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Characterization of Kenmotsu manifolds with a generalized symmetric metric connection

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Abstract. The objective of the present findings is to analyze Kenmotsu manifolds by using (α, β) type generalized symmetric metric connection. The characterization of Kenmotsu manifold by using certain curvature properties corresponding to the generalized symmetric metric connection is investigated. In the end, an example of Kenmotsu manifold with the generalized symmetric metric connection admitting Q tensor and Weyl conformal curvature tensor is constructed by using partial differential equations.

1. Introduction

In a general metric space, Hayden introduced special metric connection with non-zero torsion[11]. In a differentiable manifold, Golab studied the quarter symmetric connections which was a generalization of Hayden's connections[10]. Later, these connections was developed by many authors (see [1], [8], [9] [10], [19], [23], [24], [15]). In almost contact manifolds, semi-symmetric metric connections was introduced by Sharfuddin and Husain [25]. In next years, many characterizations of semi-symmetric and quarter-symmetric connections have been made on Kenmotsu manifolds and Sasakian manifolds [2], [13], [14], [16], [17], [18], [20], [22], [26],[27], [28], [29], [30].

A linear connection $\bar{D}$ is suggested to be generalized symmetric connection on condition that the torsion tensor connection is presented in the form as follows:

$$T(\Omega_1, \Omega_2) = \alpha\{u(\Omega_2)\Omega_1 - u(\Omega_1)\Omega_2\} + \beta\{u(\Omega_2)\psi\Omega_1 - u(\Omega_1)\psi\Omega_2\}, \tag{1}$$

for any $\Omega_1, \Omega_2 \in \Gamma(TM)$, where φ is (1,1)-type tensor, u is a 1-form and α, β are smooth functions. In addition, the connection is called to be a generalized symmetric metric connection (briefly, GSMC) if $\bar{D}g = 0$ when a Riemennian metric g on M. Otherwise, the connection is non-metric [3], [4].

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- Set $\alpha = 0$ in (1), GSMC is known as $\beta-$quarter-symmetric connection.
- Set $\beta = 0$ in (1), GSMC is known as $\alpha-$semi-symmetric connection.

The generalized Z tensor is a symmetric tensor of type $(0,2)$ defined as [31]

$$Z(\Omega_1, \Omega_2) = S(\Omega_1, \Omega_2) + \lambda g(\Omega_1, \Omega_2),$$

where $\Omega_1, \Omega_2 \in \Gamma(TM)$, λ is some scalar function and S is Ricci tensor on M. The generalized Z tensor for $\lambda = -\frac{r}{n}$ reduced to a classical Z tensor, where r is a scalar curvature and n is the dimension on M. The generalized Z tensor can be considered a generalized Einstein gravitational tensor with some scalar function λ. Mantica and Suh [32], introduced Q tensor by taking the trace of Z tensor. Q tensor is a tensor of type $(1,3)$ defined as

$$Q(\Omega_1, \Omega_2, \Omega_3) = R(\Omega_1, \Omega_2, \Omega_3) - \frac{\lambda}{n-1}\{g(\Omega_2, \Omega_3)\Omega_1 - g(\Omega_1, \Omega_3)\Omega_2\}, \tag{2}$$

where $\Omega_1, \Omega_2, \Omega_3 \in \Gamma(TM)$ and R is a Riemannian curvature tensor.

Various characterizations of Kenmotsu manifold by using Q tensor were studied in [33]. A Kenmotsu manifold is locally symmetric to a hyperbolic space $H^n(-1)$ if and only if $R(\varrho, \Omega_5)Q = 0$, where ϱ is a characteristic vector field of the contact structure and Ω_5 is any vector field on M [21]. In [34], the author proved that a Sasakian manifold is locally symmetric to a unit sphere $S^n(1)$ if and only if the condition $R(\varrho, \Omega_5)Q = 0$ holds.

The projective tensor P, the concircular curvature tensor $\tilde{Z}$ and the Weyl conformal curvature tensor C, are defined by [35]

$$P(\Omega_1, \Omega_2)\Omega_3 = R(\Omega_1, \Omega_2)\Omega_3 - \frac{1}{n-1}\Big\{S(\Omega_2, \Omega_3)\Omega_1 - S(\Omega_1, \Omega_3)\Omega_2\Big\}, \tag{3}$$

$$\tilde{Z}(\Omega_1, \Omega_2)\Omega_3 = R(\Omega_1, \Omega_2)\Omega_3 - \frac{r}{n(n-1)}\Big\{g(\Omega_2, \Omega_3)\Omega_1 - g(\Omega_1, \Omega_3)\Omega_2\Big\}, \tag{4}$$

$$\begin{aligned} C(\Omega_1, \Omega_2)\Omega_3 &= R(\Omega_1, \Omega_2)\Omega_3 - \frac{1}{n-2}\Big\{S(\Omega_2, \Omega_3)\Omega_1 - S(\Omega_1, \Omega_3)\Omega_2 + g(\Omega_2, \Omega_3)Lv_1 \\ &- g(\Omega_1, \Omega_3)Lv_2\Big\} + \frac{r}{(n-1)(n-2)}\Big\{g(\Omega_2, \Omega_3)\Omega_1 - g(\Omega_1, \Omega_3)\Omega_2\Big\}, \end{aligned} \tag{5}$$

for all $\Omega_1, \Omega_2, \Omega_3 \in \Gamma(TM)$. Let (M^n, g) be a Riemannian manifold with curvature tensor R, Projective tensor P, symmetric tensor Z, Q tensor Q, and Weyl conformal curvature tensor C, then for all $\Omega_1, \Omega_2, \Omega_3, \Omega_4, \varrho \in \Gamma(TM)$ we have [34]

$$\begin{aligned} \Big(Q(\varrho, \Omega_3)Q\Big)(\Omega_1, \Omega_2)\varrho &= Q(\varrho, \Omega_3)Q(\Omega_1, \Omega_2)\varrho - Q\Big(Q(\varrho, \Omega_3)\Omega_1, \Omega_2\Big)\varrho \\ &\quad -Q\Big(\Omega_1, Q(\varrho, \Omega_3)\Omega_2\Big)\varrho - Q(\Omega_1, \Omega_2)Q(\varrho, \Omega_3)\varrho, \\ \Big(R(\Omega_4, \Omega_3)Q\Big)(\Omega_1, \Omega_2)\varrho &= R(\Omega_4, \Omega_3)Q(\Omega_1, \Omega_2)\varrho - Q(R(\Omega_4, \Omega_3)\Omega_1, \Omega_2)\varrho. \\ &\quad -Q(\Omega_1, R(\Omega_4, \Omega_3)\Omega_2)\varrho - Q(\Omega_1, \Omega_2)R(\Omega_4, \Omega_3)\varrho. \\ \Big(Q(\varrho, \Omega_3)Z\Big)(\Omega_1, \Omega_2)\varrho &= Q(\varrho, \Omega_3)Z(\Omega_1, \Omega_2)\varrho - Z\Big(Q(\varrho, \Omega_3)\Omega_1, \Omega_2\Big)\varrho \\ &\quad -Z\Big(\Omega_1, Q(\varrho, \Omega_3)\Omega_2\Big)\varrho - Z(\Omega_1, \Omega_2)Q(\varrho, \Omega_3)\varrho \\ \Big(Q(\varrho, \Omega_3)P\Big)(\Omega_1, \Omega_2)\varrho &= Q(\varrho, \Omega_2)P(\Omega_1, \Omega_2)\varrho - P\Big(Q(\varrho, \Omega_3)\Omega_1, \Omega_2\Big)\varrho \\ &\quad -P\Big(\Omega_1, Q(\varrho, \Omega_3)\Omega_2\Big)\varrho - P(\Omega_1, \Omega_2)Q(\varrho, \Omega_3)\varrho. \end{aligned}$$

In the current article, preliminaries are presented in Section 2, and Section 3 illustrates a generalized symmetric connection for a Kenmotsu manifold. Additionaly, "the curvature tensor" and "Ricci tensor" and its scalar curvature of a Kenmotsu manifold are calculated in relation to generalized metric connection. As for Section 4, Q tensor ($\bar{Q}$), projective tensor ($\bar{P}$), concircular curvature tensor ($\bar{Z}$) and Weyl conformal curvature tensor ($\bar{C}$) are studied by using GSMC. Furthermore, we discuss the various conditions for Kenmotsu manifold.

