathcal{D}_{p_2}(x),y) + (-1)^{\alpha_2|x|}\Phi(x,\mathcal{D}_{p_2}(y)))}_{I_1}\\
&\quad - \underbrace{(-1)^{\alpha_1\alpha_2}\mathcal{D}_{p_2}((-1)^{\alpha_1(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_1} + \Phi(\mathcal{D}_{p_1}(x),y) + (-1)^{\alpha_1|x|}\Phi(x,\mathcal{D}_{p_1}(y)))}_{I_2}\\
&\quad - (-1)^{(\alpha_1+\alpha_2)(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_1}\mathcal{D}_{p_2} + (-1)^{(\alpha_1+\alpha_2)(|x|+|y|)+\alpha_1\alpha_2}\Phi(x,y)\mathcal{D}_{p_2}\mathcal{D}_{p_1},
\end{aligned} \tag{41}$$

where

$$I_1 = \mathcal{D}_{p_1}((-1)^{\alpha_2(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_2} + \Phi(\mathcal{D}_{p_2}(x),y) + (-1)^{\alpha_2|x|}\Phi(x,\mathcal{D}_{p_2}(y))),$$

$$I_2 = (-1)^{\alpha_1\alpha_2}\mathcal{D}_{p_2}((-1)^{\alpha_1(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_1} + \Phi(\mathcal{D}_{p_1}(x),y) + (-1)^{\alpha_1|x|}\Phi(x,\mathcal{D}_{p_1}(y))).$$

By Eq. (21), we get

$$\begin{aligned} I_1 = &(-1)^{\alpha_2(|x|+|y|)}((-1)^{\alpha_1(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_1} + \Phi(\mathcal{D}_{p_1}(x),y) \\ &+ (-1)^{\alpha_1|x|}\Phi(x,\mathcal{D}_{p_1}(y)))\mathcal{D}_{p_2} + ((-1)^{\alpha_1(|x|+|y|+\alpha_2)}\Phi(\mathcal{D}_{p_2}(x),y)\mathcal{D}_{p_1} \\ &+ \Phi(\mathcal{D}_{p_1}\mathcal{D}_{p_2}(x),y) + (-1)^{\alpha_1(\alpha_2+|x|)}\Phi(\mathcal{D}_{p_2}(x),\mathcal{D}_{p_1}(y))) \\ &+ (-1)^{\alpha_2|x|}((-1)^{\alpha_1(|x|+|y|+\alpha_2)}\Phi(x,\mathcal{D}_{p_2}(y))\mathcal{D}_{p_1} + \Phi(\mathcal{D}_{p_1}(x),\mathcal{D}_{p_2}(y)) \\ &+ (-1)^{\alpha_1|x|}\Phi(x,\mathcal{D}_{p_1}\mathcal{D}_{p_2}(y))), \end{aligned} \tag{42}$$

and

$$\begin{aligned} I_2 = &(-1)^{\alpha_1(\alpha_2+|x|+|y|)}((-1)^{\alpha_2(|x|+|y|)}\Phi(x,y)\mathcal{D}_{p_2} + \Phi(\mathcal{D}_{p_2}(x),y) \\ &+ (-1)^{\alpha_2|x|}\Phi(x,\mathcal{D}_{p_2}(y)))\mathcal{D}_{p_1} - (-1)^{\alpha_1\alpha_2}((-1)^{\alpha_2(|x|+|y|+\alpha_1)}\Phi(\mathcal{D}_{p_1}(x),y)\mathcal{D}_{p_2} \\ &+ \Phi(\mathcal{D}_{p_2}\mathcal{D}_{p_1}(x),y) + (-1)^{\alpha_2(\alpha_1+|x|)}\Phi(\mathcal{D}_{p_1}(x),\mathcal{D}_{p_2}(y))) \\ &- (-1)^{\alpha_1(|x|+\alpha_2)}((-1)^{\alpha_2(|x|+|y|+\alpha_1)}\Phi(x,\mathcal{D}_{p_1}(y))\mathcal{D}_{p_2} + \Phi(\mathcal{D}_{p_2}(x),\mathcal{D}_{p_1}(y)) \\ &+ (-1)^{\alpha_2|x|}\Phi(x,\mathcal{D}_{p_2}\mathcal{D}_{p_1}(y))). \end{aligned} \tag{43}$$

