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Personalized individual semantics derived consensus model in hesitant fuzzy linguistic MCGDM based on discrimination degrees and multidimensional preferences

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Abstract. In multi-criteria group decision making (MCGDM) with qualitative settings, hesitant fuzzy linguistic term sets (HFLTSs) provide a flexible way to capture decision makers (DMs)' hesitancy when eliciting linguistic expressions. However, existing studies overlook the fact that words mean different things to different people, which entails that DMs have personalized individual semantics (PISs) in terms of their expressions in linguistic MCGDM. This study develops a novel framework to address hesitant fuzzy linguistic MCGDM considering PISs of DMs. First, the concept of discrimination measure for DMs is defined. Based on the discrimination measure, a discrimination-based optimization model and a multidimensional preference-based optimization model are established to derive personalized numerical scales (NSs) of linguistic terms for DMs in different situations. Second, a consensus-reaching method based on an optimization model that aims to minimize the amount of adjustments between the original and updated linguistic decision matrices and to preserve their accuracy is constructed to yield a consensual solution. Finally, an illustrative example followed by comparative and sensitivity analysis is presented to demonstrate the application and features of the proposed framework in this study.

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Table 1: Abbreviations used in this study.

Abbreviation	Meaning
MCGDM	Multi-criteria group decision making
HFLTSs	Hesitant fuzzy linguistic term sets
DMs	Decision makers
PISs	Personalized individual semantics
GDM	Group decision making
CW	Computing with words
NS	Numerical scale
CRP	Consensus reaching process
PNSs	Personalized numerical scales
LPRs	Linguistic preference relations
LTS	Linguistic term set
NS	Numerical scale
LDAs	Linguistic distribution assessments
CLEs	Comparative linguistic expressions
HFLWA	Hesitant fuzzy linguistic weighted average
EV	Expected value

1. Introduction

Generally, decision makers (DMs) would like to express their ideas using linguistic descriptions due to the thinking and reasoning habit of human. For the sake of enriching the flexibility of linguistic evaluations in different group decision making (GDM) contexts, several famous decision-making frameworks on the basis of linguistic methods have been proposed for computing with words (CW), such as 2-tuple linguistic model (Herrera and Martínez, 2000), numerical scale (NS) model (Dong et al., 2009), linguistic calculation model based on discrete fuzzy numbers (Massanet et al., 2014), cloud-based model (Ji et al., 2024; Nie et al., 2022), and so on. Although these models have been successful in managing one particular kind of decision-making problem, there exists some limitations. In these linguistic models, DMs generally express evaluations with single linguistic terms or interval linguistic variables. However, DMs may hesitant to elicit ratings among several linguistic terms. Rodríguez et al. (2012) pioneered the concept of hesitant fuzzy linguistic term sets (HFLTSs), which consist of a set of consecutive linguistic terms. This new methodology enriches the flexibility of linguistic elicitation based on the context-free grammar and offers a novel tool for CW (Wang et al., 2018).

Recently, various methods have been developed for computing with HFLTSs, which can be roughly divided into two main groups (Wang et al., 2018). The first group considers computing with envelopes, which has to convert HFLTSs into envelopes or their extensions. For example, Rodríguez et al. (2012) proposed two symbolic aggregation operators from the pessimistic and optimistic perspective to fuse the set of envelopes of HFLTSs. Zhang et al. (2018) improved the distance measure of HFLTSs by including the width of envelope of each HFLTS. Song et al. (2024) converted HFLTSs into trapezoidal fuzzy envelopes and embedded them into an α-level-based fuzzy data development analysis cross-efficiency model for group decision-making. The second category enables computing with possible terms, which tries to consider all possible linguistic terms involved in an HFLTS. For example, Gou and Xu (2016) introduced several novel operational rules for HFLTSs on the basis of a pair of translation functions. Chen et al. (2021) introduced an N-two-stage aggregation framework for coping with the possibility distributions of HFLTSs. Chitiva-Enciso et al. (2024) directly operated on all possible terms within HFLTSs to aggregate linguistic evaluations and prioritize alternatives. Moreover, several information measures of HFLTSs have developed by different manners, including distance measures (Zhang and Dai, 2023), correlation coefficients (Liao et al., 2015), entropy measures (Yang et al., 2023), and so on. For more outcomes on the current development of HFLTSs, one can refer to Refs. (Wang et al., 2018; Zhang et al., 2024; Zhang et al., 2025).

Although HFLTS effectively expands the expressive capabilities of linguistic assessment, it reveals deeper semantic challenges in complex multi-criteria group decision making (MCGDM) practices. When dealing with linguistic GDM problems, it is argued and accepted that words mean different things for various people in CW (Mendel et al., 2010), which implies the necessity of studying the issue of personalized individual semantics (PISs). For example, two referees are invited to review a manuscript. The two referees make comments for this manuscript with "good". However, the linguistic rating "good" may have different numerical semantics for them. Recently, various attempts have been made to address this issue, which can be roughly categorized into three groups, including type-2 fuzzy set model (Mendel, 2019; Mendel and Wu, 2010), multi-granular linguistic models (Jiang et al., 2024; Yu et al., 2021), and consistency-driven models (Li et al., 2022; Li et al., 2017; Li et al., 2024; Zhang et al., 2020; Zhang et al., 2025; Zhang and Li, 2022). Given its ability of explicitly estimating personalized numerical scales (PNSs), the consistency-driven model has gained widespread adoption, whose existing works have largely been confined to linguistic preference relations (LPRs) or their extended forms. For example, Li et al. (2018) developed a consistency-driven methodology to assign PNSs based on hesitant fuzzy LPRs. Zhang et al. (2021) established a consistency-driven model to obtain PNSs, followed by an optimization-based model to improve the group consensus level within the context of comparative LPRs with self-confidence. Tang et al. (2020) constructed two consistency and consensus models to assign PNSs and support linguistic GDM with distribution LPRs. Recently, Tian et al. (2025) extended the PIS model from the traditional LPRs framework to a multi-criteria group decision matrix based on linguistic distribution assessments (LDAs). Li and Zhang (2025) proposed a multi-criteria decision method for modeling PIS under incomplete linguistic preference relations (ILPRs), further extending PIS research to multi-criteria decision scenarios. However, the aforementioned studies have unexplored the GDM problems presented in the form of multi-criteria decision matrix using HFLTSs.

Simultaneously, the consensus reaching process (CRP) is the core component of GDM, it is required to achieve a consensus-based decision and various consensus frameworks have been presented to help DMs obtain consensual solutions in different GDM situations (Gai et al., 2024; Herrera-Viedma et al., 2014; Li and Zhang, 2024; Tang et al., 2025; Tian et al., 2026; Wu et al., 2022). Accordingly, considerable studies have focused on the issue of consensus in hesitant fuzzy linguistic MCGDM problems, where a group of DMs are involved to assess a collection of alternatives with regard to a set of criteria (Kabak and Ervural, 2017; Wang et al., 2018). Wu and Xu (2016) developed a novel consensus-reaching model based on an interaction algorithm, in which a possibility distribution-based representing method is employed to describe the linguistic preferences in HFLTSs. Liu et al. (2021) investigated a new consensus-supporting model on the basis of an iteration algorithm, where the D-S evidence theory is extended to aggregate group members' hesitant fuzzy linguistic evaluation information. Different from the aforementioned interaction or iteration-based models, several optimization-based models for supporting consensus were established. Zhang et al. (2018) and Yu et al. (2021) established minimum adjustment-based models to help DMs achieve consensus in MCGDM in the context of HFLTSs and multi-solutions preserving DMs' original opinions to the maximum extent (Zhang et al., 2020). Recently, Xiao et al. (2023) proposed an optimization-based consensus model that transforms comparative linguistic expressions (CLEs) into LDAs and resolves consensus issues in hesitant linguistic environments by minimizing adjustment strategies. Li et al. (2024) proposed a minimum adjustment consensus model for ordinal classification GDM considering heterogeneous preference information, which supports flexible preference expression but cannot handle evaluation information presented in the form of hesitant fuzzy linguistic decision matrices. Although great efforts have been made to offer the assistance for the CRP in hesitant fuzzy linguistic MCGDM, semantic discrepancies caused by PIS directly affect the effectiveness of consensus reached.

Overall, despite various extensions of PIS models have been developed to address linguistic GDM problems, there still exist some limitations as follows:

(1) The existing PIS models can only work well when DMs' evaluations are represented as LPRs or their extensions (Li et al., 2022; Li et al., 2017; Li et al., 2018; Li et al., 2024; Li and Zhang, 2025; Tang et al., 2020; Tian et al., 2025; Zhang et al., 2020; Zhang et al., 2021; Zhang et al., 2025; Zhang and Li, 2022). However, currently, there is little research paying attention to modeling PISs in linguistic GDM problems where DMs' evaluations are presented in forms of hesitant fuzzy linguistic MCGDM matrices.