2. Preliminaries

To practically call a differentiable manifold M of dim ($n = 2m + 1$) as a contact metric, one should consider a $(1, 1)$ tensor field, a contravariant vector field, a 1− form and a Riemannian metric g that satisfies the following conditions

$$\psi\varrho = 0, \tag{6}$$
$$\eta(\psi\Omega_1) = 0 \tag{7}$$
$$\eta(\varrho) = 1, \tag{8}$$
$$\psi^2(\Omega_1) = -\Omega_1 + \eta(\Omega_1)\varrho, \tag{9}$$
$$g(\psi\Omega_1, \psi\Omega_2) = g(\Omega_1, \Omega_2) - \eta(\Omega_1)\eta(\Omega_2), \tag{10}$$
$$g(\Omega_1, \varrho) = \eta(\Omega_1), \tag{11}$$

for $\Omega_1, \Omega_2 \in \Gamma(TM)$. When $g(\Omega_1, \psi\Omega_2) = \Phi(\Omega_1, \Omega_2)$ is written, the tensor field ϕ is viewed as an anti-symmetric $(0, 2)$ tensor field [7]. If the contact metric manifold meets the conditions

$$(\nabla_{\Omega_1}\psi)\Omega_2 = g(\psi\Omega_1, \Omega_2)\varrho - \eta(\Omega_2)\psi\Omega_1, \tag{12}$$
$$\nabla_{\Omega_1}\varrho = \Omega_1 - \eta(\Omega_1)\varrho, \tag{13}$$

then it is called a Kenmotsu manifold. [12].

In addition, Kenmotsu manifolds have the following expressions [12]:

$$(\nabla_{\Omega_1}\eta)\Omega_2 = g(\psi\Omega_1, \psi\Omega_2) \tag{14}$$
$$\begin{aligned} g(R(\Omega_1, \Omega_2)\Omega_3, \varrho) &= \eta(R(\Omega_1, \Omega_2)\Omega_3) \\ &= g(\Omega_1, \Omega_3)\eta(\Omega_2) - g(\Omega_2, \Omega_3)\eta(\Omega_1), \end{aligned} \tag{15}$$
$$R(\varrho, \Omega_1)\Omega_2 = \eta(\Omega_2)\Omega_1 - g(\Omega_1, \Omega_2)\varrho, \tag{16}$$
$$R(\Omega_1, \Omega_2)\varrho = \eta(\Omega_1)\Omega_2 - \eta(\Omega_2)\Omega_1, \tag{17}$$
$$R(\varrho, \Omega_1)\varrho = \Omega_1 - \eta(\Omega_1)\varrho, \tag{18}$$
$$S(\Omega_1, \varrho) = -(n-1)\eta(\Omega_1), \tag{19}$$
$$S(\psi\Omega_1, \psi\Omega_2) = S(\Omega_1, \Omega_2) + (n-1)\eta(\Omega_1)\eta(\Omega_2) \tag{20}$$

for $\Omega_1, \Omega_2 \in \Gamma(TM)$, where R and S can be seen as the curvature tensor and the Ricci tensor belonging to M. A Kenmotsu manifold M is found to be generalized η- Einstein when the Ricci tensor S of it is presented as follows;

$$S(\Omega_1, \Omega_2) = ag(\Omega_1, \Omega_2) + b\eta(\Omega_1)\eta(\Omega_2) + cg(\psi\Omega_1, \Omega_2), for\Omega_1, \Omega_2 \in \Gamma(TM). \tag{21}$$

a, b and c are viewed as smooth functions. In such a way that $a \neq 0$ and $b \neq 0$ in the event that $c = 0$, M is seen as η-Einstein manifold.

3. Kenmotsu Manifolds with GSMC

Suppose that $\bar{D}$ presents a linear connection and let D be a Levi-Civita connection. Then, we have

$$\bar{D}_{\Omega_1}\Omega_2 = D_{\Omega_1}\Omega_2 + H(\Omega_1, \Omega_2), \tag{22}$$

for $\Omega_1, \Omega_2 \in \Gamma(TM)$. Using (1) and (22), H is a tensor of type $(1, 2)$, we have

$$H(\Omega_1, \Omega_2) = \frac{1}{2}[T(\Omega_1, \Omega_2) + T^{'}(\Omega_1, \Omega_2) + T^{'}(\Omega_2, \Omega_1)], \tag{23}$$

and

$$g(T^{'}(\Omega_1, \Omega_2), \Omega_3) = g(T(\Omega_3, \Omega_1), \Omega_2), \tag{24}$$

where T represents the torsion tensor of $\bar{D}$ By virtue of (1) and (24), one obtains

$$T^{'}(\Omega_1, \Omega_2) = \alpha\{\eta(\Omega_1)\Omega_2 - g(\Omega_1, \Omega_2)\varrho\} + \beta\{-\eta(\Omega_1)\phi\Omega_2 - g(\psi\Omega_1, \Omega_2)\varrho\}. \tag{25}$$

Furthermore, through use of (1), (23) and above euation, we get

$$H(\Omega_1, \Omega_2) = \alpha\{\eta(\Omega_2)\Omega_1 - g(\Omega_1, \Omega_2)\varrho\} + \beta\{-\eta(\Omega_1)\psi\Omega_2\}. \tag{26}$$

As the results, the following theorem exists:

Theorem 3.1. *[5], [6] In a Kenmotsu manifold, GSMC $\bar{D}$ is assosiated to the relation*

$$\bar{D}_{\Omega_1}\Omega_2 = D_{\Omega_1}\Omega_2 + \alpha\{\eta(\Omega_2)\Omega_1 - g(\Omega_1, \Omega_2)\varrho\} - \beta\eta(\Omega_1)\psi\Omega_2. \tag{27}$$

Corollary 3.2. *In a Kenmotsu manifolds with GSMC $\bar{D}$, semi-symmetric metric and quarter symmetric metric connections are characterized by, respectively*

$$\bar{D}_{\Omega_1}\Omega_2 = D_{\Omega_1}\Omega_2 + \eta(\Omega_2)\Omega_1 - g(\Omega_1, \Omega_2)\varrho, \tag{28}$$

$$\bar{D}_{\Omega_1}\Omega_2 = D_{\Omega_1}\Omega_2 - \eta(\Omega_1)\psi\Omega_2. \tag{29}$$

By means of (27), the following proposition exists:

Proposition 3.3. *[5], [6] In a Kenmotsu manifolds with GSMC $\bar{\nabla}$, we have*

$$(\bar{D}_{\Omega_1}\psi)\Omega_2 = (\alpha + 1)\{g(\psi\Omega_1, \Omega_2)\varrho - \eta(\Omega_2)\psi\Omega_1\}, \tag{30}$$

$$\bar{D}_{\Omega_1}\varrho = (\alpha + 1)\{\Omega_1 - \eta(\Omega_1)\varrho\}, \tag{31}$$

$$(\bar{D}_{\Omega_1}\eta)\Omega_2 = (\alpha + 1)\{g(\Omega_1, \Omega_2) - \eta(\Omega_2)\eta(\Omega_1)\}, \tag{32}$$

for every $\Omega_1, \Omega_2 \in \Gamma(TM)$.

Let $\bar{R}$ be the curvature tensor corresponding to generalized metric connection $\bar{D}$, then

$$\bar{R}(\Omega_1, \Omega_2)\Omega_3 = \bar{D}_{\Omega_1}\bar{D}_{\Omega_2}\Omega_3 - \bar{D}_{\Omega_2}\bar{D}_{\Omega_1}\Omega_3 - \bar{D}_{[\Omega_1,\Omega_2]}\Omega_3. \tag{33}$$