Then, inserting Eqs. (42) and (43) into Eq. (41), we get

$$\begin{aligned} &(\mathcal{D}_{p_1}\mathcal{D}_{p_2} - (-1)^{\alpha_1\alpha_2}\mathcal{D}_{p_2}\mathcal{D}_{p_1})\Phi(x,y) - (-1)^{(\alpha_1+\alpha_2)(|x|+|y|)}\Phi(x,y)(\mathcal{D}_{p_1}\mathcal{D}_{p_2} \\ &- (-1)^{\alpha_1\alpha_2}\mathcal{D}_{p_2}\mathcal{D}_{p_1}) = \Phi((\mathcal{D}_{q_1}\mathcal{D}_{q_2} - (-1)^{\alpha_1\alpha_2}\mathcal{D}_{q_2}\mathcal{D}_{q_1})(x),y) \\ &\quad + (-1)^{|x|(\alpha_1+\alpha_2)}\Phi(x,(\mathcal{D}_{q_1}\mathcal{D}_{q_2} - (-1)^{\alpha_1\alpha_2}\mathcal{D}_{q_2}\mathcal{D}_{q_1})(y)), \end{aligned} \tag{44}$$

which implies that $[(\mathcal{D}_{p_1},\mathcal{D}_{q_1}),(\mathcal{D}_{p_2},\mathcal{D}_{q_2})]$ is compatible by Definition 3.2.
To prove the second part, refer [15]. □

4. Abelian Extensions and Extensibility of superderivations

In this section, we construct obstruction classes for extensibility of superderivatios by Lemma 3.1 and also give a representation of $\mathcal{T}_\Phi$ in terms of extensibility of superderivations.

Lemma 4.1. *Keep notations as above. The cohomology class $[Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}] \in \mathcal{H}^1(Q;\mathcal{P})$ does not depend on the choice of the section of π.*

Proof. Let s_1 and s_2 be sections of π and Ω_1, Ω_2 be defined by Eq. (9) while $Ob^{\Omega,1}_{(\mathcal{D}_p,\mathcal{D}_q)}$ and $Ob^{\Omega,2}_{(\mathcal{D}_p,\mathcal{D}_q)}$ are defined by Eq. (20) with respect to Ω_1, Ω_2. Then

$$\begin{aligned} &Ob^{\Omega,1}_{(\mathcal{D}_p,\mathcal{D}_q)}(x,y,z) - Ob^{\Omega,2}_{(\mathcal{D}_p,\mathcal{D}_q)}(x,y,z) \\ &= \mathcal{D}_p(\Omega_1(x,y,z)) - \Omega_1(\mathcal{D}_q(x),y,z) - (-1)^{\alpha|x|}\Omega_1(x,\mathcal{D}_q(y),z) - (-1)^{\alpha(|x|+|y|)}\Omega_1(x,y,\mathcal{D}_q(z)) \\ &\quad - \mathcal{D}_p(\Omega_2(x,y,z)) + \Omega_2(\mathcal{D}_q(x),y,z) + (-1)^{\alpha|x|}\Omega_2(x,\mathcal{D}_q(y),z) + (-1)^{\alpha(|x|+|y|)}\Omega_2(x,y,\mathcal{D}_q(z)) \\ &= \underbrace{\mathcal{D}_p(\Omega_1(x,y,z) - \Omega_2(x,y,z))}_{I_1} - \underbrace{(\Omega_1(\mathcal{D}_q(x),y,z) - \Omega_2(\mathcal{D}_q(x),y,z))}_{I_2} \\ &\quad - (-1)^{\alpha|x|}\underbrace{(\Omega_1(x,\mathcal{D}_q(y),z) - \Omega_2(x,\mathcal{D}_q(y),z))}_{I_3} \\ &\quad - (-1)^{\alpha(|x|+|y|)}\underbrace{(\Omega_1(x,y,\mathcal{D}_q(z)) - \Omega_2(x,y,\mathcal{D}_q(z)))}_{I_4}, \end{aligned} \tag{45}$$

for $x, y, z \in \mathcal{Q}$ and $|\mathcal{D}_p| = |\mathcal{D}_q| = \alpha$. Define an even linear map $\lambda : \mathcal{Q} \to \mathcal{P}$ by $\lambda(x) := s_1(x) - s_2(x)$ where $x \in \mathcal{Q}$. From Corollary 2.5, we have

$$\begin{aligned}\Omega_1(x, y, z) - \Omega_2(x, y, z) = &- \lambda([x, y, z]_{\mathcal{Q}}) + \Phi(x, y)\lambda(z) + (-1)^{|x|(|y|+|z|)}\Phi(y, z)\lambda(x) \\ &+ (-1)^{|z|(|x|+|y|)}\Phi(z, x)\lambda(y),\end{aligned} \tag{46}$$

for any $x, y, z \in \mathcal{Q}$. Therefore, we get

$$\begin{aligned}I_1 &= \mathcal{D}_p(\Omega_1(x, y, z) - \Omega_2(x, y, z)) \\ &= \mathcal{D}_p(-\lambda([x, y, z]_{\mathcal{Q}}) + \Phi(x, y)\lambda(z) + (-1)^{|x|(|y|+|z|)}\Phi(y, z)\lambda(x) \\ &\quad + (-1)^{|z|(|x|+|y|)}\Phi(z, x)\lambda(y)),\end{aligned}$$