(2) As with the minimum adjustment-based consensus models in Refs. (Li et al., 2024; Xiao et al., 2023;

Yu et al., 2021; Zhang et al., 2018), DMs usually would like to keep their original judgments as much as possible. However, the existing studies overlooked the reality that words indicate deviant meanings for various individuals. Furthermore, some consensus models (Liu et al., 2021; Wu and Xu, 2016) may fail to remain the accurate degrees of the updated judgements in MCGDM with HFLTSs.

To overcome the above limitations, this study aims to propose a novel framework for handling PISs and consensus in MCGDM with HFLTSs. The main contributions of this study are summarized as follows:

(1) Two kinds of PIS models including a discrimination-based and a multidimensional preference-based optimization models are established. The proposed framework is the first attempt to manage PISs in linguistic GDM where DMs' evaluations are presented in forms of hesitant fuzzy linguistic MCGDM matrices. The established PIS models can provide effective and alternative chances to assign PNSs of linguistic terms for DMs in different situations.

(2) An improved optimization model on the basis of the idea of minimizing the amount of adjustments is constructed to determine the updated individual evaluations. After obtaining the PNSs, an optimization model that minimizes the adjustment distance between the original and updated decision matrices is constructed to support consensus. The established consensus model not only consider the PISs of DMs, but also cautiously preserve the accurate degrees of the updated judgements through adding necessary accuracy constraints.

This paper is organized as follows. Section 2 reviews some basic concepts, including 2-tuple linguistic model, NS model, HFLTSs and consistency-driven model for managing PISs. Section 3 provides two kinds of optimization models to obtain PNSs. Section 4 introduces a consensus framework for MCGDM with HFLTSs. Section 5 presents an illustrative example, comparative analysis and sensitivity analysis to validate the application and feasibility of the proposed framework. Finally, Section 6 concludes this study.

2. Preliminaries

2.1. 2-tuple linguistic model and numerical scale model

Definition 1 (Herrera and Martínez, 2000). Let $S = \{s_0, s_1, ..., s_g\}$ be a linguistic term set (LTS) and $\beta \in [0, g]$ indicate the result of a symbolic aggregation operation. The conversion functions between 2-tuples and numerical values are defined as follows:

$$\Delta\text{: } [0, g] \to \bar{S} \text{ being } \Delta(\beta) = (s_\tau, \alpha) \text{ with} \tag{1}$$

$$\begin{cases} s_\tau, & \tau = round(\beta) \\ \alpha = \beta - \tau, & \alpha \in [-0.5, 0.5) \end{cases}. \tag{2}$$

The inverse function of Δ, $\Delta^{-1} : \bar{S} \to [0, g]$ is defined as $\Delta^{-1}(s_\tau, \alpha) = \tau + \alpha$. A computational model for linguistic 2-tuple was discussed in Ref. (Herrera and Martínez, 2000), where the corresponding negation operator is $Neg(\Delta^{-1}(s_\tau, \alpha)) = \Delta(g - \Delta^{-1}(s_\tau, \alpha))$.

Dong et al. (2009) introduced the concept of NS, which was as an extension of 2-tuple linguistic model.

Definition 2 (Dong et al., 2009). Let $S = \{s_0, s_1, ..., s_g\}$ be an LTS and R be a set of real numbers. A function $NS : S \to R$ is called a NS of S, and $NS(s_\tau)$ is the numerical index of s_τ.

Definition 3 (Dong et al., 2009). The numerical scale NS for (s_τ, α) is defined as follows:

$$NS(s_\tau, \alpha) = \begin{cases} NS(s_\tau) + \alpha \times (NS(s_{\tau+1}) - NS(s_\tau)), & \alpha \geqslant 0 \\ NS(s_\tau) + \alpha \times (NS(s_\tau) - NS(s_{\tau-1})), & \alpha < 0 \end{cases}. \tag{3}$$

If $NS(s_\tau) < NS(s_{\tau+1})$ for $\tau = 0, 1, ..., g-1$, then the numerical scale NS on S is ordered.

The inverse operator of NS is defined as (Dong et al., 2009):

$$NS^{-1} : R \to \bar{S}\text{with} \tag{4}$$

$$NS^{-1}(r) = \begin{cases} \left(s_\tau, \frac{r - NS(s_\tau)}{NS(s_{\tau+1}) - NS(s_\tau)}\right), & NS(s_\tau) < r < \frac{NS(s_\tau) + NS(s_{\tau+1})}{2} \\ \left(s_\tau, \frac{r - NS(s_\tau)}{NS(s_\tau) - NS(s_{\tau-1})}\right), & \frac{NS(s_{\tau-1}) + NS(s_\tau)}{2} \leqslant r \leqslant NS(s_\tau) \end{cases}. \tag{5}$$

2.2. HFLTSs

Definition 4 (Rodríguez et al., 2012). Let $S = \{s_0, s_1, ..., s_g\}$ be an LTS. An HFLTS $H = \{s_L, s_{L+1}, ..., s_U\}$ is an ordered finite subset of the consecutive linguistic terms of S

To effectively managing HFLTSs with different numbers of linguistic terms, Wu and Xu (2016) proposed a novel methodology on the basis of possibility distribution.

Definition 5 (Wu and Xu, 2016). Let $H = \{s_L, s_{L+1}, ..., s_U\}$ be an HFLTS on LTS $S = \{s_0, s_1, ..., s_g\}$. The possibility distribution for H on S is represented by $P = (p_0, p_1, ..., p_g)$, where p_τ is calculated as follows:

$$p_\tau = \begin{cases} 0, & \tau = 0, 1, ..., L-1 \\ \frac{1}{U-L+1}, & \tau = L, L+1, ..., U \\ 0, & \tau = U, U+1, ..., g \end{cases}, \tag{6}$$

where p_τ is the possibility degree under which the alternative has an evaluation value s_τ provided by the expert, such that $p_\tau \in [0, 1]$ and $\sum_{\tau=0}^{g} p_\tau = 1$

Definition 6 (Wu and Xu, 2016). Let $\Lambda = \{H_1, H_2, ..., H_n\}$ be a set of HFLTSs on LTS $S = \{s_0, s_1, ..., s_g\}$ and $w = (w_1, w_2, ..., w_n)^T$ be their associated weight vector, such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Each HFLTS H_j is transformed into a possibility distribution $P_j = (p_{j,0}, p_{j,1}, ..., p_{j,g})$. The hesitant fuzzy linguistic weighted average (HFLWA) operator is defined as a possibility distribution $P = (p_0, p_1, ..., p_g)$.

$$\begin{aligned} HFLWA(H_1, H_2, ..., H_n) &= HFLWA(P_1, P_2, ..., P_n) \\ &= (p_0, p_1, ..., p_g) \end{aligned}, \tag{7}$$

where $p_\tau = \sum_{j=1}^{n} w_j p_{j,\tau}$

It should be noted that the possibilities of linguistic terms may not be equal after calculation. The heterogeneous possibilities associated with linguistic terms in HFLTSs can indicate the subtle differences in expressing evaluations (Pang et al., 2016). The distance between two HFLTSs are defined based on the idea of LDAs (Zhang et al., 2014).

Definition 7 (Zhang et al., 2017). Let H_j $(j = 1, 2)$ be two HFLTSs on S. The distance between H_1 and H_2 indicated by $P_j = (p_{j,0}, p_{j,2}, ..., p_{j,g})$ $(j = 1, 2)$ is defined as follows:

$$d(H_1, H_2) = \left| \sum_{\tau=0}^{g} (p_{1,\tau} - p_{2,\tau}) NS(s_\tau) \right|, \tag{8}$$

where $d(H_1, H_2) \in [0, 1]$ and $NS(s_\tau)$ is the numerical scale as per defined in Definition 2.

Definition 8 (Wu and Xu, 2016). Let $H = \{s_L, s_{L+1}, ..., s_U\}$ be an HFLTS on LTS $S = \{s_0, s_1, ..., s_g\}$. The expected value (EV) for H denoted by $P = (p_0, p_1, ..., p_g)$ is defined as follows:

$$EV(H) = \sum_{\tau=0}^{g} NS(s_\tau) p_\tau, \tag{9}$$

where $EV(H) \in [0, 1]$ and large value of $EV(H)$ indicates high performance of H.

2.3. Consistency-based PIS method

Definition 9 (Dong et al., 2009). Let $L = [l_{ij}]_{n\times n}$ be an LPR with respect to NS on LTS S. Then, the consistency index of L with respect to NS is defined as follows:

$$CI(L) = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i<j<k}^{n} \left| NS(l_{ij}) + NS(l_{jk}) - NS(l_{ik}) - 0.5 \right|, \tag{10}$$

where $NS(l_{ij}) \in [0, 1]$ represents the numerical scale value of the linguistic term l_{ij} for all $i, j = 1, 2, ..., n$.