Using (3.3), (27) and (33), we have

$$\begin{aligned}\bar{R}(\Omega_1, \Omega_2)\Omega_3 = {} & R(\Omega_1, \Omega_2)\Omega_3 + \{\alpha(\alpha + 2)\Big(g(\Omega_1, \Omega_3)\Omega_2 - g(\Omega_2, \Omega_3)\Omega_1\Big) \\ & +\alpha(\alpha + 1)\Big(\eta(\Omega_2)\eta(\Omega_3)\Omega_1 - \eta(\Omega_1)\eta(\Omega_3)\Omega_2 \\ & +g(\Omega_2, \Omega_3)\eta(\Omega_1) - g(\Omega_1, \Omega_3)\eta(\Omega_2)\Big) + \beta(\alpha + 1)[\eta(\Omega_2)\eta(\Omega_3)\psi\Omega_1 \\ & -\eta(\Omega_1)\eta(\Omega_3)\psi\Omega_2 + g(\Omega_1, \psi\Omega_3)\eta(\Omega_2)\varrho - g(\Omega_2, \psi\Omega_3)\eta(\Omega_1)\varrho \\ & = R(\Omega_1, \Omega_2)\Omega_3 + \alpha(\alpha + 2)(g(\Omega_1, \Omega_3)\Omega_2 - g(\Omega_2, \Omega_3)\Omega_1) \\ & +(1 + \alpha)\Big\{\eta(V)R(\varrho, \alpha\Omega_1 + \beta\psi\Omega_1)\Omega_3 \\ & -\eta(\Omega_1)R(\varrho, \alpha\Omega_2 + \beta\psi\Omega_2)\Omega_3\Big\},\end{aligned} \tag{34}$$

where $R(\Omega_1, \Omega_2)\Omega_3$ is a Riemannian curvature tensor corresponding to the Levi-Civita connection D. When (35) and the first Bianchi identity are paid attention, we have

$$\begin{aligned}\bar{R}(v_1v_2)\Omega_3 & + \bar{R}(\Omega_2, \Omega_3)\Omega_1 + \bar{R}(\Omega_3, \Omega_1)\Omega_2 = 2(\beta + \alpha\beta)\Big\{\eta(\Omega_1)g(\psi\Omega_2, \Omega_3) \\ & + \eta(\Omega_2)g(\Omega_1, \psi\Omega_3) + \eta(\Omega_3)g(\Omega_2, \psi\Omega_1)\Big\}.\end{aligned} \tag{35}$$

Thus, the following proposition is obtained

Proposition 3.4. *[5], [6] Let us consider that M be a Kenmotsu manifold (dim= n) together with GSMC of type* (α, β)*. When* $\alpha = -1$ *or* $\beta = 0$ *it provides the first Bianchi identity of* $\bar{D}$ *on M.*

The following proposition is presented using (15), (16), (17), (18) and (35).

Proposition 3.5. *[5], [6] When M be a Kenmotsu manifold (dim= n) with GSMC of type* (α, β)*, the obtained expressions are:*

$$\begin{aligned}
\bar{R}(\Omega_1, \Omega_2)\varrho &= (\alpha+1)\{R(\Omega_1, \Omega_2)\varrho + \beta[\eta(\Omega_2)\psi\Omega_2 - \eta(\Omega_1)\psi\Omega_2]\}, && (36)\\
\bar{R}(\varrho, \Omega_1)\Omega_2 &= (\alpha+1)\{R(\varrho, \Omega_1)\Omega_2 - \beta[\eta(\Omega_2)\psi\Omega_1 + g(\Omega_1, \psi\Omega_2)\varrho]\}, && (37)\\
\bar{R}(\varrho, \Omega_2)\varrho &= (\alpha+1)\{R(\varrho, \Omega_2)\varrho - \beta\psi\Omega_2\}, && (38)\\
\eta(\bar{R}(\Omega_1, \Omega_2)\Omega_3) &= (\alpha+1)\{\eta(\Omega_2)g(\Omega_1, \Omega_3) - \eta(\Omega_1)g(\Omega_2, \Omega_3) \\
&+ \beta[\eta(\Omega_2)g(\Omega_1, \psi\Omega_3) - \eta(\Omega_1)g(\Omega_2, \psi\Omega_3)]\} && (39)
\end{aligned}$$

$\forall \Omega_1, \Omega_2, \Omega_3 \in \Gamma(TM)$.

The Ricci tensor $\bar{S}$ and the scalar curvature $\bar{r}$ of a Kenmotsu manifold is presented with GSMC $\bar{D}$

$$\bar{S}(\Omega_1, \Omega_2) = \sum_{i=1}^{n} g(\bar{R}(e_i, \Omega_1)\Omega_2, e_i),$$

$$\bar{r} = \sum_{i=1}^{n} \bar{S}(e_i, e_i),$$

$\forall \Omega_1, \Omega_2 \in \Gamma(TM)$. Let $\{e_1, e_2, ..., e_n\}$ be an orthonormal frame. In the view of (10) and (35), the obtained equation is

$$\begin{aligned}
\bar{S}(\Omega_2, \Omega_3) &= S(\Omega_2, \Omega_3) + \{(2-n)\alpha^2 + (3-2n)\alpha\}g(\Omega_2, \Omega_3) \\
&+(n-2)(\alpha^2+\alpha)\eta(\Omega_2)\eta(\Omega_3) - (\beta + \alpha\beta)g(\Omega_2, \phi\Omega_3). && (40)
\end{aligned}$$

By the symmetric property of the Ricci tensor S, (40) gives

$$\bar{S}(\Omega_2, \Omega_3) - \bar{S}(\Omega_3, \Omega_2) = -2(\beta + \alpha\beta)g(\Omega_2, \psi\Omega_3). \tag{41}$$

Thus, the theorem below is obtained.

Theorem 3.6. *[5], [6] Let us consider that M be a Kenmotsu manifold (dim= n). The Ricci tensor* $\bar{S}$ *of GSMC* $\bar{D}$ *is symmetric if and only if* $\alpha = -1$ *or* $\beta = 0$*.*

The following theorem is enabled through use of (40)

Theorem 3.7. *[5], [6] When M is an* $n-$ *dimensional Kenmotsu manifold with GSMC of type* (α, β)*, The scalar curvature concerning the connection is*

$$\bar{r} = r + (n-2)(1-n)\alpha^2 - 2(n-1)^2\alpha. \tag{42}$$

4. Some characterizations of Kenmotsu manifold with respect to GSMC

In this section, we study the Q tensor ($\bar{Q}$), concircular curvature tensor ($\bar{Z}$) and projective tensor ($\bar{P}$) by using GSMC. Corresponding to the equations (2), (3) and (4), the Q tensor ($\bar{Q}$), projective curvature tensor ($\bar{P}$) and concircular curvature tensor ($\bar{Z}$) with respect to GSMC are defined as

$$\bar{Q}(\Omega_1, \Omega_2, \Omega_3) = \bar{R}(\Omega_1, \Omega_2, \Omega_3) - \tfrac{\lambda}{n-1}\{(\Omega_2, \Omega_3)\Omega_1 - g(\Omega_1, \Omega_3)\Omega_2\}, \tag{43}$$

$$\bar{P}(\Omega_1, \Omega_2)\Omega_3 = \bar{R}(\Omega_1, \Omega_2)\Omega_3 - \tfrac{1}{n-1}\Big\{\bar{S}(\Omega_2, \Omega_3)\Omega_1 - \bar{S}(\Omega_1, \Omega_3)\Omega_2\Big\}, \tag{44}$$

$$\bar{Z}(\Omega_1, \Omega_2)\Omega_3 = \bar{R}(\Omega_1, \Omega_2)\Omega_3 - \tfrac{\bar{r}}{n(n-1)}\Big\{g(\Omega_2, \Omega_3)\Omega_1 - g(\Omega_1, \Omega_3)\Omega_2\Big\}, \tag{45}$$

$$\begin{aligned}
\bar{C}(\Omega_1, \Omega_2)\Omega_3 = \bar{R}(\Omega_1, \Omega_2)\Omega_3 &- \tfrac{1}{n-2}\Big\{\bar{S}(\Omega_2, \Omega_3)\Omega_1 - \bar{S}(\Omega_1, \Omega_3)\Omega_2 + g(\Omega_2, \Omega_3)L\Omega_1 \\
&-g(\Omega_1, \Omega_3)L\Omega_2\Big\} + \tfrac{\bar{r}}{(n-1)(n-2)}\Big\{g(\Omega_2, \Omega_3)\Omega_1 - g(\Omega_1, \Omega_3)\Omega_2\Big\}, && (46)
\end{aligned}$$

where L is a Ricci operator such that $g(L\Omega_1, \Omega_2) = S(\Omega_1, \Omega_2)$ for all $\Omega_1, \Omega_2, \Omega_3 \in \Gamma(TM)$. By using, (36) - (38), (16) - (18) and (43), we have

Proposition 4.1. *Let (M^n, g) be a Kenmotsu manifold, then with respect to GSMC the Q tensor ($\bar{Q}$), satisfy the following relations*

$$\bar{Q}(\Omega_1, \Omega_2)\varrho = (1 + \alpha + \frac{\lambda}{n-1})R(\Omega_1, \Omega_2)\varrho + \beta(1 + \alpha)\Big(\eta(\Omega_2)\psi\Omega_1 - \eta(\Omega_1)\psi\Omega_2\Big), \tag{47}$$

$$\bar{Q}(\varrho, \Omega_1)\Omega_2 = (1 + \alpha + \frac{\lambda}{n-1})R(\varrho, \Omega_1)\Omega_2 - \beta(1 + \alpha)\Big(\eta(\Omega_2)\psi\Omega_1 + g(\Omega_1, \psi\Omega_2)\varrho\Big), \tag{48}$$

$$\bar{Q}(\varrho, \Omega_1)\varrho = (1 + \alpha + \frac{\lambda}{n-1})R(\varrho, \Omega_1)\varrho - \beta(1 + \alpha)\psi\Omega_1, \tag{49}$$

for all $\Omega_1, \Omega_2 \in \Gamma(TM)$.