$$\begin{aligned}I_2 &= \Omega_1(\mathcal{D}_q(x), y, z) - \Omega_2(\mathcal{D}_q(x), y, z) \\ &= -\lambda([\mathcal{D}_q(x), y, z]_{\mathcal{Q}}) + \Phi(\mathcal{D}_q(x), y)\lambda(z) + (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y, z)\lambda(\mathcal{D}_q(x)) \\ &\quad + (-1)^{|z|(\alpha+|x|+|y|)}\Phi(z, \mathcal{D}_q(x))\lambda(y),\end{aligned}$$

$$\begin{aligned}I_3 &= \Omega_1(x, \mathcal{D}_q(y), z) - \Omega_2(x, \mathcal{D}_q(y), z) \\ &= -\lambda([x, \mathcal{D}_q(y), z]_{\mathcal{Q}}) + \Phi(x, \mathcal{D}_q(y))\lambda(z) + (-1)^{|x|(\alpha+|y|+|z|)}\Phi(\mathcal{D}_q(y), z)\lambda(x) \\ &\quad + (-1)^{|z|(\alpha+|x|+|y|)}\Phi(z, x)\lambda(\mathcal{D}_q(y)),\end{aligned}$$

and

$$\begin{aligned}I_4 &= \Omega_1(x, y, \mathcal{D}_q(z)) - \Omega_2(x, y, \mathcal{D}_q(z)) \\ &= -\lambda([x, y, \mathcal{D}_q(z)]_{\mathcal{Q}}) + \Phi(x, y)\lambda(\mathcal{D}_q(z)) + (-1)^{|x|(\alpha+|y|+|z|)}\Phi(y, \mathcal{D}_q(z))\lambda(x) \\ &\quad + (-1)^{(\alpha+|z|)(|x|+|y|)}\Phi(\mathcal{D}_q(z), x)\lambda(y).\end{aligned}$$

By I_1, I_2, I_3, I_4, and Eq. (45), we get

$$\begin{aligned}&Ob^{\Omega,1}_{(\mathcal{D}_p,\mathcal{D}_q)}(x, y, z) - Ob^{\Omega,2}_{(\mathcal{D}_p,\mathcal{D}_q)}(x, y, z) \\ &= (\mathcal{D}_p(\Phi(x, y))\lambda(z) - \Phi(\mathcal{D}_q(x), y)\lambda(z) - (-1)^{\alpha|x|}\Phi(x, \mathcal{D}_q(y))\lambda(z)) \\ &\quad + (-1)^{|x|(|y|+|z|)}(\mathcal{D}_p(\Phi(y, z))\lambda(x) - \Phi(\mathcal{D}_q(y), z)\lambda(x) - (-1)^{\alpha|y|}\Phi(y, \mathcal{D}_q(z))\lambda(x)) \\ &\quad + (-1)^{|z|(|x|+|y|)}(\mathcal{D}_p(\Phi(z, x))\lambda(y) - \Phi(\mathcal{D}_q(z), x)\lambda(y) - (-1)^{\alpha|z|}\Phi(z, \mathcal{D}_q(x))\lambda(y)) \\ &\quad - \mathcal{D}_p(\lambda([x, y, z]_{\mathcal{Q}})) - (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y, z)\lambda(\mathcal{D}_q(x)) \\ &\quad - (-1)^{\alpha|x|+|z|(\alpha+|x|+|y|)}\Phi(z, x)\lambda(\mathcal{D}_q(y)) - (-1)^{\alpha(|x|+|y|)}\Phi(x, y)\lambda(\mathcal{D}_q(z)) + \lambda(\mathcal{D}_q([x, y, z]_{\mathcal{Q}})),\end{aligned}$$

where $\mathcal{D}_q$ is a superderivation. Since $(\mathcal{D}_p, \mathcal{D}_q)$ is compatible, by Eq. (21) it follows that

$$\begin{aligned}&(\mathcal{D}_p(\Phi(x, y))\lambda(z) - \Phi(\mathcal{D}_q(x), y)\lambda(z) - (-1)^{\alpha|x|}\Phi(x, \mathcal{D}_q(y))\lambda(z)) \\ &= (-1)^{\alpha(|x|+|y|)}\Phi(x, y)(\mathcal{D}_p(\lambda(z))),\end{aligned}$$

$$\begin{aligned}&(-1)^{|x|(|y|+|z|)}(\mathcal{D}_p(\Phi(y, z))\lambda(x) - \Phi(\mathcal{D}_q(y), z)\lambda(x) - (-1)^{\alpha|y|}\Phi(y, \mathcal{D}_q(z))\lambda(x)) \\ &= (-1)^{(|x|+\alpha)(|y|+|z|)}\Phi(y, z)(\mathcal{D}_p(\lambda(x))),\end{aligned}$$

and

$$\begin{aligned}&(-1)^{|z|(|x|+|y|)}(\mathcal{D}_p(\Phi(z, x))\lambda(y) - \Phi(\mathcal{D}_q(z), x)\lambda(y) - (-1)^{\alpha|z|}\Phi(z, \mathcal{D}_q(x))\lambda(y)) \\ &= (-1)^{|z|(|x|+|y|+\alpha)+\alpha|x|}\Phi(z, x)\mathcal{D}_p(\lambda(y)).\end{aligned}$$