When deriving the set of possible PNSs for an LPR, an optimization model with the objective function of maximizing the consistency level of LPR $L^h = [l_{ij}^h]_{n\times n}$ provided by e_h is constructed as follows:

$$\begin{aligned} & Max CI(L^h) \\ & s.t.\ \Gamma_{NS}^h \end{aligned}, \tag{11}$$

where Γ^h_{NS} be the constraint set that determines the range of NSs, as described in Eq. (12).

$$NS^h(s_\tau)\begin{cases} = 0, & \tau = 0 \\ \in (\frac{\tau-1}{g}, \frac{\tau+1}{g}], & \tau = 1,2,...,g-1;\ \tau \neq \frac{g}{2} \\ > NS^h(s_{\tau-1}), & \tau = 1,2,...,g \\ = 0.5, & \tau = \frac{g}{2} \\ = 1, & \tau = g \end{cases}. \tag{12}$$

By solving Model (11), the set of possible PNSs of linguistic terms for e_h can be obtained, i.e., $APS^h = \{(NS^h(s_0), NS^h(s_1), ..., NS^h(s_g)), ...\}$. The obtained NSs can guarantee the maximum consistency of the given LPR $L^h = [l^h_{ij}]_{n\times n}$.

Remark 1. Model (11) is valid to derive PNSs in the context of LPRs or extended LPRs because the core of the consistency-driven model is to manage PISs by maximizing the consistency level of an individual LPR or its extensions. However, it will be invalid in the situations where DMs provide linguistic evaluations in forms of multi-criteria decision matrices.

3. Optimization models to obtain PNSs in hesitant fuzzy linguistic MCGDM

This section presents two kinds of optimization models to obtain PNSs in MCGDM with HFLTSs.

For simplicity, let $A = \{a_1, a_2, ..., a_m\}$ be a finite set of m alternatives, $C = \{c_1, c_2, ..., c_n\}$ be a finite set of n criteria, $w = (w_1, w_2, ..., w_n)^T$ be the criterion weight vector, such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$ and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_q)^T$ be the weight vector of DMs, such that $\lambda_h \in [0, 1]$ and $\sum_{h=1}^q \lambda_h = 1$. Suppose that $R^h = [r^h_{ij}]_{m\times n}$ $(h = 1, 2, ..., q)$ is the linguistic decision matrix, where r^h_{ij} represents the comparative linguistic expression of alternative a_i with regard to criterion c_j given by DM e_h. For convenience, let $M = \{1, 2, ..., m\}$, $N = \{1, 2, ..., n\}$ and $Q = \{1, 2, ..., q\}$.

In GDM, several experts form a decision group and may be in charge of performing an evaluation involving a set of alternatives. It is required that an expert should be qualified with the ability to distinguish between cases which are similar but not identical (Herowati et al., 2017). Motivated by the idea of maximizing deviation method (Wang, 1997), the total deviation among all alternatives is considered as an effective tool to measure the discrimination level of an expert (Tian et al., 2019).

Definition 10. The discrimination measure $Dis(e_k)$ of e_k is defined as follows:

$$Dis(e_h) = \frac{1}{m(m-1)} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1,k\neq i}^{m} w_j d(r^h_{ij}, r^h_{kj}), \tag{13}$$

where $Dis(e_h) \in [0, 1]$, w_j is the weight of criterion c_j and $d(r^h_{ij}, r^h_{kj})$ is the distance between comparative linguistic expressions r^h_{ij} and r^h_{kj} as per Eq. (7).

Example 1. Assume that $R^1 = [r^1_{ij}]_{4\times 4}$ is a hesitant fuzzy linguistic decision matrix given by e_1 based on LTS $S = \{s_0, s_1, ..., s_6\}$. The corresponding criterion weight vector is $w = (0.2, 0.25, 0.2, 0.35)^T$. Assume that the NSs on LTS S are predetermined as $NS(s_\tau) = \frac{\tau}{6}$ $(\tau = 0, 1, ..., 6)$, Then, according to Eq. (13), the discrimination degree is $Dis(e_1) = 0.157$.

$$R^1 = \begin{bmatrix} \{s_2, s_3\} & \{s_3, s_4\} & \{s_2, s_3, s_4\} & \{s_3\} \\ \{s_1, s_2, s_3\} & \{s_2, s_3\} & \{s_3, s_4\} & \{s_4, s_5, s_6\} \\ \{s_2, s_3, s_4\} & \{s_4\} & \{s_3\} & \{s_3, s_4\} \\ \{s_4, s_5\} & \{s_2, s_3\} & \{s_3, s_4\} & \{s_4\} \end{bmatrix}.$$

In practice, different people may elicit various semantics when providing linguistic assessments. In other words, the NSs on LTS of different DMs may be various when they provide their linguistic evaluations. Thus, a solution is required to identify the PNSs for different DMs.

3.1. Discrimination-based optimization model to obtain PNSs

As mentioned before, an expert is expected to be qualified as the discriminatory ability to differentiate between cases. When experts are required to provide linguistic ratings, their PISs of linguistic terms are embedded in the evaluations, which implicitly indicate the subtle differences among alternatives distinguished by experts. Therefore, an optimization model by maximizing the discrimination degree can be considered as a good solution to obtain the PISs of experts.

$$\begin{aligned} & Max\ Dis(e_h) = \tfrac{1}{m(m-1)} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1,k\neq i}^{m} w_j d(r_{ij}^h, r_{kj}^h) \\ & s.t.\ \Gamma_{NS}^h \end{aligned} \tag{14}$$

where $d(r_{ij}^h, r_{kj}^h) = \left|\sum_{\tau=0}^{g} (p_{ij,\tau}^h - p_{ik,\tau}^h) NS^h(s_\tau)\right|$ and Γ_{NS}^h is the constraint set, as described in Eq.(12).

Example 2. (continuation of Example 1). Assume that $R^1 = [r_{ij}^1]_{4\times4}$ and w are the same as Example 1. The PNSs of linguistic terms on LTS S for e_1 can be identified as follows:

By resolving Model (14), the PNSs of linguistic terms for e_1 can be obtained, i.e., $NS^1(s_0) = 0$, $NS^1(s_1) = 0.001$, $NS^1(s_2) = 0.167$, $NS^1(s_3) = 0.5$, $NS^1(s_4) = 0.833$, $NS^1(s_5) = 0.999$ and $NS^1(s_6) = 1$. According to Eq. (13), the discrimination degree is $Dis'(e_1) = 0.261$, which is higher than that in Example 1.

Remark 2. The result of Example 2 reveals the non-uniformly distributive semantics of linguistic terms when DM e_1 provides the hesitant fuzzy linguistic decision information.

Remark 3. The discrimination-based optimization Model (14) is a linear programming (LP) model with g + 1 decision variables and linear constraints, where g + 1 is the number of linguistic terms S. The number of constraints is also determined by the size of the linguistic term set and the feasible region, which is independent of the number of alternatives m or criteria n. This structure implies that the computational complexity of solving Model (14) does not scale with the size of the decision problem (i.e., the number of alternatives or criteria) but only with the number of the linguistic term set. Since the number of linguistic terms is typically small, this model can be efficiently solved using mature LP solvers even for large-scale group decision-making problems with many alternatives and criteria. Therefore, the discrimination-based approach computationally feasible for large-scale group decision-making problems.

3.2. Multidimensional preference-based optimization model to obtain PNSs

In MCGDM, multiple DMs must provide hesitant fuzzy linguistic ratings $R^h = [r_{ij}^h]_{m\times n}$ ($h \in Q$) of alternatives with regard to each criterion. In addition to the data of hesitant fuzzy linguistic ratings in R^h, DMs are asked to directly compare any two alternatives in pairs. However, DMs may provide different, or even conflicting judgements regarding the preferences of alternatives. Moreover, PISs of linguistic evaluations given by DMs may deviate from the overall evaluations of alternatives. These possibilities imply that certain preference relations have chances of resulting in violations over alternatives due to the inconsistent preference information and PISs of individual DMs. Under this circumstance, an attempt based on the comprehensive evaluations of alternatives considering PISs of DMs is made to measure the consistency level and inconsistency level between the orders of alternatives.

(1) Consistency and inconsistency indices

Suppose that DM e_h may give the comparison preference relations in terms of pairwise alternatives using $\Omega^h = \left\{(\alpha,\beta) \middle| a_\alpha \mathrel{\underset{-}{\rightarrow}} \succ a_\beta \text{ for } a_\alpha, a_\beta \in A\right\}$ ($h \in Q$) based on his/her knowledge and experience. Here, Ω^h is called multidimensional preference information that represents the set of ordered pairs (α,β) provided by e_h.