Next, if we take $\bar{Q}(\varrho, \Omega_5)\bar{Q} = 0$, then

$$\begin{aligned}\Big(\bar{Q}(\varrho, \Omega_5)\bar{Q}\Big)(\Omega_1, \Omega_2)\varrho &= \bar{Q}(\varrho, \Omega_5)\bar{Q}(\Omega_1, \Omega_2)\varrho - \bar{Q}\Big(\bar{Q}(\varrho, \Omega_5)\Omega_1, \Omega_2\Big)\varrho \\ &\quad -\bar{Q}\Big(\Omega_1, \bar{Q}(\varrho, \Omega_5)\Omega_2\Big)\varrho - \bar{Q}(\Omega_1, \Omega_2)\bar{Q}(\varrho, \Omega_5)\varrho = 0,\end{aligned} \tag{50}$$

for all $\Omega_1, \Omega_2, \Omega_5 \in \Gamma(TM)$. Using equations (16) - (18) and Proposition 4.1, in above equation, we get

$$\begin{aligned}\Big(\bar{Q}(\varrho, \Omega_5)\bar{Q}\Big)(\Omega_1, \Omega_2)\varrho &= \Big(1 + \alpha + \frac{\lambda}{n-1}\Big)\Big\{(1 + \alpha)\Big(g(\Omega_5, \Omega_1)\Omega_2 \\ &\quad -g(\Omega_5, \Omega_2)\Omega_1\Big) - \bar{R}(\Omega_1.\Omega_2)\Omega_5\Big\} \\ &\quad +\beta(1 + \alpha)\Bigg\{\Big(1 + \alpha + \frac{\lambda}{n-1}\Big)\Big(g(\Omega_5, \Omega_2)\psi\Omega_1 \\ &\quad -g(\Omega_5, \Omega_1)\psi\Omega_2 + g(\Omega_5, \psi\Omega_2)\eta(\Omega_1)\varrho \\ &\quad -g(\Omega_5, \psi\Omega_1)\eta(\Omega_2)\varrho\Big) + \bar{R}(\Omega_1, \Omega_2)\psi\Omega_5 \\ &\quad -(1 + \alpha)(g(\Omega_5, \psi\Omega_2)\Omega_1 - g(\Omega_5, \psi\Omega_1)\Omega_2) \\ &\quad +\beta(1 + \alpha)\Big(\eta(\Omega_2)g(\Omega_5, \Omega_1)\varrho - \eta(\Omega_1)g(\Omega_5, \Omega_2)\varrho\Big) \\ &\quad -g(\Omega_5, \phi\Omega_1)\phi\Omega_2 + g(\Omega_5, \phi\Omega_2)\phi\Omega_1\Bigg\},\end{aligned} \tag{51}$$

where

$$\begin{aligned}\bar{R}(\Omega_1, \Omega_2)\psi\Omega_5 &= \psi R(\Omega_1, \Omega_2)\Omega_5 + g(Y, \Omega_2)\psi\Omega_5 - g(\Omega_5, \Omega_1)\psi\Omega_2 \\ &\quad +\Big(\alpha(\alpha + 2) + 1\Big)(g(\Omega_1, \psi\Omega_5)\Omega_2 - g(\Omega_2, \psi\Omega_5)\Omega_1) \\ &\quad +(1 + \alpha)\Big\{\eta(\Omega_1)g(\alpha\Omega_2 + \beta\psi\Omega_2, \psi\Omega_5)\varrho \\ &\quad -\eta(\Omega_2)g(\alpha\Omega_1 + \beta\psi\Omega_1, \psi\Omega_5)\varrho\Big\}.\end{aligned} \tag{52}$$

Let

$$\bar{K}(\Omega_1, \Omega_2) = \frac{\bar{R}(\Omega_1, \Omega_2, \Omega_1, \Omega_2)}{g(\Omega_1, \Omega_1)g(\Omega_2, \Omega_2) - |g(\Omega_1, \Omega_2)|^2}, \tag{53}$$

be the sectional curvature with respect to GSMC, then

$$\bar{K} = K + \alpha(\alpha + 2) - \alpha(\alpha + 1)\frac{|\eta(\Omega_1)|^2 g(\Omega_1, \Omega_2) + |\eta(\Omega_2)|^2 g(\Omega_1, \Omega_1) - 2\eta(\Omega_1)\eta(\Omega_2)g(\Omega_1, \Omega_2)}{g(\Omega_1, \Omega_1)g(\Omega_2, \Omega_2) - |g(\Omega_1, \Omega_2)|^2}. \tag{54}$$

From equations (51), it is easy to analyze that characterization of Kenmotsu manifold by Q tensor ($\bar{Q}$) for $\alpha \neq -1$ and $\beta \neq 0$ is not in compact form and much meaning full; hence we are only discussing the cases where ($\alpha \neq -1, \beta = 0$) and ($\alpha = -1, \beta \neq 0$).

Theorem 4.2. *The Q tensor ($\bar{Q}$) with $\alpha-$ semi-symmetric connection on Kenmotsu manifold satisfies the condition $\bar{Q}(\varrho, \Omega_5)\bar{Q} = 0$ if and only if either $\lambda = -(1+\alpha)(n-1)$ or manifold is with $\bar{K} = -(1+\alpha)$, where λ is the scalar function and $\bar{K}$ is the sectional curvature.*

Proof. Equations (51) and (53), completes the proof. □

Theorem 4.3. *Let us consider that M be a Kenmotsu manifold (dim= n) with $(-1, \beta)$ GSMC then the Q tensor ($\bar{Q}$) will satisfy the condition $\bar{Q}(\varrho, \Omega_5)\bar{Q} = 0$ if and only if curvature tensor vanishes ($\bar{R} = 0$).*

Proof. Equations (51) and (53), completes the proof. □

Theorem 4.4. *Let (M, g) be a Kenmotsu manifold with curvature tensor ($\bar{R}$) and Q tensor ($\bar{Q}$), then $\left(\bar{Q}(\varrho, \Omega_5)\bar{Q}\right)(\Omega_1, \Omega_2)\varrho = \left(\bar{Q}(\varrho, \Omega_5)\bar{R}\right)(\Omega_1, \Omega_2)\varrho \ \forall \ \Omega_1, \Omega_2, \Omega_5 \in \Gamma(TM)$.*

Proof. Using equation (43) in equation (50), we have

$$\begin{aligned} \left(\bar{Q}(\varrho, \Omega_5)\bar{Q}\right)(\Omega_1, \Omega_2)\varrho = & \left(\bar{Q}(\varrho, \Omega_5)\bar{R}\right)(\Omega_1, \Omega_2)\varrho \\ & -\frac{\lambda}{n-1}\Big\{\eta(\Omega_2)\bar{Q}(\varrho, \Omega_5)\Omega_1 - \eta(\Omega_1)\bar{Q}(\varrho, \Omega_5)\Omega_2 \\ & +g(\bar{Q}(\varrho, \Omega_5)\Omega_1, \varrho)\Omega_2 - g(\Omega_2, \varrho)\bar{Q}(\varrho, \Omega_5)\Omega_1 \\ & -g(\bar{Q}(\varrho, \Omega_5)\Omega_2, \varrho)\Omega_1 + g(\Omega_1, \varrho)\bar{Q}(\varrho, \Omega_5)\Omega_2 \\ & -g(\Omega_2, \bar{Q}(\varrho, \Omega_5)\varrho)\Omega_1 + g(\Omega_1, \bar{Q}(\varrho, \Omega_5)\varrho)\Omega_2\Big\}. \end{aligned} \tag{55}$$