Therefore, we obtain

$$
\begin{aligned}
& Ob^{\Omega,1}_{(\mathcal{D}_p,\mathcal{D}_q)}(x,y,z) - Ob^{\Omega,2}_{(\mathcal{D}_p,\mathcal{D}_q)}(x,y,z) \\
&= (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\mathcal{D}_p(\lambda(z)) - \mathcal{D}_p(\lambda([x,y,z]_{\mathcal{Q}})) + (-1)^{(|x|+\alpha)(|y|+|z|)}\Phi(y,z)\mathcal{D}_p(\lambda(x)) \\
&\quad + (-1)^{|z|(|x|+|y|+\alpha)+\alpha|x|}\Phi(z,x)\mathcal{D}_p(\lambda(y)) - (-1)^{(\alpha+|x|)(|y|+|z|)}\Phi(y,z)\lambda(\mathcal{D}_q(x)) \\
&\quad - (-1)^{|z|(\alpha+|x|+|y|)+\alpha|x|}\Phi(z,x)\lambda(\mathcal{D}_q(y)) - (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\lambda(\mathcal{D}_q(z)) + \lambda(\mathcal{D}_q([x,y,z]_{\mathcal{Q}})) \\
&= \delta_\Phi(\mathcal{D}_p\lambda - \lambda\mathcal{D}_q)(x,y,z),
\end{aligned}
$$

which implies that $[Ob^{\Omega,1}_{(\mathcal{D}_p,\mathcal{D}_q)}] = [Ob^{\Omega,2}_{(\mathcal{D}_p,\mathcal{D}_q)}] \in \mathcal{H}^1(\mathcal{Q};\mathcal{P})$ as required. □

Now, we will define extensibility of superderivations.

Definition 4.2. *Let $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} \mathcal{Q} \to 0$ be an abelian extension of 3-Lie superalgebras. A pair $(\mathcal{D}_p,\mathcal{D}_q) \in Der(\mathcal{P}) \times Der(\mathcal{Q})$ is called extensible if there is a superderivation $\mathcal{D}_l \in Der(\mathcal{L})$ such that the following diagram commutes:*

$$
\begin{array}{ccccccccc}
0 & \longrightarrow & \mathcal{P} & \xrightarrow{i} & \mathcal{L} & \xrightarrow{\pi} & \mathcal{Q} & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle \mathcal{D}_p} & & \downarrow{\scriptstyle \mathcal{D}_l} & & \downarrow{\scriptstyle \mathcal{D}_q} & & \\
0 & \longrightarrow & \mathcal{P} & \xrightarrow{i} & \mathcal{L} & \xrightarrow{\pi} & \mathcal{Q} & \longrightarrow & 0,
\end{array}
$$

where $i : \mathcal{P} \to \mathcal{L}$ is the inclusion map.

The following result shows that extensibility implies compatibility.

Proposition 4.3. *Let $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} \mathcal{Q} \to 0$ be an abelian extension of 3-Lie superalgebras. If a pair of superderivations $(\mathcal{D}_p,\mathcal{D}_q) \in Der(\mathcal{P}) \times Der(\mathcal{Q})$ is extensible, then $(\mathcal{D}_p,\mathcal{D}_q)$ is compatible with respect to Φ given by Eq. (7).*

Proof. Since $(\mathcal{D}_p,\mathcal{D}_q)$ is extensible, there exists a superderivation $\mathcal{D}_l \in Der(\mathcal{L})$ such that $i\mathcal{D}_p = \mathcal{D}_l i$, $\pi\mathcal{D}_l = \mathcal{D}_q\pi$, and $|\mathcal{D}_p| = |\mathcal{D}_q| = |\mathcal{D}_l| = \alpha$. Then

$$\mathcal{D}_l s(x) - s(\mathcal{D}_q(x)) \in \mathcal{P} \text{ for } x \in \mathcal{Q}.$$