For each ordered pair $(\alpha,\beta) \in \Omega^h$, if a_α and a_β are ranked in the same order within the final preference order yielded by the decision matrices and the multidimensional preference information, there is concordance; if they have the same rank, then an ex aequo situation exists; if they are counter-ranked in order within the two preorders, then there is discordance (SrinivasanAllan and Shocker, 1973). To check the rank consistency in regard to each ordered pair (α, β), the overall evaluations of each alternative are calculated to facilitate the comparison between a_α and (α, β) based on the HFLWA operator and EVs of HFLTSs. By coupling the criterion weights, the comprehensive evaluation r_i^h of alternative a_i is calculated as follows:

$$r_i^h = HFLWA(r_{i1}^h, r_{i2}^h, ..., r_{in}^h) = \sum_{j=1}^{n} w_j r_{ij}^h = (p_{i,0}^h, p_{i,1}^h, ..., p_{i,g}^h), \tag{15}$$

where $r_{ij}^h = (p_{ij,0}^h, p_{ij,1}^h, ..., p_{ij,g}^h)$ and $p_{i,\tau}^h = \sum_{j=1}^{n} w_j p_{ij,\tau}^h$.

Then, the EV of r_i^h is calculated as follows:

$$EV^h(a_i) = \sum_{\tau=0}^{g} NS(s_\tau) p_{i,\tau}^h, \tag{16}$$

where $p_{i,\tau}^h$ is calculated by Eq. (15). Large value of $EV^h(a_i)$ denotes great preference of alternative a_i presented by e_h.

Assume that the NSs of LTS for e_h is unknown a priori in GDM. The collective set Ω^h of pairwise comparison judgments with regard to pairs of alternatives given by DMs are identified as inputs, and a set of PNSs is considered as outputs. For each ordered pair $(\alpha, \beta) \in \Omega^h$ in the linear programming technique for multidimensional analysis of preference (LINMAP), it is expected that the comprehensive EVs $EV^h(a_\alpha)$ and $EV^h(a_\beta)$ of alternatives a_α and a_β, respectively, are associated with the solution of the PNSs of LTSs, satisfying the inequality of $EV^h(a_\alpha) \geqslant EV^h(a_\beta)$ (SrinivasanAllan and Shocker, 1973). Following this rule, the consistency and inconsistency indices in the PNS-based LINMAP can be defined.

Consider the solution of PNSs obtained by the multidimensional preference analysis. For each $(\alpha, \beta) \in \Omega^h$, if $EV^h(a_\alpha) \geqslant EV^h(a_\beta)$, then no error can be attributed to the solution. Assume that $CI_{\alpha\beta}^{h+}$ measures the degree of consistency between the ranking orders of a_α and a_β for the ordered pair $(\alpha, \beta) \in \Omega^h$. Then, CI $CI_{\alpha\beta}^{h+}$ can be defined as follows:

$$CI_{\alpha\beta}^{h+} = \begin{cases} EV^h(a_\alpha) - EV^h(a_\beta) & \text{if } EV^h(a_\alpha) \geqslant EV^h(a_\beta) \\ 0 & \text{if } EV^h(a_\alpha) < EV^h(a_\beta) \end{cases}. \tag{17}$$

Based on Eq. (16), $CI_{\alpha\beta}^{h+}$ can be rewritten as

$$CI_{\alpha\beta}^{h+} = Max\left\{0, \sum_{\tau=0}^{g} NS(s_\tau) \sum_{j=1}^{n} w_j (p_{\alpha j,\tau}^h - p_{\beta j,\tau}^h)\right\}. \tag{18}$$

Thus, the total CI CI^{h^+} can be calculated as follows:

$$ICI^{h^-} = \sum_{(\alpha,\beta)\in\Omega^h} ICI_{\alpha\beta}^{h-}, with ICI^{h^-} \geqslant 0. \tag{19}$$

Conversely, for each ordered pair $(\alpha, \beta) \in \Omega^h$, assume that $ICI_{\alpha\beta}^{h-}$ measures the degree of inconsistency between the ranking orders of a_α and a_β. Then, inconsistency index (ICI) ICI can be defined as follows:

$$ICI_{\alpha\beta}^{h-} = \begin{cases} EV^h(a_\beta) - EV^h(a_\alpha) & \text{if } EV^h(a_\alpha) < EV^h(a_\beta) \\ 0 & \text{if } EV^h(a_\alpha) \geqslant EV^h(a_\beta) \end{cases}, \tag{20}$$

Based on Eq. (20), $ICI_{\alpha\beta}^{h-}$ can be rewritten as

$$ICI_{\alpha\beta}^{h-} = Max\left\{0, \sum_{\tau=0}^{g} NS(s_\tau) \sum_{j=1}^{n} w_j (p_{\beta j,\tau}^h - p_{\alpha j,\tau}^h)\right\}. \tag{21}$$

Thus, the total ICI ICI^{h^+} can be calculated as follows:

$$ICI^{h^-} = \sum_{(\alpha,\beta)\in\Omega^h} ICI^{h-}_{\alpha\beta}, with ICI^{h^-} \geqslant 0. \tag{22}$$

(2) Double objective optimization model

An optimization problem is formulated to minimize ICI^{h^-} to find a reliable solution that is close to the paired comparison judgments given by expert e_h. Moreover, it is required to maximize the discrimination degree $Dis(e_h)$ of the decision matrix given by expert e_h. Thus, a double objective optimization model is built to derive the PNSs as follows:

$$\begin{aligned} & Min\ ICI^{h^-} \\ & Max\ Dis(e_h) \\ & s.t. \begin{cases} CI^{h^+} - ICI^{h^-} \geqslant \rho \\ \Gamma^h_{NS} \end{cases} . \end{aligned} \tag{23}$$

In Model (23), the first objective is to minimize the total ICI and the second objective is to maximize the discrimination degree. The parameter ρ is a priori threshold provided by the DMs, which is regarded as the DMs' lowest acceptable level toward the difference of $CI^{h^+} - ICI^{h^-}$.

For each ordered pair $(\alpha,\beta) \in \Omega^h$, it has $ICI^{h-}_{\alpha\beta} \geqslant 0$ and $ICI^{h-}_{\alpha\beta} \geqslant \sum_{\tau=0}^{g} NS(s_\tau) \sum_{j=1}^{n} w_j (p^h_{\beta j,\tau} - p^h_{\alpha j,\tau})$ based on Eq. (21). Therefore, Model (23) can be rewritten as follows:

$$\begin{aligned} & Min\ ICI^{h^-} = \sum_{(\alpha,\beta)\in\Omega^h} ICI^{h-}_{\alpha\beta} \\ & Max\ Dis(e_h) = \tfrac{1}{m(m-1)} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1,k\neq i}^{m} w_j d(r^h_{ij}, r^h_{kj}) \\ & s.t. \begin{cases} \sum_{(\alpha,\beta)\in\Omega^h} \sum_{\tau=0}^{g} NS(s_\tau) \sum_{j=1}^{n} w_j (p^h_{\alpha j,\tau} - p^h_{\beta j,\tau}) \geqslant \rho \\ ICI^{h-}_{\alpha\beta} + \sum_{\tau=0}^{g} NS(s_\tau) \sum_{j=1}^{n} w_j (p^h_{\alpha j,\tau} - p^h_{\beta j,\tau}) \geqslant 0 \\ ICI^{h-}_{\alpha\beta} \geqslant 0 & (\alpha,\beta) \in \Omega^h \\ Gamma^h_{NS} & (\alpha,\beta) \in \Omega^h \end{cases} . \end{aligned} \tag{24}$$

Remark 4. In Model (24), the decision variables are $ICI^{h-}_{\alpha\beta}$ $(\alpha,\beta) \in \Omega^h$ and $NS^h(s_\tau)$ $(\tau = 0, 1, 2, ..., g)$. Model (24) is decomposed as Model (14) if DM e_h does not provide any paired comparison judgments, i.e., $\Omega^h = \Phi$. When resolving multi-objective problems, the weighted-sums appraoch is considered as the most straightforward technique (Kaddani et al., 2017).

Example 3 (continuation of Example 2). Assume that $R^1 = [r^1_{ij}]_{4\times 4}$ and w are the same as Example 1. DM e_1 provides the paired comparison judgments of alternatives are $\Omega^h = \{(a_2, a_1), (a_2, a_3), (a_4, a_2), (a_4, a_3)\}$. The PNSs of linguistic terms on LTS S for e_1 can be identified as follows:

Without loss of generality, ρ is set to be 0.1. By resolving Model (24), the PNSs of linguistic terms for e_1 can be obtained, i.e., $NS^1(s_0) = 0$, $NS^1(s_1) = 0.333$, $NS^1(s_2) = 0.334$, $NS^1(s_3) = 0.5$, $NS^1(s_4) = 0.833$, $NS^1(s_5) = 0.999$ and $NS^1(s_6) = 1$.

4. Hesitant fuzzy linguistic MCGDM considering PIS and consensus

This section presents an optimization model with minimum adjustments to generate the consensual solutions.

4.1. Consensus measures

Assume that $R^c = [r^c_{ij}]_{m\times n}$ is the overall linguistic decision matrix of the group members, where r^c_{ij} can be presented in forms of a possibility distribution, denoted by $P^c_{ij} = (p^c_{ij,0}, p^c_{ij,1}, ..., p^c_{ij,g})$. Moreover, assume that the collective NSs of linguistic terms for the group is $APS^c = \left\{NS^c(s_0), NS^c(s_1), ..., NS^c(s_g)\right\}$.