The various terms of above equation, are

$$\begin{cases} \bar{Q}(\varrho, \Omega_5)\Omega_1 = \left(1+\alpha+\frac{\lambda}{n-1}\right)\left(\eta(\Omega_1)\Omega_5 - g(\Omega_5, \Omega_1)\varrho\right) - \beta(1+\alpha)\left(\eta(\Omega_1)\psi\Omega_5 + g(\Omega_5, \psi\Omega_1)\varrho\right), \\ \bar{Q}(\varrho, \Omega_5)\Omega_2 = \left(1+\alpha+\frac{\lambda}{n-1}\right)\left(\eta(\Omega_2)\Omega_5 - g(\Omega_5, \Omega_2)\varrho\right) - \beta(1+\alpha)\left(\eta(\Omega_2)\psi\Omega_5 + g(\Omega_5, \psi\Omega_2)\varrho\right), \\ g(\bar{Q}(\varrho, \Omega_5)\Omega_1, \varrho)\Omega_2 = \left(1+\alpha+\frac{\lambda}{n-1}\right)\left(\eta(\Omega_1)\eta(\Omega_5) - g(\Omega_5, \Omega_1)\right)\Omega_2 - \beta(1+\alpha)g(\Omega_5, \psi\Omega_1)\Omega_2, \\ g(\bar{Q}(\varrho, \Omega_5)\Omega_2, \varrho)\Omega_1 = \left(1+\alpha+\frac{\lambda}{n-1}\right)\left(\eta(\Omega_2)\eta(\Omega_5) - g(\Omega_5, \Omega_2)\right)\Omega_1 - \beta(1+\alpha)g(\Omega_5, \psi\Omega_2)\Omega_1, \\ g(\bar{Q}(\varrho, \Omega_5)\varrho, \Omega_2)\Omega_1 = \left(1+\alpha+\frac{\lambda}{n-1}\right)\left(g(\Omega_5, \Omega_2) - \eta(\Omega_5)\eta(\Omega_2)\right)\Omega_1 - \beta(1+\alpha)g(\psi\Omega_5, \Omega_2)\Omega_1, \\ g(\bar{Q}(\varrho, \Omega_5)\varrho, \Omega_1)\Omega_2 = \left(1+\alpha+\frac{\lambda}{n-1}\right)\left(g(\Omega_5, \Omega_1) - \eta(\Omega_5)\eta(\Omega_1)\right)\Omega_2 - \beta(1+\alpha)g(\psi\Omega_5, \Omega_1)\Omega_2. \end{cases} \tag{56}$$

Using Proposition 4.1 and set of equations (56) in equation (55), we obtain

$$\begin{aligned} & \eta(\Omega_2)\bar{Q}(\varrho, \Omega_5)\Omega_1 - \eta(\Omega_1)\bar{Q}(\varrho, \Omega_5)\Omega_1 + g(\bar{Q}(\varrho, \Omega_5)\Omega_1, \varrho)\Omega_2 - g(\Omega_2, \varrho)\bar{Q}(\varrho, \Omega_5)\Omega_1 \\ & -g(\bar{Q}(\varrho, \Omega_5)\Omega_2, \varrho)\Omega_1 + g(\Omega_1, \varrho)\bar{Q}(\varrho, \Omega_5)\Omega_2 - g(\Omega_2, \bar{Q}(\varrho, \Omega_5)\varrho)\Omega_1 + g(\Omega_1, \bar{Q}(\varrho, \Omega_5)\varrho)\Omega_2 = 0, \end{aligned}$$

hence $\left(\bar{Q}(\varrho, \Omega_5)\bar{Q}\right)(\Omega_1, \Omega_2)\varrho = \left(\bar{Q}(\varrho, \Omega_5)\bar{R}\right)(\Omega_1, \Omega_2)\varrho \ \forall \ \Omega_1, \Omega_2, \Omega_5 \in \Gamma(TM)$. □

Thus from Theorem 4.2 and Theorem 4.3, we can say that

Corollary 4.5. *The curvature tensor ($\bar{R}$) with $\alpha-$ semi-symmetric connection on Kenmotsu manifold satisfy the condition $\bar{Q}(\varrho, \Omega_5)\bar{R} = 0$ if and only if either $\lambda = -(1+\alpha)(n-1)$ or manifold is with $\bar{K} = -(1+\alpha)$, where λ is the scalar function and $\bar{K}$ is the sectional curvature.*

Remark 4.6. *[33] If $\alpha = 0$, then $\bar{Q}(\varrho, \Omega_5)\bar{R} = 0$ if and only if manifold is locally isometric to $H^n(-1)$.*

Theorem 4.7. *Let (M, g) be a Kenmotsu manifold with curvature tensor ($\bar{R}$) and Q tensor ($\bar{Q}$), then $\bar{R}.\bar{Q} = \bar{R}.\bar{R}$.*

Proof. Let $\Omega_1, \Omega_2, \Omega_3, \Omega_4$ and Ω_5 are vector fields on M, then

$$\begin{aligned}\Big(\bar{R}(\Omega_4, \Omega_5)\bar{Q}\Big)(\Omega_1, \Omega_2)\Omega_3 &= \bar{R}(\Omega_4, \Omega_5)\bar{Q}(\Omega_1, \Omega_2)\Omega_3 - \bar{Q}(\bar{R}(\Omega_4, \Omega_5)\Omega_1, \Omega_2)\Omega_3 \\ &\quad -\bar{Q}(\Omega_1, \bar{R}(\Omega_4, \Omega_5)\Omega_2)\Omega_3 - \bar{Q}(\Omega_1, \Omega_2)\bar{R}(\Omega_4, \Omega_5)\Omega_3 \\ &= \Big(\bar{R}(\Omega_4, \Omega_5)\bar{R}\Big)(\Omega_1, \Omega_2)\Omega_3 - \frac{\lambda}{n-1}\Big\{g(\bar{R}(\Omega_4, \Omega_5)\Omega_1, \Omega_3)\Omega_2 \\ &\quad -g(\bar{R}(\Omega_4, \Omega_5)\Omega_2, \Omega_3)\Omega_1 - g(\bar{R}(\Omega_4, \Omega_5)\Omega_3, \Omega_2)\Omega_1 \\ &\quad -g(\bar{R}(\Omega_4, \Omega_5)\Omega_3, \Omega_1)\Omega_2\Big\}. \end{aligned} \tag{57}$$

From equation (35), we have

$$\begin{cases} g(\bar{R}(\Omega_1, \Omega_2)\Omega_3, \Omega_5) = g(\bar{R}(\Omega_1, \Omega_2)\Omega_5, \Omega_3), \\ \bar{R}(\Omega_1, \Omega_2)\Omega_3 = -\bar{R}(\Omega_2, \Omega_1)\Omega_3. \end{cases} \tag{58}$$

Thus equations (57) and (58), implies that $\bar{R}.\bar{Q} = \bar{R}.\bar{R}$. □

Now, if $\bar{Q}(\varrho, \Omega_5)\bar{Z} = 0$, then for all $\Omega_1, \Omega_2, \Omega_5 \in \Gamma(TM)$, we have

$$\begin{aligned}\Big(\bar{Q}(\varrho, \Omega_5)\bar{Z}\Big)(\Omega_1, \Omega_2)\varrho &= \bar{Q}(\varrho, \Omega_5)\bar{Z}(\Omega_1, \Omega_2)\varrho - \bar{Z}\Big(\bar{Q}(\varrho, \Omega_5)\Omega_1, \Omega_2\Big)\varrho \\ &\quad -\bar{Z}\Big(\Omega_1, \bar{Q}(\varrho, \Omega_5)\Omega_2\Big)\varrho - \bar{Z}(\Omega_1, \Omega_2)\bar{Q}(\varrho, \Omega_5)\varrho = 0. \end{aligned} \tag{59}$$

Using equations (16) - (18) and Proposition 4.1 in equation (60), we have

$$\begin{aligned}\Big(\bar{Q}(\varrho, \Omega_5)\bar{Z}\Big)(\Omega_1, \Omega_2)\varrho &= \Big(1 + \alpha + \frac{\lambda}{n-1}\Big)\Big\{(1+\alpha)\Big(g(\Omega_5, \Omega_1)\Omega_2 \\ &\quad -g(\Omega_5, \Omega_2)\Omega_1\Big) - \bar{R}(\Omega_1 . \Omega_2)\Omega_5\Big\} \\ &\quad +\beta(1+\alpha)\Big\{\Big(1 + \alpha + \frac{\lambda}{n-1}\Big)\Big(g(\Omega_5, \Omega_2)\psi\Omega_1 - g(\Omega_5, \Omega_1)\psi\Omega_2\Big) \\ &\quad +\Big(1 + \alpha + \frac{\bar{r}}{n-1}\Big)\Big(g(\Omega_5, \psi\Omega_2)\eta(\Omega_1)\varrho - g(\Omega_5, \psi\Omega_1)\eta(\Omega_2)\varrho\Big) \\ &\quad +(1+\alpha)(g(\Omega_5, \psi\Omega_1)\Omega_2 - g(\Omega_5, \psi\Omega_2)\Omega_1) + \bar{R}(\Omega_1, \Omega_2)\psi\Omega_5 \\ &\quad +\beta(1+\alpha)\Big(\eta(\Omega_2)g(\Omega_5, \Omega_1)\varrho - \eta(\Omega_1)g(\Omega_5, \Omega_2)\varrho\Big) \\ &\quad -g(\Omega_5, \phi\Omega_1)\phi\Omega_2 + g(\Omega_5, \phi\Omega_2)\phi\Omega_1\Big\}. \end{aligned} \tag{60}$$