So, there is an even linear map $\mu : \mathcal{Q} \to \mathcal{P}$ given by

$$\mu(x) := \mathcal{D}_l s(x) - s(\mathcal{D}_q(x)). \tag{47}$$

Since $[\mathcal{P},\mathcal{P},\mathcal{L}] = 0$, we have

$$[\mu(x), s(y), v]_{\mathcal{L}} = [s(x), \mu(y), v]_{\mathcal{L}} = 0,$$

for $x, y \in \mathcal{Q}$ and $v \in \mathcal{P}$. Since $i\mathcal{D}_p = \mathcal{D}_l i$ and $\mathcal{D}_l \in Der(\mathcal{L})$, we get

$$
\begin{aligned}
& \mathcal{D}_p(\Phi(x,y)(v)) - (-1)^{\alpha(|x|+|y|)}\Phi(x,y)\mathcal{D}_p(v) \\
&= \mathcal{D}_p([s(x),s(y),v]_{\mathcal{L}}) - (-1)^{\alpha(|x|+|y|)}[s(x),s(y),\mathcal{D}_p(v)]_{\mathcal{L}} \\
&= \mathcal{D}_l([s(x),s(y),v]_{\mathcal{L}}) - (-1)^{\alpha(|x|+|y|)}[s(x),s(y),\mathcal{D}_p(v)]_{\mathcal{L}} \\
&= [\mathcal{D}_l(s(x)),s(y),v]_{\mathcal{L}} + (-1)^{\alpha|x|}[s(x),\mathcal{D}_l(s(y)),v]_{\mathcal{L}} + (-1)^{\alpha(|x|+|y|)}[s(x),s(y),\mathcal{D}_l(v)]_{\mathcal{L}} \\
&\quad - (-1)^{\alpha(|x|+|y|)}[s(x),s(y),\mathcal{D}_p(v)]_{\mathcal{L}} \\
&= [s(\mathcal{D}_q(x)),s(y),v]_{\mathcal{L}} + [\mu(x),s(y),v]_{\mathcal{L}} + (-1)^{\alpha|x|}([s(x),s(\mathcal{D}_q(y)),v]_{\mathcal{L}} \\
&\quad + [s(x),\mu(y),v]_{\mathcal{L}}) + (-1)^{\alpha(|x|+|y|)}[s(x),s(y),\mathcal{D}_l(v)]_{\mathcal{L}} - (-1)^{\alpha(|x|+|y|)}[s(x),s(y),\mathcal{D}_p(v)]_{\mathcal{L}} \\
&= [s(\mathcal{D}_q(x)),s(y),v]_{\mathcal{L}} + (-1)^{\alpha|x|}([s(x),s(\mathcal{D}_q(y)),v]_{\mathcal{L}} \\
&= \Phi(\mathcal{D}_q(x),y)(v) + (-1)^{\alpha|x|}\Phi(x,\mathcal{D}_q(y))(v).
\end{aligned}
$$

□

Proposition 4.4. *Let* $0 \to \mathcal{P} \xrightarrow{i} \mathcal{L} \xrightarrow{\pi} \mathcal{Q} \to 0$ *be an abelian extension of* 3*-Lie superalgebras. Assume that* $(\mathcal{D}_p, \mathcal{D}_q) \in Der_{\bar{0}}(\mathcal{P}) \times Der_{\bar{0}}(\mathcal{Q})$ *is compatible with respect to* Φ *given by Eq. (7). Then* $(\mathcal{D}_p, \mathcal{D}_q)_{\bar{0}}$ *is extensible if and only if* $[Ob^{\mathcal{L}}_{(\mathcal{D}_p,\mathcal{D}_q)}] \in \mathcal{H}^1(\mathcal{Q};\mathcal{P})$ *is trivial.*

Proof. Suppose that $(\mathcal{D}_p, \mathcal{D}_q)$ is extensible. Then there exists a superderivation $\mathcal{D}_l \in Der(\mathcal{L})$ such that the associative diagram 4.2 is commutative. Since $\pi\mathcal{D}_l = \mathcal{D}_q\pi$ and $|\mathcal{D}_p| = |\mathcal{D}_q| = |\mathcal{D}_l| = \alpha$, we have

$$\mathcal{D}_l s(x) - s(\mathcal{D}_q(x)) \in \mathcal{P} \text{ for } x \in \mathcal{Q}.$$

So there is an even linear map $\mu : \mathcal{Q} \to \mathcal{P}$ given by Eq. (47). It is sufficient to show that

$$Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, x_3) = (\delta_\Phi\mu)(x_1, x_2, x_3), \quad x_i \in \mathcal{Q}. \tag{48}$$