Thus, R^c and APS^c can be calculated based on the weighted aggregation operator as follows:

$$r^c_{ij} = HFLWA(r^1_{ij}, r^2_{ij}, ..., r^q_{ij}), \tag{25}$$

where $p^c_{ij,\tau} = \sum_{h=1}^{q} \lambda_h p^h_{ij,\tau}$ by referring to Eq. (8).

$$NS^c(s_\tau) = \sum_{h=1}^{q} \lambda_h NS^h(s_\tau), \tag{26}$$

where $NS^h(s_\tau) \in APS^h$ is obtained on the basis of the built optimization models in Section 3.

Based on Eq. (7), the consensus measure between the individual members and the group is presented as follows:

$$GCL = 1 - \frac{1}{qmn} \sum_{h=1}^{q} \sum_{i=1}^{m} \sum_{j=1}^{n} d(r^h_{ij}, r^c_{ij}), \tag{27}$$

where $GCL \in [0, 1]$.

4.2. Managing group consensus based on optimization model with minimum adjustments

The basic idea of the established optimization model is to seek the minimum distance between the original and adjusted decision matrices, to keep the modified assessments remain HFLTSs and to preserve their accuracy. Let $\bar{R}^h = [\bar{r}^h_{ij}]_{m\times n}$ $(h \in Q)$ be the adjusted decision matrices and Λ be the set of all HFLTSs. Thus, an optimization-based consensus model is designed as follows:

$$\begin{aligned} &Min\ Z = \frac{1}{qmn} \sum_{h=1}^{q} \sum_{i=1}^{m} \sum_{j=1}^{n} d(r^h_{ij}, \bar{r}^h_{ij}) \\ &s.t. \begin{cases} GCL = 1 - \frac{1}{qmn} \sum_{h=1}^{q} \sum_{i=1}^{m} \sum_{j=1}^{n} d(\bar{r}^h_{ij}, \bar{r}^c_{ij}) \geqslant \eta \\ \bar{r}^c_{ij} = HFLWA(\bar{r}^1_{ij}, \bar{r}^2_{ij}, ..., \bar{r}^q_{ij})\ (i \in M,\ j \in N) \\ \bar{r}^h_{ij} \in \Lambda\ (i \in M,\ j \in N,\ h \in Q) \\ \underset{i\in M,\ j\in N}{Min} \{\#r^h_{ij}\} \leqslant \#\bar{r}^h_{ij} \leqslant \underset{i\in M,\ j\in N}{Max} \{\#r^h_{ij}\} \end{cases}, \end{aligned} \tag{28}$$

where the first three constraints guarantee that the group achieve the predefined consensus level and the last constraint keeps the accuracy levels of each revised decision matrix.

To convert Model (28) into a 0-1 linear programming model, a binary variable $x^h_{ij,\tau} \in \{0, 1\}$ is introduced as follows:

$$x^h_{ij,\tau} = \begin{cases} 1,\ p^h_{ij,\tau} > 0 \\ 0,\ p^h_{ij,\tau} = 0 \end{cases} \qquad (\tau = 0, 1, \ldots, g;\ i \in M,\ j \in N,\ h \in Q). \tag{29}$$

Lemma 1. $\underset{i\in M,\ j\in N}{Min} \{\#r^h_{ij}\} \leqslant \#\bar{r}^h_{ij} \leqslant \underset{i\in M,\ j\in N}{Max} \{\#r^h_{ij}\}$ is equivalent to $\underset{i\in M,\ j\in N}{Min} \left\{\sum_{\tau=0}^{g} x^h_{ij,\tau}\right\} \leqslant \sum_{\tau=0}^{g} \bar{x}^h_{ij,\tau} \leqslant \underset{i\in M,\ j\in N}{Max} \left\{\sum_{\tau=0}^{g} x^h_{ij,\tau}\right\}$.

Proof. According to the conditions in Eq. (29), it has $\#r^h_{ij} = \sum_{\tau=0}^{g} x^h_{ij,\tau}$ and $\#\bar{r}^h_{ij} = \sum_{\tau=0}^{g} \bar{x}^h_{ij,\tau}$. This completes the proof of Lemma 1.

Lemma 2. (Dong et al., 2015). r^h_{ij} is an HFLTS if and only if (a) $\sum_{\tau=0}^{g-1} \left|x^h_{ij,\tau+1} - x^h_{ij,\tau}\right| \leqslant 2$; and (b) $x^h_{ij,0} - x^h_{ij,g} \leqslant 1$.

Theorem 1. Let $u_{ij}^h = \left|\sum_{\tau=0}^{g}(p_{ij,\tau}^h - \bar{p}_{ij,\tau}^h)NS^h(s_\tau)\right|$, $v_{ij}^h = \left|\sum_{\tau=0}^{g}\left(\bar{p}_{ij,\tau}^h NS^h(s_\tau) - \bar{p}_{ij,\tau}^c NS^c(s_\tau)\right)\right|$ and $y_{ij,\tau}^h = \left|\bar{x}_{ij,\tau+1}^h - \bar{x}_{ij,\tau}^h\right|$. Then, Model (28) can be equally converted into Model (30).

$$Min\ Z = \frac{1}{qmn}\sum_{h=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} u_{ij}^h$$

$$s.t.\begin{cases}
\sum_{\tau=0}^{g}(p_{ij,\tau}^h - \bar{p}_{ij,\tau}^h)NS^h(s_\tau) \leqslant u_{ij}^h\ (i \in M,\ j \in N,\ h \in Q) & (a)\\
\sum_{\tau=0}^{g}(\bar{p}_{ij,\tau}^h - p_{ij,\tau}^h)NS^h(s_\tau) \leqslant u_{ij}^h\ (i \in M,\ j \in N,\ h \in Q) & (b)\\
\frac{1}{qmn}\sum_{h=1}^{q}\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}^h \leqslant 1-\eta & (c)\\
\sum_{\tau=0}^{g}\left(\bar{p}_{ij,\tau}^h NS^h(s_\tau) - \bar{p}_{ij,\tau}^c NS^c(s_\tau)\right) \leqslant v_{ij}^h\ (i \in M,\ j \in N,\ h \in Q) & (d)\\
\sum_{\tau=0}^{g}\left(\bar{p}_{ij,\tau}^c NS^c(s_\tau) - \bar{p}_{ij,\tau}^h NS^h(s_\tau)\right) \leqslant v_{ij}^h\ (i \in M,\ j \in N,\ h \in Q) & (e)\\
\bar{p}_{ij,\tau}^c = \sum_{h=1}^{q}\lambda_h \bar{p}_{ij,\tau}^h\ (\tau = 0,1,\dots,g;\ i \in M,\ j \in N) & (f)\\
NS^c(s_\tau) = \sum_{h=1}^{q}\lambda_h NS^h(s_\tau)\ (\tau = 0,1,\dots,g) & (g)\\
\sum_{\tau=0}^{g}\bar{p}_{ij,\tau}^h = 1\ (i \in M,\ j \in N,\ h \in Q) & (h)\\
\sum_{\tau=0}^{g-1} y_{ij,\tau}^h \leqslant 2\ (i \in M,\ j \in N,\ h \in Q) & (i)\\
\bar{x}_{ij,\tau+1}^h - \bar{x}_{ij,\tau}^h \leqslant y_{ij,\tau}^h\ (\tau = 0,1,\dots,g-1;\ i \in M,\ j \in N,\ h \in Q) & (j)\\
\bar{x}_{ij,\tau}^h - \bar{x}_{ij,\tau+1}^h \leqslant y_{ij,\tau}^h\ (\tau = 0,1,\dots,g-1;\ i \in M,\ j \in N,\ h \in Q) & (k)\\
\bar{x}_{ij,0}^h + \bar{x}_{ij,g}^h \leqslant 1\ (i \in M,\ j \in N,\ h \in Q) & (l)\\
\underset{i\in M,\ j\in N}{Min}\left\{\sum_{\tau=0}^{g} x_{ij,\tau}^h\right\} \leqslant \sum_{\tau=0}^{g}\bar{x}_{ij,\tau}^h \leqslant \underset{i\in M,\ j\in N}{Max}\left\{\sum_{\tau=0}^{g} x_{ij,\tau}^h\right\} & (m)\\
\bar{x}_{ij,\tau}^h - \bar{p}_{ij,\tau}^h < 1\ (\tau = 0,1,\dots,g;\ i \in M,\ j \in N,\ h \in Q) & (n)\\
\bar{p}_{ij,\tau}^h - \bar{x}_{ij,\tau}^h \leqslant 0\ (\tau = 0,1,\dots,g;\ i \in M,\ j \in N,\ h \in Q) & (o)\\
\bar{x}_{ij,\tau}^h \in \{0,1\}\ (\tau = 0,1,\dots,g;\ i \in M,\ j \in N,\ h \in Q) & (p)\\
y_{ij,\tau}^h \in [0,1]\ (\tau = 0,1,\dots,g-1;\ i \in M,\ j \in N,\ h \in Q) & (q)\\
\bar{p}_{ij,\tau}^h \in [0,1]\ (\tau = 0,1,\dots,g;\ i \in M,\ j \in N,\ h \in Q) & (r)
\end{cases}. \tag{30}$$

Proof. Constraints (a) and (b) guarantee that $\left|\sum_{\tau=0}^{g}(p_{ij,\tau}^h - \bar{p}_{ij,\tau}^h)NS^h(s_\tau)\right| \leqslant u_{ij}^h$ based on the establishment of the theorem with the property $|x| = Max\{x, -x\}$; constraints (c)-(h) guarantee that $GCL \geqslant \eta$; constraints (i)-(l) guarantee that $\bar{r}_{ij}^h \in \Lambda$; constraint (m) guarantees that the adjusted decision matrices have the same accuracy as the original ones; constraints (n) and (o) guarantee that if $\bar{p}_{ij,\tau}^h > 0$, then $\bar{x}_{ij,\tau}^h = 1$; otherwise, $\bar{x}_{ij,\tau}^h = 0$. Thus, Model (28) can be equally converted into Model (30). This completes the proof of Theorem 1.