Similarly, for projective curvature tensor ($\bar{P}$), if $\bar{Q}(\varrho, \Omega_5)\bar{P} = 0$, then

$$\Big(\bar{Q}(\varrho,\Omega_5)\bar{P}\Big)(\Omega_1,\Omega_2)\varrho = \bar{Q}(\varrho,\Omega_2)\bar{P}(\Omega_1,\Omega_2)\varrho - \bar{P}\Big(\bar{Q}(\varrho,\Omega_5)\Omega_1,\Omega_2\Big)\varrho - \bar{P}\Big(\Omega_1,\bar{Q}(\varrho,\Omega_5)\Omega_2\Big)\varrho - \bar{P}(\Omega_1,\Omega_2)\bar{Q}(\varrho,\Omega_5)\varrho = 0. \tag{61}$$

Using equations (16) - (18) and Proposition 4.1 in equation (62), we have

$$\begin{aligned}\Big(\bar{Q}(\varrho,\Omega_5)\bar{P}\Big)(\Omega_1,\Omega_2)\varrho &= \beta(1+\alpha)\bigg\{\Big(1+\alpha+\frac{\lambda}{n-1}\Big)\Big(g(\Omega_5,\Omega_2)\psi\Omega_1 \\ &-g(\Omega_5,\Omega_1)\psi\Omega_2 + g(\Omega_5,\psi\Omega_2)\eta(\Omega_1)\varrho \\ &-g(\Omega_5,\psi\Omega_1)\eta(\Omega_2)\varrho\Big) + \bar{P}(\Omega_1,\Omega_2)\psi\Omega_5 \\ &+\beta(1+\alpha)\Big(\eta(\Omega_2)g(\Omega_5,\Omega_1)\varrho - \eta(\Omega_1)g(\Omega_5,\Omega_2)\varrho\Big) \\ &-g(\Omega_5,\phi\Omega_1)\phi\Omega_2 + g(\Omega_5,\phi\Omega_2)\phi\Omega_1\bigg\} \\ &+\Big(1+\alpha+\frac{\lambda}{n-1}\Big)\bar{P}(\Omega_1,\Omega_2)\Omega_5.\end{aligned} \tag{62}$$

So from equations (60) and (62), for $\alpha-$ semi-symmetric connection and $(1,\beta)$ type GSMC on Kenmotsu manifold we obtain the following results.

Theorem 4.8. *The concircular curvature tensor ($\bar{Z}$) on Kenmotsu manifold M, with $\alpha-$ semi-symmetric connection holds the condition $\bar{Q}(\varrho,\Omega_5)\bar{Z}=0$ if and only if either $\lambda=-(1+\alpha)(n-1)$ or manifold is with $\bar{K}=-(1+\alpha)$, where λ is the scalar function and $\bar{K}$ is the sectional curvature..*

Corollary 4.9. *The concircular curvature tensor ($\bar{Z}$) on Kenmotsu manifold M, with $(-1,\beta)$ type GSMC satisfy $\bar{Q}(\varrho,\Omega_5)\bar{Z}=0$ if and only if curvature tensor is zero ($\bar{R}=0$).*

Theorem 4.10. *The projective curvature tensor ($\bar{P}$) on Kenmotsu manifold M, with $\alpha-$ semi-symmetric connection holds the condition $\bar{Q}(\varrho,\Omega_5)\bar{P}=0$ if and only if either $\lambda=-(1+\alpha)(n-1)$ or $\bar{P}(\Omega_1,\Omega_2)\Omega_5=0$,*

where λ is a scalar function.

Corollary 4.11. *The projective curvature tensor ($\bar{P}$) on Kenmotsu manifold M,with $(-1,\beta)$ type GSMC holds $\bar{Q}(\varrho,\Omega_5)\bar{P}=0$ if and only if projective tensor is zero $\bar{P}(\Omega_1,\Omega_2)\Omega_5=0$.*

Let the Q tensor ($\bar{Q}$) and Weyl conformal curvature tensor ($\bar{C}$) on Kenmotsu manifold corresponding to GSMC satisfying the condition $\bar{Q}(\varrho,\Omega_5)\bar{C}=0$, then

$$\Big(\bar{Q}(\varrho,\Omega_5)\bar{C}\Big)(\Omega_1,\Omega_2)\varrho = \bar{Q}(\varrho,\Omega_5)\bar{C}(\Omega_1,\Omega_2)\varrho - \bar{C}\Big(\bar{Q}(\varrho,\Omega_5)\Omega_1,\Omega_2\Big)\varrho - \bar{C}\Big(\Omega_1,\bar{Q}(\varrho,\Omega_5)\Omega_2\Big)\varrho - \bar{C}(\Omega_1,\Omega_2)\bar{Q}(\varrho,\Omega_5)\varrho = 0. \tag{63}$$

From equations (16) - (18), (64) and Proposition 4.1, we obtain

$$\begin{aligned}\Big(\bar{Q}(\varrho,\Omega_5)\bar{C}\Big)(\Omega_1,\Omega_2)\varrho &= \Big(1+\alpha+\frac{\lambda}{n-1}\Big)\Big\{\eta(\bar{C}(\Omega_1,\Omega_2)\varrho)\Omega_5 - g(\Omega_5,\bar{C}(\Omega_1,\Omega_2)\varrho)\varrho \\ &-\eta(\Omega_1)\bar{C}(\Omega_5,\Omega_2)\varrho + g(\Omega_5,\Omega_1)\bar{C}(\varrho,\Omega_2)\varrho - \eta(\Omega_2)\bar{C}(\Omega_1,\Omega_5)\varrho \\ &+g(\Omega_5,\Omega_2)\bar{C}(\Omega_1,\varrho)\varrho - \bar{C}(\Omega_1,\Omega_2)\Omega_5 + \eta(\Omega_5)\bar{C}(\Omega_1,\Omega_2)\varrho\Big\} \\ &-\beta(1+\alpha)\bigg\{\eta(\bar{C}(\Omega_1,\Omega_2)\varrho)\psi\Omega_5 - g(\psi\Omega_5,\bar{C}(\Omega_1,\Omega_2)\varrho)\varrho \\ &-\eta(\Omega_1)\bar{C}(\psi\Omega_5,\Omega_2)\varrho + g(\psi\Omega_5,\Omega_1)\bar{C}(\varrho,\Omega_2)\varrho - \eta(\Omega_2)\bar{C}(\Omega_1,\psi\Omega_5)\varrho \\ &+g(\psi\Omega_5,\Omega_2)\bar{C}(\Omega_1,\varrho)\varrho - \bar{C}(\Omega_1,\Omega_2)\psi\Omega_5\bigg\}.\end{aligned} \tag{64}$$

Taking the inner product of equation (64) with any vector field $G \in \Gamma(TM)$, we have

$$\begin{aligned}
&\Big(1+\alpha+\frac{\lambda}{n-1}\Big)\Big\{\eta(\bar{C}(\Omega_1,\Omega_2)\varrho)g(\Omega_5,G)+\eta(\bar{C}(\Omega_1,\Omega_2)\Omega_5)\eta(G)\\
&+\eta(\Omega_1)\eta(\bar{C}(\Omega_5,\Omega_2)G)-g(\Omega_5,\Omega_1)\eta(\bar{C}(\varrho,\Omega_2)G)+\eta(\Omega_2)\eta(\bar{C}(\Omega_1,\Omega_5)G)\\
&-g(\Omega_5,\Omega_2)\eta(\bar{C}(\Omega_1,\varrho)G)-\bar{C}(\Omega_1,\Omega_2,\Omega_5,G)-\eta(\Omega_5)\eta(\bar{C}(\Omega_1,\Omega_2)G)\Big\}\\
&-\beta(1+\alpha)\Big\{\eta(\bar{C}(\Omega_1,\Omega_2)\varrho)g(\psi\Omega_5,\Omega_2)+\eta(G)\eta(\bar{C}(\Omega_1,\Omega_2)\psi\Omega_5)\\
&+\eta(\Omega_1)\eta(\bar{C}(\psi\Omega_5,\Omega_2)G)-g(\psi\Omega_5,\Omega_1)\eta(\bar{C}(\varrho,\Omega_2)G)+\eta(\Omega_2)\eta(\bar{C}(\Omega_1,\psi\Omega_5)G)\\
&+g(\psi\Omega_5,\Omega_2)\eta(\bar{C}(\Omega_1,\varrho)G)-\bar{C}(\Omega_1,\Omega_2,\psi\Omega_5,G)\Big\}=0. \qquad (65)
\end{aligned}$$