Now

$$\begin{aligned}
&\mathcal{D}_l([s(x_1) + v_1, s(x_2) + v_2, s(x_3) + v_3]_{\mathcal{L}}) \\
&= [\mathcal{D}_l(s(x_1) + v_1)), s(x_2) + v_2, s(x_3) + v_3]_{\mathcal{L}} \\
&\quad + (-1)^{\alpha|x_1|}[s(x_1) + v_1, \mathcal{D}_l(s(x_2) + v_2)), s(x_3) + v_3]_{\mathcal{L}} \\
&\quad + (-1)^{\alpha(|x_1|+|x_2|)}[s(x_1) + v_1, s(x_2) + v_2, \mathcal{D}_l(s(x_3) + v_3)]_{\mathcal{L}},
\end{aligned} \tag{49}$$

for $x_i \in \mathcal{Q}$ and $v_i \in \mathcal{P}$. Since $[\mathcal{P}, \mathcal{P}, \mathcal{L}] = 0$, we get

$$\begin{aligned}
&[s(x_1) + v_1, s(x_2) + v_2, s(x_3) + v_3]_{\mathcal{L}} \\
&= [s(x_1), s(x_2), s(x_3)]_{\mathcal{L}} + [s(x_1), s(x_2), v_3]_{\mathcal{L}} + [v_1, s(x_2), s(x_3)]_{\mathcal{L}} + [s(x_1), v_2, s(x_3)]_{\mathcal{L}} \\
&= [s(x_1), s(x_2), s(x_3)]_{\mathcal{L}} + \Phi(x_1, x_2)(v_3) + (-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2, x_3)(v_1) \\
&\quad + (-1)^{|x_3|(|x_1|+|x_2|)}\Phi(x_3, x_1)(v_2),
\end{aligned}$$

and hence the left-hand side of Eq. (49) is

$$\begin{aligned}
&\mathcal{D}_l([s(x_1), s(x_2), s(x_3)]_{\mathcal{L}} + \Phi(x_1, x_2)(v_3) + (-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2, x_3)(v_1) + (-1)^{|x_3|(|x_1|+|x_2|)}\Phi(x_3, x_1)(v_2)) \\
&= \mathcal{D}_l(s([x_1, x_2, x_3]_{\mathcal{Q}}) + \Omega(x_1, x_2, x_3) + \Phi(x_1, x_2)(v_3) + (-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2, x_3)(v_1) \\
&\quad + (-1)^{|x_3|(|x_1|+|x_2|)}\Phi(x_3, x_1)(v_2)).
\end{aligned}$$

Since $\mathcal{D}_l i = i\mathcal{D}_p$ where i is the inclusion map, and $\Omega(x_1, x_2, x_3)$, $\Phi(x_1, x_2)(v_3)$, $\Phi(x_2, x_3)(v_1)$, $\Phi(x_3, x_1)(v_2) \in \mathcal{P}$, by the definition of μ as in Eq. (47), it follows that

$$\begin{aligned}
&s(\mathcal{D}_q([x_1, x_2, x_3]_{\mathcal{Q}}) + \mu([x_1, x_2, x_3]_{\mathcal{Q}}) + \mathcal{D}_p(\Omega(x_1, x_2, x_3)) + \mathcal{D}_p(\Phi(x_1, x_2)(v_3)) \\
&\quad + (-1)^{|x_1|(|x_2|+|x_3|)}\mathcal{D}_p(\Phi(x_2, x_3)(v_1)) + (-1)^{|x_3|(|x_1|+|x_2|)}\mathcal{D}_p(\Phi(x_3, x_1)(v_2))) \\
&= s([\mathcal{D}_q(x_1), x_2, x_3]_{\mathcal{Q}}) + (-1)^{\alpha|x_1|}s([x_1, \mathcal{D}_q(x_2), x_3]_{\mathcal{Q}}) \\
&\quad + (-1)^{\alpha(|x_2|+|x_3|)}s([x_1, x_2, \mathcal{D}_q(x_3)]_{\mathcal{Q}}) + \mu([x_1, x_2, x_3]_{\mathcal{Q}}) + \mathcal{D}_p(\Omega(x_1, x_2, x_3)) \\
&\quad + \mathcal{D}_p(\Phi(x_1, x_2)(v_3)) + (-1)^{|x_1|(|x_2|+|x_3|)}\mathcal{D}_p(\Phi(x_2, x_3)(v_1)) \\
&\quad + (-1)^{|x_3|(|x_1|+|x_2|)}\mathcal{D}_p(\Phi(x_3, x_1)(v_2)).
\end{aligned} \tag{50}$$

Now, we compute the right-hand side of Eq. (49). Since $\mathcal{D}_l|_{\mathcal{P}} = \mathcal{D}_p$,

$$\begin{aligned}
\mathcal{D}_l(s(x_i) + v_i) &= \mathcal{D}_l(s(x_i)) + \mathcal{D}_p(v_i) \\
&= \mathcal{D}_l(s(x_i)) - s(\mathcal{D}_q(x_i)) + s(\mathcal{D}_q(x_i)) + \mathcal{D}_p(v_i) \\
&= s(\mathcal{D}_q(x_i)) + \mu(x_i) + \mathcal{D}_p(v_i) \in s(\mathcal{Q}) \oplus \mathcal{P}.
\end{aligned}$$