4.3. Proposed framework for MCGDM with HFLTSs

This subsection presents a framework for managing hesitant fuzzy linguistic MCGDM considering PIS and consensus, which is described in Fig. 1. The detailed steps of the proposed approach are as follows:

Step 1: Collect the decision information of DMs.

Each DM provide decision information with comparative linguistic expressions, which are transformed into hesitant fuzzy decision matrices R^h $(h \in Q)$ by referring Ref. (Rodríguez et al., 2012). Moreover, the multidimensional preference information in terms of alternatives is Ω^h $(h \in Q)$.

Step 2: Identify PNSs of linguistic terms for each DM.

Case 1: if $\Omega^h = \Phi$, then Model (14) can be used to obtain PNSs ASP^h of linguistic terms on LTS for DM e_h.

Case 2: if $\Omega^h \neq \Phi$, then Model (24) can be used to obtain PNSs ASP^h of linguistic terms on LTS for DM e_h.

Step 3: Calculate the group consensus level.

Calculate the group consensus level GCL. If $GCL < \eta$, then go to Step 4; otherwise go to Step 5.

Step 4: Improve the group consensus level.

Establish the optimization model based on Model (30). Then, the adjusted decision matrices $\bar{R}^h$ $(h \in Q)$ can be derived.

Step 5: Derive the collective evaluations of each alternative.

The overall evaluations of each alternative can be calculated based on the HFLWA operator.

Step 6: Calculate the EVs of each alternative and identify the ranking of them.

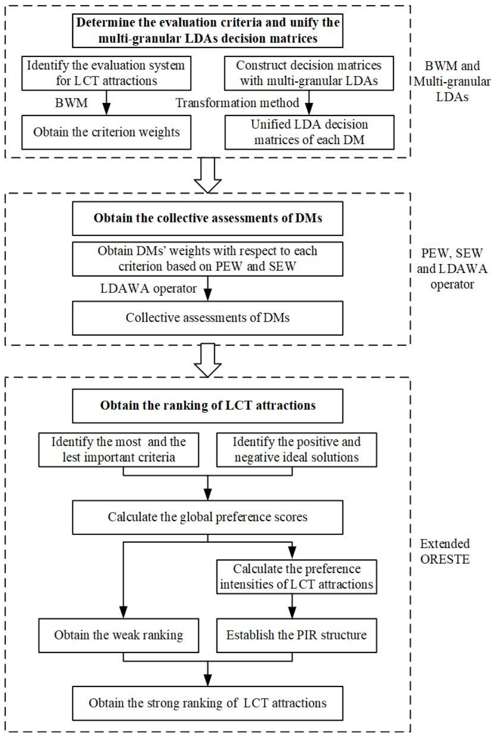

Fig. 1. Flowchart of the proposed framework.

5. Numerical example and performance analysis

This section presents a numerical example that is adapted from (Tian et al., 2021) to show the application of the proposed approach. Moreover, a comparative analysis and a sensitivity analysis are conducted to reveal the outstanding features of the proposed framework.

5.1. Illustrative example

Company ABC tends to select the optimal investment objective(s) among the member countries of the Belt and Road Initiative (BPI). Four countries are considered as alternatives, namely, a_i $(i = 1, 2, 3, 4)$ through comprehensive preliminary investigation. A panel consisting of four experts, represented as e_h $(h = 1, 2, 3, 4)$ are invited to assess and choose the optimal alternative(s) regarding to four criteria, namely, local infrastructure level (c_1), credit risk (c_2), regulation and law environment (c_3) and political stability (c_4). Assume that the weight vectors of criteria and experts are $w = (0.2, 0.25, 0.2, 0.35)^T$ and $\lambda = (0.25, 0.25, 0.25, 0.25)^T$, respectively. Moreover, the LTS for generating linguistic evaluations is $S = \{$ s_0 = extremely bad, s_1 = very bad, s_2 = bad, s_3 = medium, s_4 = good, s_5 = very good, s_6 = extremely good$\}$.

The proposed approach is employed to help DMs tackle the above linguistic MCGDM problem. Let the parameter $\rho = 0.1$ and the consensus threshold $\eta = 0.85$.

Step 1: Collect the decision information of DMs.

Each DM provides decision information with comparative linguistic expressions, which are translated into hesitant fuzzy linguistic decision matrices R^h $(h = 1, 2, 3, 4)$ by referring Ref. (Rodríguez et al., 2012). Moreover, only three DMs provide the multidimensional preference information as follows:

$\Omega^1 = \{(a_1, a_2), (a_3, a_2), (a_4, a_3), (a_4, a_2)\}$, $\Omega^2 = \{(a_4, a_1), (a_1, a_3), (a_2, a_1), (a_2, a_3)\}$ and $\Omega^4 = \{(a_1, a_2), (a_3, a_2), (a_4, a_3)\}$.

$$R^1 = \begin{bmatrix} \{s_3, s_4\} & \{s_5\} & \{s_5, s_6\} & \{s_4, s_5\} \\ \{s_4, s_5, s_6\} & \{s_1, s_2\} & \{s_3\} & \{s_5, s_6\} \\ \{s_0, s_1, s_2\} & \{s_4, s_5, s_6\} & \{s_4, s_5\} & \{s_3, s_4, s_5\} \\ \{s_5, s_6\} & \{s_4, s_5\} & \{s_3, s_4\} & \{s_4\} \end{bmatrix},$$

$$R^2 = \begin{bmatrix} \{s_5, s_6\} & \{s_5, s_6\} & \{s_3\} & \{s_2, s_3\} \\ \{s_0, s_1, s_2\} & \{s_5, s_6\} & \{s_5, s_6\} & \{s_2, s_3\} \\ \{s_3, s_4\} & \{s_2, s_3\} & \{s_1, s_2\} & \{s_4\} \\ \{s_4, s_5, s_6\} & \{s_1, s_2\} & \{s_3, s_4, s_5\} & \{s_5, s_6\} \end{bmatrix},$$

$$R^3 = \begin{bmatrix} \{s_0, s_1, s_2\} & \{s_3, s_4\} & \{s_4, s_5\} & \{s_4, s_5\} \\ \{s_5, s_6\} & \{s_4, s_5\} & \{s_4, s_5\} & \{s_1, s_2\} \\ \{s_4\} & \{s_3\} & \{s_4, s_5, s_6\} & \{s_1, s_2\} \\ \{s_3, s_4, s_5\} & \{s_3, s_4\} & \{s_2, s_3\} & \{s_2, s_3, s_4\} \end{bmatrix},$$

$$R^4 = \begin{bmatrix} \{s_5\} & \{s_3\} & \{s_1, s_2\} & \{s_3, s_4\} \\ \{s_3, s_4\} & \{s_4, s_5\} & \{s_1, s_2, s_3\} & \{s_4\} \\ \{s_5, s_6\} & \{s_1, s_2\} & \{s_5, s_6\} & \{s_5, s_6\} \\ \{s_1, s_2\} & \{s_4, s_5, s_6\} & \{s_5, s_6\} & \{s_4, s_5\} \end{bmatrix}.$$

Step 2: Identify PNSs of linguistic terms for each DM.

Since $\Omega^3 = \Phi$, the discrimination-based optimization model is established based on Model (14) and the PNSs ASP^3 can be obtained. Since $\Omega^h \neq \Phi$ $(h = 1, 2, 4)$, the multidimensional preference-based optimization models are established based on model (24) and the PNSs ASP^h $(h = 1, 2, 4)$ can be obtained. Moreover, the collective PNSs ASP^c for the group can be obtained, as shown in Table 2. The PNSs of linguistic terms for each DM are vividly presented in Fig. 2.

Table 2: PNSs of linguistic terms.