Since the inner product of equation (5) with ϱ, yields

$$\begin{aligned}
\bar{C}(\Omega_1,\Omega_2,\Omega_3,\varrho) &= (1+\alpha)\Big(\frac{1}{n-2}+\frac{\bar{r}}{(n-1)(n-2)}\Big)\Big(\eta(\Omega_1)g(\Omega_2,\Omega_3)-\eta(\Omega_2)g(\Omega_1,\Omega_3)\Big)\\
&-\frac{1}{n-2}\Big(\bar{S}(\Omega_2,\Omega_3)\eta(\Omega_1)-\bar{S}(\Omega_1,\Omega_3)\eta(\Omega_2)\Big)-\beta(1+\alpha)\Big(\eta(\Omega_2)g(\psi\Omega_1,\Omega_3)\\
&\eta(\Omega_1)g(\psi\Omega_2,\Omega_3)\Big). \qquad (66)
\end{aligned}$$

So, using the equation (66) in (65), we obtain

$$\begin{aligned}
&\Big(1+\alpha+\frac{\lambda}{n-1}\Big)\Big\{\bar{C}(\Omega_1,\Omega_2,G,\Omega_5)+(1+\alpha)\Big(\frac{1}{n-2}+\frac{\bar{r}}{(n-1)(n-2)}\Big)\Big(g(\Omega_5,\Omega_2)g(\Omega_1,G)\\
&-g(\Omega_5,\Omega_1)g(\Omega_2,G)\Big)+\frac{1}{n-2}\Big(\bar{S}(\Omega_5,G)g(\Omega_5,\Omega_1)-\bar{S}(\Omega_1,G)g(\Omega_5,\Omega_2)+\bar{S}(\Omega_1,\Omega_5)\eta(\Omega_2)\eta(G)\\
&-\bar{S}(\Omega_2,\Omega_5)\eta(\Omega_1)\eta(G)\Big)+(1+\alpha)\frac{n-1}{n-2}\Big(\eta(\Omega_2)\eta(G)g(\Omega_5,\Omega_1)-\eta(\Omega_1)\eta(G)g(\Omega_5,\Omega_2)\Big)\\
&-\beta(1+\alpha)\Big(\eta(\Omega_2)\eta(G)g(\psi\Omega_1,\Omega_5)-\eta(\Omega_1)\eta(G)g(\psi\Omega_2,\Omega_5)+g(\psi\Omega_2,G)g(\Omega_5,\Omega_1)\\
&-g(\psi\Omega_1,G)g(\Omega_5,\Omega_2)\Big)\Big\}-\beta\Big(1+\alpha\Big)\Big\{\bar{C}(\Omega_1,\Omega_2,G,\psi\Omega_5)+\frac{1}{n-2}\Big(\bar{S}(\Omega_2,G)g(\psi\Omega_5,\Omega_1)\\
&-\bar{S}(\Omega_1,G)g(\psi\Omega_5,\Omega_2)\Big)+(1+\alpha)\Big(\frac{1}{n-2}+\frac{\bar{r}}{(n-1)(n-2)}\Big)\Big(g(\psi\Omega_5,\Omega_2)g(\Omega_1,G)\\
&-g(\psi\Omega_5,\Omega_1)g(\Omega_2,G)\Big)+\frac{1}{n-2}\Big(\bar{S}(\Omega_1,\psi\Omega_5)\eta(\Omega_2)\eta(G)-\bar{S}(\Omega_2,\psi\Omega_5)\eta(\Omega_1)\eta(G)\Big)\\
&+(1+\alpha)\frac{n-1}{n-2}\Big(\eta(\Omega_2)\eta(G)g(\psi\Omega_5,\Omega_1)-\eta(\Omega_1)\eta(G)g(\psi\Omega_5,\Omega_2)\Big)\\
&-\beta(1+\alpha)\Big(\eta(\Omega_2)\eta(G)g(\Omega_1,\Omega_5)-\eta(\Omega_1)\eta(G)g(\Omega_2,\Omega_5)+g(\psi\Omega_2,G)g(\psi\Omega_5,\Omega_1)\\
&-g(\psi\Omega_1,G)g(\psi\Omega_5,\Omega_2)\Big)\Big\}=0. \qquad (67)
\end{aligned}$$

Let $\{e_j; j = 1, 2 \ldots n\}$ be an orthonormal frame on tangent bundle of Kenmotsu manifold, then taking

$\Omega_1 = \Omega_5 = e_j$ in equation (67) and taking sum over j, for $1 \leq j \leq n$, we get

$$\Big(1+\alpha+\frac{\lambda}{n-1}\Big)\Big\{-\frac{n-1}{n-2}\bar{S}(\Omega_2,G)+(1+\alpha)\Big(\frac{n-1}{n-2}+\frac{\bar{r}}{n-2}\Big)g(\Omega_2,G)$$
$$-\frac{1}{n-2}\Big(\bar{r}+(1+\alpha)\frac{n(n-1)}{n-2}\Big)\eta(\Omega_2)\eta(G)+\beta(1+\alpha)(n-2)g(\psi\Omega_2,G)\Big\}$$
$$-\beta\Big(1+\alpha\Big)\Big\{-\Big(\frac{n-3}{n-2}\Big)\bar{S}(\Omega_2,\psi G)+\Big((3-n)\alpha^2+(5-2n)\alpha-(n-2)+\frac{1+\alpha}{n-2}$$
$$+\frac{\bar{r}}{(n-1)(n-2)}\alpha\Big)g(\Omega_2,\psi G)-\frac{1}{n-2}\Big(\bar{S}(e_j,\psi e_j)-2\beta(1+\alpha)(n-2)\Big)g(\Omega_2,G)$$
$$+\frac{1}{n-2}\Big(\bar{S}(e_j,\psi e_j)-2\beta(1+\alpha)(n-2)\Big)\eta(\Omega_2)\eta(G)\Big\}=0. \tag{68}$$

Theorem 4.12. *Let (M,g) be a Kenmotsu manifold with $\alpha-$ semi-symmetric metric connection. Then Q tensor ($\bar{Q}$) of (M,g) satisfies $\bar{Q}(\varrho,\Omega_5)\bar{C}=0$ if and only if the manifold is generalized η Einstein or $\lambda=-(1+\alpha)(n-1)$.*

Proof. Taking $\beta = 0$ in equation (68), we have

$$\Big(1+\alpha+\frac{\lambda}{n-1}\Big)\Big\{-\frac{n-1}{n-2}\bar{S}(\Omega_2,G)+(1+\alpha)\Big(\frac{n-1}{n-2}+\frac{\bar{r}}{n-2}\Big)g(\Omega_2,G)$$
$$-\frac{1}{n-2}\Big(\bar{r}+(1+\alpha)\frac{n(n-1)}{n-2}\Big)\eta(\Omega_2)\eta(G)\Big\}=0. \tag{69}$$

Therefore from equations (69), we have either $\lambda = -(1+\alpha)(n-1)$ or

$$\bar{S}(\Omega_2,G)=(1+\alpha)\Big(\frac{\bar{r}+n-1}{n-1}\Big)g(\Omega_2,G)-\frac{1}{n-1}\Big(\bar{r}+(1+\alpha)\frac{n(n-1)}{n-2}\Big)\eta(\Omega_2)\eta(G).$$

Now using equation (40) in above equation, we obtain

$$S(\Omega_2,G)=\Big((1+\alpha)\Big(\frac{\bar{r}+n-1}{n-1}\Big)-\{(2-n)\alpha^2+(3-2n)\alpha\}\Big)g(\Omega_2,G)$$
$$-\Big((n-2)(\alpha^2+\alpha)+\frac{1}{n-1}\Big[\bar{r}+(1+\alpha)\frac{n(n-1)}{n-2}\Big]\Big)\eta(\Omega_2)\eta(G). \tag{70}$$

Comparing equation (70) and (21), we get the required result. □

Corollary 4.13. *Let (M,g) be a Kenmotsu manifold with a GSMC of type $(1,\beta)$. Then Q tensor ($\bar{Q}$) of (M,g) satisfies $\bar{Q}(\varrho,\Omega_5)\bar{C}=0$ if and only if the manifold is generalized η Einstein.*