From above and $[\mathcal{P},\mathcal{P},\mathcal{L}]=0$, the right-hand side of Eq. (49) is

$$\begin{aligned}
&[s(\mathcal{D}_q(x_1))+\mu(x_1)+\mathcal{D}_p(v_1),s(x_2)+v_2,s(x_3)+v_3]_{\mathcal{L}}\\
&\quad+(-1)^{\alpha|x_1|}[s(x_1)+v_1,s(\mathcal{D}_q(x_2))+\mu(x_2)+\mathcal{D}_p(v_2),s(x_3)+v_3]_{\mathcal{L}}\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|)}[s(x_1)+v_1,s(x_2)+v_2,s(\mathcal{D}_q(x_3))+\mu(x_3)+\mathcal{D}_p(v_3)]_{\mathcal{L}}\\
&=[s(\mathcal{D}_q(x_1)),s(x_2),s(x_3)]_{\mathcal{L}}+[s(\mathcal{D}_q(x_1)),s(x_2),v_3]_{\mathcal{L}}+[s(\mathcal{D}_q(x_1)),v_2,s(x_3)]_{\mathcal{L}}\\
&\quad+[\mu(x_1),s(x_2),s(x_3)]_{\mathcal{L}}+[\mathcal{D}_p(v_1),s(x_2),s(x_3)]_{\mathcal{L}}+(-1)^{\alpha|x_1|}([s(x_1),s(\mathcal{D}_q(x_2)),s(x_3)]_{\mathcal{L}}\\
&\quad+[s(x_1),s(\mathcal{D}_q(x_2)),v_3]_{\mathcal{L}}+[s(x_1),\mu(x_2),s(x_3)]_{\mathcal{L}}+[s(x_1),\mathcal{D}_p(v_2),s(x_3)]_{\mathcal{L}}\\
&\quad+[v_1,s(\mathcal{D}_q(x_2)),s(x_3)]_{\mathcal{L}})+(-1)^{\alpha(|x_1|+|x_2|)}([s(x_1),s(x_2),s(\mathcal{D}_q(x_3))]_{\mathcal{L}}\\
&\quad+[v_1,s(x_2),s(\mathcal{D}_q(x_3))]_{\mathcal{L}}+[s(x_1),s(x_2),\mu(x_3)]_{\mathcal{L}}+[s(x_1),s(x_2),\mathcal{D}_p(v_3)]_{\mathcal{L}}\\
&\quad+[s(x_1),v_2,s(\mathcal{D}_q(x_3))]_{\mathcal{L}}).
\end{aligned}\tag{51}$$

By Eqs. (50) and (51) it follows that

$$\begin{aligned}
&s([\mathcal{D}_q(x_1),x_2,x_3]_{\mathcal{Q}})+(-1)^{\alpha|x_1|}s([x_1,\mathcal{D}_q(x_2),x_3]_{\mathcal{Q}})+(-1)^{\alpha(|x_1|+|x_2|)}s([x_1,x_2,\mathcal{D}_q(x_3)]_{\mathcal{Q}})\\
&\quad+\mu([x_1,x_2,x_3]_{\mathcal{Q}})+\mathcal{D}_p(\Omega(x_1,x_2,x_3))+\mathcal{D}_p(\Phi(x_1,x_2)(v_3))\\
&\quad+(-1)^{|x_1|(|x_2|+|x_3|)}\mathcal{D}_p(\Phi(x_2,x_3)(v_1))+(-1)^{|x_3|(|x_1|+|x_2|)}\mathcal{D}_p(\Phi(x_3,x_1)(v_2))\\
&=[s(\mathcal{D}_q(x_1)),s(x_2),s(x_3)]_{\mathcal{L}}+\Phi(\mathcal{D}_q(x_1),x_2)(v_3)+(-1)^{|x_3|(\alpha+|x_1|+|x_2|)}\Phi(x_3,\mathcal{D}_q(x_1))(v_2)\\
&\quad+(-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2,x_3)\mu(x_1)+(-1)^{(\alpha+|x_1|)(|x_2|+|x_3|)}\Phi(x_2,x_3)\mathcal{D}_p(v_1)\\
&\quad+(-1)^{\alpha|x_1|}([s(x_1),s(\mathcal{D}_q(x_2)),s(x_3)]_{\mathcal{L}}+(-1)^{\alpha|x_1|}\Phi(x_1,\mathcal{D}_q(x_2))(v_3)\\
&\quad+(-1)^{\alpha(|x_1|+|x_3|)+|x_3|(|x_1|+|x_2|)}\Phi(x_3,x_1)\mathcal{D}_p(v_2)+(-1)^{|x_3|(|x_1|+|x_2|)+\alpha|x_1|}\Phi(x_3,x_1)\mu(x_2)\\
&\quad+(-1)^{|x_1|(|x_2|+|x_3|)}\Phi(\mathcal{D}_q(x_2),x_3)(v_1)+(-1)^{\alpha(|x_1|+|x_2|)}([s(x_1),s(x_2),s(\mathcal{D}_q(x_3))]_{\mathcal{L}}\\
&\quad+(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)\mu(x_3)+(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)\mathcal{D}_p(v_3)\\
&\quad+(-1)^{|x_3|(|x_1|+|x_2|)}\Phi(\mathcal{D}_q(x_3),x_1)(v_2)+(-1)^{\alpha|x_2|+|x_1|(|x_2|+|x_3|)}\Phi(x_2,\mathcal{D}_q(x_3))(v_1)).
\end{aligned}$$