	APS^1	APS^2	APS^3	APS^4	APS^c
$NS(s_0)$	0	0	0	0	0
$NS(s_1)$	0.01	0.01	0.01	0.01	0.01
$NS(s_2)$	0.167	0.167	0.167	0.167	0.167
$NS(s_3)$	0.5	0.5	0.5	0.5	0.5
$NS(s_4)$	0.654	0.654	0.654	0.654	0.654
$NS(s_5)$	0.99	0.99	0.99	0.99	0.99
$NS(s_6)$	1	1	1	1	1

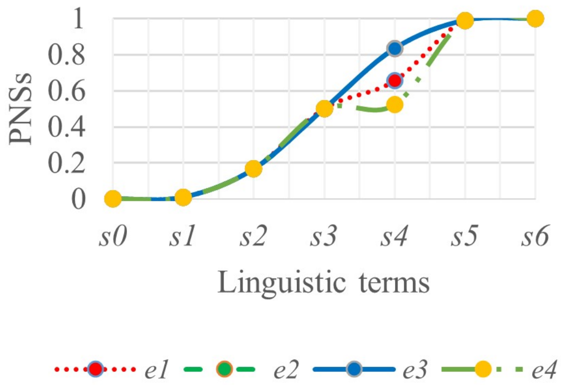

Fig. 2. PNSs of linguistic terms for each DM.

Step 3: Calculate the group consensus level.

The group consensus level can be calculated by referring to Eq. (27), i.e., $GCL = 0.723 < 0.85$. Thus, the CRP is required.

Step 4: Improve the group consensus level.

The consensus-reaching model can be established based on Model (30). After resolving the constructed model, the revised decision matrices $\bar{R}^h$ $(h = 1,2,3,4)$ are derived with the group consensus level $GCL = 0.85$.

$$\bar{R}^1 = \begin{bmatrix} \{s_5\} & \{s_5\} & \{s_5\} & \{s_2,s_3\} \\ \{s_4,s_5\} & \{s_6\} & \{s_4,s_5\} & \{s_4,s_5,s_6\} \\ \{s_0,s_1,s_2\} & \{s_2\} & \{s_4\} & \{s_6\} \\ \{s_5,s_6\} & \{s_4,s_5\} & \{s_3,s_4,s_5\} & \{s_3\} \end{bmatrix},$$

$$\bar{R}^2 = \begin{bmatrix} \{s_5,s_6\} & \{s_2,s_3,s_4\} & \{s_4,s_5\} & \{s_1,s_2,s_3\} \\ \{s_0,s_1,s_2\} & \{s_6\} & \{s_2,s_3,s_4\} & \{s_3,s_4,s_5\} \\ \{s_5,s_6\} & \{s_2\} & \{s_0\} & \{s_3,s_4,s_5\} \\ \{s_3,s_4,s_5\} & \{s_0,s_1,s_2\} & \{s_2,s_3,s_4\} & \{s_6\} \end{bmatrix},$$

$$\bar{R}^3 = \begin{bmatrix} \{s_5,s_6\} & \{s_3,s_4,s_5\} & \{s_4\} & \{s_2,s_3\} \\ \{s_6\} & \{s_6\} & \{s_3,s_4,s_5\} & \{s_0,s_1,s_2\} \\ \{s_3,s_4,s_5\} & \{s_2,s_3,s_4\} & \{s_3,s_4,s_5\} & \{s_4\} \\ \{s_4,s_5\} & \{s_4,s_5,s_6\} & \{s_1,s_2,s_3\} & \{s_2,s_3,s_4\} \end{bmatrix},$$

$$\bar{R}^4 = \begin{bmatrix} \{s_6\} & \{s_2,s_3,s_4\} & \{s_4,s_5,s_6\} & \{s_3\} \\ \{s_3,s_4\} & \{s_6\} & \{s_1,s_2,s_3\} & \{s_3\} \\ \{s_4,s_5,s_6\} & \{s_1,s_2,s_3\} & \{s_4,s_5,s_6\} & \{s_5,s_6\} \\ \{s_4\} & \{s_4\} & \{s_5,s_6\} & \{s_4,s_5\} \end{bmatrix}.$$

The corresponding possibility distributions $\bar{P}^h$ $(h = 1,2,3,4)$ of $\bar{R}^h$ are presented. It should be noted that the elements that are zero are omitted due to the limited layout.

$$\bar{P}^1 = \begin{bmatrix} (1) & (1) & (1) & (0.1,0.9) \\ (0.332,0.668) & (1) & (0.9,0.1) & (0.1,0.1,0.8) \\ (0.1,0.1,0.8) & (1) & (1) & (1) \\ (0.499,0.501) & (0.5,0.5) & (0.78,0.1,0.12) & (1) \end{bmatrix},$$

$$\bar{P}^2=\begin{bmatrix} (0.499,0.501) & (0.1,0.1,0.8) & (0.9,0.1) & (0.1,0.1,0.8) \\ (0.1,0.568,0.332) & (1) & (0.1,0.1,0.8) & (0.8,0.1,0.1) \\ (0.9,0.1) & (1) & (1) & (0.299,0.1,0.601) \\ (0.1,0.1,0.8) & (0.1,0.4,0.5) & (0.1,0.1,0.8) & (1) \end{bmatrix},$$

$$\bar{P}^3=\begin{bmatrix} (0.9,0.1) & (0.633,0.1,0.267) & (1) & (0.1,0.9) \\ (1) & (1) & (0.133,0.1,0.767) & (0.397,0.1,0.503) \\ (0.299,0.1,0.601) & (0.8,0.1,0.1) & (0.1,0.1,0.8) & (1) \\ (0.9,0.1) & (0.8,0.1,0.1) & (0.267,0.1,0.633) & (0.1,0.8,0.1) \end{bmatrix},$$

$$\bar{P}^4=\begin{bmatrix} (1) & (0.1,0.1,0.8) & (0.403,0.1,0.497) & (1) \\ (0.5,0.5) & (1) & (0.1,0.1,0.8) & (1) \\ (0.1,0.1,0.8) & (0.767,0.1,0.133) & (0.1,0.1,0.8) & (0.499,0.501) \\ (1) & (1) & (0.499,0.501) & (0.5,0.5) \end{bmatrix}.$$

Step 5: Derive the collective evaluations of each alternative.

The collective decision matrix $\bar{R}^c = [\bar{r}^c_{ij}]_{4\times 4}$ can be calculated based on the HFLWA operator. Moreover, the overall evaluations $\bar{r}^c_i$ $(i = 1, 2, 3, 4)$ of each alternative can be obtained. Their corresponding possibility distributions are as follows:

$\bar{p}^c_1 = (0, 0.009, 0.039, 0.367, 0.284, 0.197, 0.105),$

$\bar{p}^c_2 = (0.04, 0.042, 0.071, 0.234, 0.149, 0.094, 0.37),$

$\bar{p}^c_3 = (0.055, 0.053, 0.221, 0.061, 0.173, 0.221, 0.216),$

$\bar{p}^c_4 = (0.006, 0.038, 0.05, 0.238, 0.341, 0.182, 0.144).$

Step 6: Calculate the EVs of alternatives and identify the ranking of them.

The EVs $EV(\bar{r}^c_i)$ $(i = 1, 2, 3, 4)$ of each alternative are calculated based on Eq. (9). $EV(\bar{r}^c_1) = 0.6915$, $EV(\bar{r}^c_2) = 0.6989$, $EV(\bar{r}^c_3) = 0.6258$ and $EV(\bar{r}^c_4) = 0.6946$. The ranking of alternatives is $a_2 > a_4 > a_1 > a_3$ and a_2 is identified as the optimal alternative.

5.2. Comparative analysis

This subsection performs the comparative analysis regarding the existing methods in two aspects, including PIS models and consensus-supporting methods. From the first perspective, the PIS models in this study are compared with the existing ones. Existing PIS models only can well manage DMs' representations denoted by LPRs or their extensions. Compared with the existing PIS models for GDM with LPRs or their extensions (Li et al., 2022; Li et al., 2017; Li et al., 2022; Li et al., 2018; Li and Zhang, 2025; Tang et al., 2020; Tian et al., 2025; Tian et al., 2026; Zhang et al., 2020; Zhang et al., 2021; Zhang and Li, 2022), the developed PIS models can model PISs in linguistic GDM problems where DMs' evaluations are presented in forms of hesitant fuzz linguistic MCGDM matrices. This indicates that this study extends existing research with respect to PIS models. In addition, to discuss the difference between considering PISs and not considering PISs, the discrimination degrees of decision matrices are calculated. Assume that the fixed NSs for S are set as $FNS(s_\tau) = \frac{\tau}{6}$ $(\tau = 0, 1, ..., 6)$, where S is the same as used in Subsection 5.1. Then, the discrimination degrees of decision matrices R^h $(h = 1, 2, 3, 4)$ based on the fixed and personalized NSs are calculated by using Eq. (13). The results are presented in Fig. 3. It shows that the decision matrices present higher discrimination degrees considering PISs than those without considering PISs. At this point, the proposed PIS determination method can provide an effective solution to reveal the implicit semantics in linguistic ratings elicited by DMs when distinguishing the difference among alternatives.