Proof. Taking $\alpha = -1$ and $\beta = 0$ in equation (68) and then using (40), provides the required result. □

5. Example

Let $M = \{(x_1,x_2,x_3) \in R^3\}$ be a 3− dimensional Riemannian manifold with Riemannian metric $g = dx_1^2 + e^{2x_1}(dx_2^2 + dx_3^2)$, then the vector fields in the term of partial differential equations

$$e_1=e^{-x_1}\frac{\partial}{\partial x_3},\ e_2=e^{-x_1}\frac{\partial}{\partial x_2},\ e_3=\frac{\partial}{\partial x_1}=\varrho, \tag{71}$$

are linearly independent and making basis for global frame of $\Gamma(TM)$. Now considering a 1-form η (dx_1) and $(1,1)$-tensor field ψ, such that $\eta(\Omega_1) = g(\Omega_1, e_3)$ for every $\Omega_1 \in \Gamma(TM)$, and $\psi e_1 = e_2, \psi e_2 = -e_1$ and $\psi e_3 = 0$. Also, from equation (71),

$$[e_1, e_2] = 0, \qquad [e_1, e_3] = e_1, \qquad [e_2, e_3] = e_2. \tag{72}$$

The following equations are obtained using equation (72) in Koszul's formula.

$$\begin{aligned} &D_{e_1}e_1 = -e_3, && D_{e_1}e_2 = 0. && D_{e_1}e_3 = e_1, \\ &D_{e_2}e_1 = 0, && D_{e_2}e_2 = -e_3, && D_{e_2}e_3 = e_2, \\ &D_{e_3}e_1 = 0, && D_{e_3}e_2 = 0, && D_{e_3}e_3 = 0, \end{aligned} \tag{73}$$

Since M is satisfying the conditions

$$\eta(e_3) = 1, \ \psi^2 Z = -Z + \eta(Z)e_3,$$

$$g(\psi Z, \psi\Omega_3) = g(Z, \Omega_3) - \eta(Z)\eta(\Omega_3),$$

$$(D_{\Omega_1}\psi)\Omega_2 = g(\psi\Omega_1, \Omega_2)\varrho - \eta(\Omega_2)\psi\Omega_1,$$

$$D_{\Omega_1}e_3 = \Omega_1 - \eta(\Omega_1)\varrho,$$

for all $\Omega_1, \Omega_2, Z, \Omega_3 \in \Gamma(TM)$ and $e_3 = \varrho$. So, (M, g) is a Kenmotsu manifold. Using equation (72), the components of the Riemann curvature tensor are

$$\begin{aligned} &R(e_1, e_2)e_1 = e_2, \quad R(e_1, e_2)e_2 = -e_1, \quad R(e_1, e_3)e_1 = e_3, \\ &R(e_1, e_3)e_3 = -e_1, \quad R(e_2, e_3)e_2 = e_3, \quad R(e_2, e_3)e_3 = -e_2, \\ &R(e_1, e_2)e_3 = R(e_1, e_3)e_2 = R(e_2, e_3)e_1 = 0. \end{aligned} \tag{74}$$

The sectional curvature for various planes spanned by orthonormal frame fields (e_1, e_2, e_3) and the components of Ricci tensor, are given by

$$K(e_1, e_2) = K(e_1, e_3) = K(e_2, e_3) = 1, \tag{75}$$

and,

$$\begin{cases} S(e_1, e_1) = -2, \ S(e_2, e_2) = -2, \ S(e_3, e_3) = -2, \\ S(e_1, e_2) = S(e_1, e_3) = S(e_2, e_3) = 0. \end{cases} \tag{76}$$

The covariant derivative with respect to GSMC $\bar{D}$ for orthonormal fields e_1, e_2 and e_3, by using equations (73) and (27), are

$$\begin{aligned} &\bar{D}_{e_1}e_1 = -(1+\alpha)e_3, && \bar{D}_{e_1}e_2 = 0, && \bar{D}_{e_1}e_3 = (1+\alpha)e_1, \\ &\bar{D}_{e_2}e_1 = 0, && \bar{D}_{e_2}e_2 = -(1+\alpha)e_3, && \bar{D}_{e_2}e_3 = \alpha e_2, \\ &\bar{D}_{e_3}e_1 = -\beta e_2, && \bar{D}_{e_3}e_2 = \beta e_1, && \bar{D}_{e_3}e_3 = 0. \end{aligned} \tag{77}$$

From above results, the following expressions are obtained.

$$\begin{aligned} &\bar{R}(e_1, e_2)e_1 = (1+\alpha)^2 e_2, && \bar{R}(e_1, e_2)e_2 = -(1+\alpha)^2 e_1, \\ &\bar{R}(e_1, e_3)e_1 = (1+\alpha)e_3, && \bar{R}(e_1, e_3)e_3 = (1+\alpha)(\beta e_2 - e_1), \\ &\bar{R}(e_2, e_3)e_2 = (1+\alpha)e_3, && \bar{R}(e_2, e_3)e_3 = -(1+\alpha)(-\beta e_1 + e_2)\,, \\ &\bar{R}(e_3, e_2)e_1 = -(1+\alpha)\beta e_3, && \bar{R}(e_3, e_1)e_2 = (1+\alpha)\beta e_3. \end{aligned} \tag{78}$$

The sectional curvature ($\bar{K}$) and the non zero components of Ricci tensor ($\bar{S}$) by using (78), are respectively

$$\bar{K}(e_1, e_2) = (1+\alpha)^2, \ \bar{K}(e_1, e_3) = (1+\alpha) = \bar{K}(e_2, e_3), \tag{79}$$

and,

$$\bar{S}(e_1,e_1) = -(1+\alpha)(2+\alpha), \qquad \bar{S}(e_2,e_2) = -(1+\alpha)(2+\alpha),$$
$$\bar{S}(e_3,e_3) = -2(1+\alpha), \qquad \bar{S}(e_2,e_1) = -(1+\alpha)\beta. \tag{80}$$

Equations (78), (79) and (80), can be verified by the equations (34), (54) and (40). Further, the scalar curvature corresponding to D ($r = -6$) and generalized-metric connection ($\bar{r} = -2(1+\alpha)(3+\alpha)$), verifies our Theorem 3.6 and Theorem 3.7.

From above results, the non zero components of the Q tensor ($\bar{Q}$) and Weyl conformal curvature tensor, are given by

$$\begin{cases} \bar{Q}(e_1,e_2)e_1 = \left((1+\alpha)^2 + \frac{\lambda}{2}\right)e_2, \\ \bar{Q}(e_1,e_2)e_2 = -\left((1+\alpha)^2 + \frac{\lambda}{2}\right)e_1, \\ \bar{Q}(e_1,e_3)e_1 = \left((1+\alpha) + \frac{\lambda}{2}\right)e_3, \\ \bar{Q}(e_1,e_3)e_3 = -\left((1+\alpha) + \frac{\lambda}{2}\right)e_1 + \beta(1+\alpha)e_2, \\ \bar{Q}(e_2,e_3)e_2 = \left((1+\alpha) + \frac{\lambda}{2}\right)e_3, \\ \bar{Q}(e_2,e_3)e_3 = \beta(1+\alpha)e_1 - \left((1+\alpha) + \frac{\lambda}{2}\right)e_2), \\ \bar{Q}(e_3,e_1)e_2 = -\beta(1+\alpha)e_3, \\ \bar{Q}(e_3,e_1)e_2 = \beta(1+\alpha)e_3, \end{cases} \tag{81}$$

and,

$$\begin{cases} \bar{C}(e_1,e_2)e_1 = \beta(1+\alpha)e_1 + (1+\alpha)(2+\alpha)e_2 + Le_2, \\ \bar{C}(e_1,e_2)e_2 = -(1+\alpha)(2+\alpha)e_1 - \beta(1+\alpha)e_2 - Le_1, \\ \bar{C}(e_1,e_3)e_1 = 2(1+\alpha)e_3 + Le_3, \\ \bar{C}(e_1,e_3)e_3 = -\beta(1+\alpha)(2+\alpha)e_1 + \beta(1+\alpha)e_2 + Le_1, \\ \bar{C}(e_2,e_3)e_2 = 2(1+\alpha)e_3 + Le_3, \\ \bar{C}(e_2,e_3)e_3 = \beta(1+\alpha)e_1 - \beta(1+\alpha)(2+\alpha)e_2 - Le_2. \end{cases} \tag{82}$$

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Author contribution

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