Then, we get

$$\begin{aligned}
0&=-\Omega(\mathcal{D}_q(x_1),x_2,x_3)-(-1)^{\alpha|x_1|}\Omega(x_1,\mathcal{D}_q(x_2),x_3)-(-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1,x_2,\mathcal{D}_q(x_3))\\
&\quad+\mathcal{D}_p(\Omega(x_1,x_2,x_3))+(-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2,x_3)\mu(x_1)-(-1)^{|x_3|(|x_1|+|x_2|)}\Phi(x_3,x_1)\mu(x_2)\\
&\quad-(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)\mu(x_3)+\mu([x_1,x_2,x_3])+(\mathcal{D}_p\Phi(x_1,x_2)\\
&\quad-(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)\mathcal{D}_p-\Phi(\mathcal{D}_q(x_1),x_2)-(-1)^{\alpha|x_1|}\Phi(x_1,\mathcal{D}_q(x_2)))(v_3)\\
&\quad+(-1)^{|x_1|(|x_2|+|x_3|)}(\mathcal{D}_p\Phi(x_2,x_3)-(-1)^{\alpha(|x_2|+|x_3|)}\Phi(x_2,x_3)\mathcal{D}_p-\Phi(\mathcal{D}_q(x_2),x_3)\\
&\quad-(-1)^{\alpha|x_2|}\Phi(x_2,\mathcal{D}_q(x_3)))(v_1)+(-1)^{|x_3|(|x_1|+|x_2|)}(\mathcal{D}_p\Phi(x_3,x_1)\\
&\quad-(-1)^{\alpha(|x_3|+|x_1|)}\Phi(x_3,x_1)\mathcal{D}_p-\Phi(\mathcal{D}_q(x_3),x_1)-(-1)^{\alpha|x_1|}\Phi(x_3,\mathcal{D}_q(x_1)))(v_2).
\end{aligned}$$

Since $(\mathcal{D}_p,\mathcal{D}_q)$ is compatible with respect to Φ,

$$(\mathcal{D}_p\Phi(x_1,x_2)-(-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1,x_2)\mathcal{D}_p-\Phi(\mathcal{D}_q(x_1),x_2)-(-1)^{\alpha|x_1|}\Phi(x_1,\mathcal{D}_q(x_2)))(v_3)=0,$$

$$(\mathcal{D}_p\Phi(x_2,x_3)-(-1)^{\alpha(|x_2|+|x_3|)}\Phi(x_2,x_3)\mathcal{D}_p-\Phi(\mathcal{D}_q(x_2),x_3)-(-1)^{\alpha|x_2|}\Phi(x_2,\mathcal{D}_q(x_3)))(v_1)=0,$$

and

$$(\mathcal{D}_p\Phi(x_3,x_1)-(-1)^{\alpha(|x_3|+|x_1|)}\Phi(x_3,x_1)\mathcal{D}_p-\Phi(\mathcal{D}_q(x_3),x_1)-(-1)^{\alpha|x_1|}\Phi(x_3,\mathcal{D}_q(x_1)))(v_2)=0.$$

Thus, we have

$$
\begin{aligned}
& - \Omega(\mathcal{D}_q(x_1), x_2, x_3) - (-1)^{\alpha|x_1|}\Omega(x_1, \mathcal{D}_q(x_2), x_3) - (-1)^{\alpha(|x_1|+|x_2|)}\Omega(x_1, x_2, \mathcal{D}_q(x_3)) \\
& \quad + \mathcal{D}_p(\Omega(x_1, x_2, x_3)) - (-1)^{|x_1|(|x_2|+|x_3|)}\Phi(x_2, x_3)\mu(x_1) - (-1)^{|x_3|(|x_1|+|x_2|)}\Phi(x_3, x_1)\mu(x_2) \\
& \quad - (-1)^{\alpha(|x_1|+|x_2|)}\Phi(x_1, x_2)\mu(x_3) + \mu([x_1, x_2, x_3]) \\
& = 0,
\end{aligned}
$$

since $|\mathcal{D}_p| = |\mathcal{D}_q| = |\mathcal{D}_l| = \alpha = 0$, hence we have $Ob^{\Omega}_{(\mathcal{D}_p,\mathcal{D}_q)}(x_1, x_2, x_3) = (\delta_\Phi\mu)(x_1, x_2, x_3)$ due to Eqs. (5) and (20). To prove the converse part, refer [15]. □

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