Another aspect is consensus-supporting methods, comparisons between the developed consensus-supporting method and the existing ones are summarized in Table 3. Minimum adjustment-based consensus models, such as method in Refs. (Li et al., 2024; Xiao et al., 2023; Yu et al., 2021; Zhang et al., 2018), aim to maintain DMs' original evaluations as much as possible, however, neglect the reality that individuals do not deem the same words express identical meanings. Existing consensus models (Wu and Xu, 2016) and (Liu et al., 2021) fail to guarantee the accuracy of the updated judgments in MCGDM with HFLTSs. To determine updated individual evaluations, this study presents an improved optimization model based on minimum adjustments, after which a consensus-supporting optimization is constructed to minimize the distance between the original and updated decision matrices. In this manner, the proposed method conserves the accurate degrees of the updated adjustments by inserting necessary constraints, meanwhile considering the PISs of DMs.

Referring to discussions in the numerical example and the comparative analysis, the outstanding features of the developed framework are summarized as follows:

(1) Effective and alternative solutions for managing PISs. The proposed PIS models can provide effective and alternative solutions to assign PNSs of linguistic terms on LTS for each DM, guaranteeing the optimal discrimination degrees regarding the linguistic decision matrices in MCGDM with HFLTSs.

(2) A straightforward approach for achieving consensual solutions. The established consensus model shares advantages of minimum adjustment-based consensus model, omitting the tedious modification process and preserving DMs' original judgements as much as possible.

(3) A cautious method for addressing DMs' evaluations. The proposed approach can not only consider the PISs of DMs, but also cautiously preserve the accurate degrees of the updated judgements through adding necessary accuracy constraints.

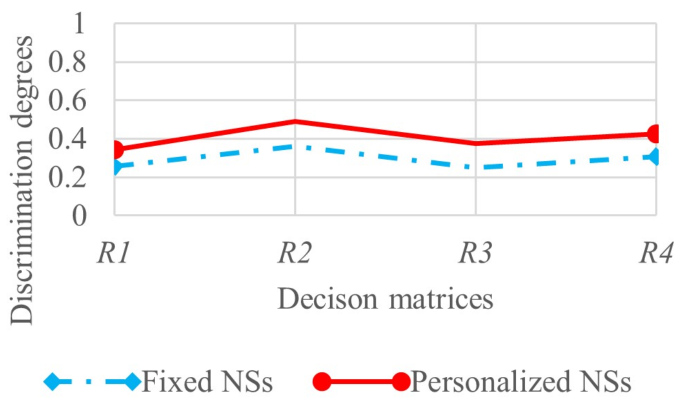

Fig. 3. Discrimination degrees of decision matrices based on Fixed and personalized NSs.

Table 3: Comparisons between the existing methods and the proposed approach.

Consensus-supporting methods	Models	Ways of addressing HFLTSs	PISs considered	Accuracy of the adjusted judgments
Method in (Wu and Xu, 2016)	Interaction algorithm	Consider the possibility distribution of HFLTSs	No	No
Method in (Liu et al., 2021)	Iteration algorithm	Regard HFLTSs as propositions in the framework of D–S evidence theory	No	No
Method in (Zhang et al., 2018)	Minimum adjustment-based 0-1 linear programming	Consider interval envelopes of HFLTSs	No	No
Method in (Yu et al., 2021)	Minimum adjustment-based linear programming	Consider fuzzy envelopes of multi-granular HFLTSs	No	No
Method in (Li et al., 2024)	Minimum adjustment-based linear programming	Not applicable (handles ordinal classification, not HFLTSs)	No	Yes
Method in (Xiao et al., 2023)	Minimum adjustment element consensus model	Transform CLEs (a natural linguistic expression of HFLTS.) into LDAs	No	Yes
The proposed approach	Minimum adjustment-based 0-1 linear programming	Consider the possibility distribution of HFLTSs	Yes	Yes

5.3. Sensitivity analysis

In the illustrative example presented in Subsection 5.1, all experts are assigned equal weights $\lambda = (0.25, 0.25, 0.25, 0.25)^T$ for fairness in the aggregation process. However, in real-world decision-making scenarios, experts often possess different levels of expertise, authority, or reliability, which may lead to asymmetric weight assignments. To investigate the influence of experts' weight distributions on the final ranking of alternatives, a sensitivity analysis is conducted by systematically perturbing the weight vector. The study designs 16 sets of weight combinations by introducing deviations of ±20% and ±30% from original equal weights, while ensuring that the weights remain normalized (i.e., $\sum_{h=1}^{4} \lambda_h = 1$). For each weight set, we recalculate the EVs of alternatives and identify the ranking of them accordingly. The final ranking results of the alternatives with different weight vectors are shown in Fig. 4. The X-axis represents different weight combinations (λ), where $case_0$ denotes the equal weights, and $case_1 \sim case_4$, $case_5 \sim case_8$, $case_9 \sim case_{12}$, $case_{13} \sim case_{16}$ represent ±20% and ±30% deviations applied respectively to the weights of experts 1, 2, 3, and 4. The Y-axis represents the four alternatives, while the Z-axis indicates ranking results.

Fig. 4. shows that when different weight vectors of experts are adopted, alternative a_2 remains the highest ranking in most cases, while alternative a_3 consistently remains the lowest ranking, with a relatively stable ranking. The ranking of a_1 exhibit minor fluctuations under certain weight settings, but no significant rank reversal occurs. This fact reveals that DMs can select a_2 as the optimal alternative, and a_3 as the worst choice. Furthermore, the rankings of other alternatives show considerable sensitivity to weight variations, suggesting that their selection order may change depending on the specific weighting scheme. Therefore, determining an appropriate λ value is both important and necessary.

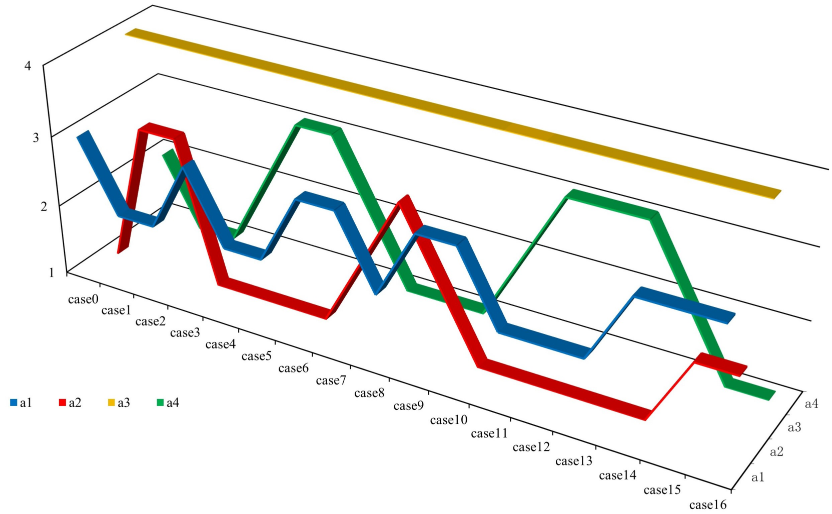

Fig. 4. Ranking results of different weight vectors of experts.

6. Conclusion

In realistic life, words may be assigned various meanings by different people, that is, DMs have PISs with respect to linguistic terms. Considering PISs of DMs can lead to a realistic and effective methodology for coping with hesitant fuzzy linguistic MCGDM problems. This study probes the issues of PIS and consensus in MCGDM with HFLTSs, which hardly has been explored in the existing research. For the issue of deriving PIS in the context of HFLTSs, this study establishes a discrimination-based and a multi-dimensional preference-based optimization models to assign PNSs of linguistic terms on LTS for DMs in two cases. To preserve accuracy of updated adjustments for consensus-supporting, this study provides an optimization-based consensus model based on minimum adjustments. The results of illustrative example and comparative study show that the developed PIS models can effectively derive the PNSs of linguistic terms on LTS for DMs in different situations and lead to higher discrimination degrees than those without considering PISs. Moreover, in addition to considering DMs' PISs, the proposed consensus-reaching framework can provide a fast and straightforward solution to achieve group consensus in MCGDM with HFLTSs and preserve accuracy of the updated judgments.

Though the proposed model presents superiorities in managing PISs and reaching consensual solutions with accuracy, complicated situations in the practical applications of MCGDM lead to deeper research in the future. Due to DMs' limited knowledge and judgment, missing values commonly exist in MCGDM. Therefore, it would be an interesting topic to explore the PIS-based method for managing incomplete linguistic MCGDM problems. Moreover, complex GDM involves large-scale members to ensure scientific decisions. This kind of studies have attracted wide attention (Cao et al., 2025; Tu et al., 2024; Wang et al., 2025), thereby requiring further extensions of the proposed framework to the situations of large-scale GDM.

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