



# Corrigendum to "Stability of Stochastic Model for Hepatitis C Transmission with an Isolation Stage"[Filomat 34 (14) (2020) 2407–2432]

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**Abstract.** The authors correct the proof of Theorem 2.1 and Theorem 3.2 in the above titled paper. Also, the remark about the conditions for stability in probability of system (9) is added. Finally, the numerical simulation, based on the new conditions for stability in probability of system (21) is given. The authors apologize for any inconvenience that may have been caused.

## 1. Corrigendum

In addition, all labels that are denoted with \* are related to the corresponding labels from [2]. The labels from this paper are without \*.

The corrigendum is as follows:

**I Theorem 2.1\*** (page 4799, the line 16 from below). In [2] is stated that  $LV \leq K$  what is not true. In order to correct this, we use different approach in proving the existence and uniqueness of the solution of system (3)\*. We use Theorem A, that can be found in [4]\*, and follow idea of [1].

**Theorem A (ID - invariance)** Let  $\mathbb{D}$  and  $\mathbb{D}_n$  be open sets in  $\mathbb{R}^n$  with

$$\mathbb{D}_n \subseteq \mathbb{D}_{n+1}, \mathbb{D} = \bigcup_n \mathbb{D}_n \text{ and } \overline{\mathbb{D}_n} \subseteq \mathbb{D}$$

and suppose  $f$  and  $g$  satisfy the existence and uniqueness conditions for solutions of (4)\* on each set  $\{(x, t) : x \in \mathbb{D}_n, t > 0\}$ . Suppose there is a nonnegative continuous function  $V : \mathbb{D} \times [0, T] \rightarrow \mathbb{R}_+$  with continuous partial derivatives of the first-order with respect to  $t$  and first and second-order with respect to  $x$  and satisfying  $LV \leq cV$  for some positive constant  $c$  and  $t > 0, x \in \mathbb{D}$ . If also,

$$\inf_{t>0, x \in \mathbb{D} \setminus \mathbb{D}_n} V(x, t) \rightarrow \infty \text{ when } n \rightarrow \infty,$$

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then, for any  $x_0 \in \mathbb{D}$ , there is a unique Markovian continuous time solution  $x(t)$  of (4)\* with  $x(0) = x_0$  and  $x(t) \in \mathbb{D}$ , for all  $t > 0$ , a.s.

Let  $\mathbb{D}$  be the set

$$\mathbb{D} = \{(S, A, C, Q, R) : (S, A, C, Q, R) \in \mathbb{R}_+^5\}, \quad (1)$$

where  $\mathbb{R}_+^5 = \{x \in \mathbb{R}^5, x_i > 0, 1 \leq i \leq 5\}$ .

In order to ensure biological justification for the model, it is necessary to show that the solutions of the system (3)\* are positive and do not explode in finite time. Thus, our goal is to prove that set  $\mathbb{D}$  defined in (1) is invariant, which will imply that the solutions of system (3)\* remain positive and non-explosive within set  $\mathbb{D}$ .

**Theorem 2.1** For any initial condition  $(S_0, A_0, C_0, Q_0, R_0) \in \mathbb{D}$  there is a unique continuous time, Markovian global solution  $(S(t), A(t), C(t), Q(t), R(t))$ , of system (3)\* on  $t \geq 0$ . This solution is invariant with respect to  $\mathbb{D}$  with probability 1.

*Proof.* The coefficients of system (3)\* are locally Lipschitz continuous on  $\mathbb{D}$ , for any initial condition  $(S_0, A_0, C_0, Q_0, R_0) \in \mathbb{D}$ . It means that there is unique local maximal solution  $(S(t), A(t), C(t), Q(t), R(t))$  for  $t \in [0, \tau(\mathbb{D})]$ , where  $\tau(\mathbb{D})$  represents the random time related to the first moment when solution  $(S(t), A(t), C(t), Q(t), R(t))$  leaves region  $\mathbb{D}$ , i.e.

$$\tau(\mathbb{D}) = \inf\{t \geq 0 : S(t) \notin (0, \infty) \vee A(t) \notin (0, \infty) \vee C(t) \notin (0, \infty) \vee Q(t) \notin (0, \infty) \vee R(t) \notin (0, \infty)\}.$$

To show that this solution is global, we need to prove that  $\tau(\mathbb{D}) = \infty$  a.s.

For  $n \in \mathbb{N}$ , let  $\mathbb{D}_n$  be the sets

$$\mathbb{D}_n = \{(S, A, C, Q, R) : e^{-n} < S < e^n, e^{-n} < A < e^n, e^{-n} < C < e^n, e^{-n} < Q < e^n, e^{-n} < R < e^n\}.$$

Let us define function  $V \in C^2(\mathbb{D}, \mathbb{R}_+)$  in the following way

$$V(S, A, C, Q, R) = S + A - \ln A + C - \ln C + Q - \ln Q + R - \ln R.$$

For  $(S, A, C, Q, R) \in \mathbb{D}$ , by using inequality  $u - 1 - \ln u > 0$ , that holds for any  $u > 0$ , we obtain  $V(S, A, C, Q, R) \geq 4$ . By applying the differential operator  $L$  on function  $V(S, A, C, Q, R)$ , yields

$$\begin{aligned} LV(S, A, C, Q, R) = & \Pi + \omega R - (\lambda + \mu)S + \frac{A-1}{A}(\lambda S - k_1 A) + \frac{C-1}{C}(\xi A - k_2 C) \\ & + \frac{Q-1}{Q}(\alpha C - k_3 Q) + \frac{R-1}{R}(\kappa A + \psi C + \gamma Q - k_4 R) + \frac{1}{2} \sum_{i=2}^5 \sigma_i^2, \end{aligned}$$

where  $k_1 = \xi + \kappa + \mu + \delta_a$ ,  $k_2 = \alpha + \psi + \mu + \delta_c$ ,  $k_3 = \gamma + \mu + \delta_q$  and  $k_4 = \omega + \gamma$ . Since the solution of system (3)\* belongs to  $\mathbb{D}$ , if we remove some nonpositive terms, we conclude,

$$LV(S, A, C, Q, R) \leq 4c,$$

where  $c = \frac{1}{4}(\Pi + k_1 + k_2 + k_3 + k_4 + \frac{1}{2} \sum_{i=2}^5 \sigma_i^2)$  is a positive constant.

Since  $V(S, A, C, Q, R) \geq 4$  for  $(S, A, C, Q, R) \in \mathbb{D}$ , then  $LV(S, A, C, Q, R) \leq cV(S, A, C, Q, R)$ . Note that,  $\inf_{(S, A, C, Q, R) \in \mathbb{D} \setminus \mathbb{D}_n} V(S, A, C, Q, R) > 3e^n$  for  $n \in \mathbb{N}$ .

Let  $W(S, A, C, Q, R, t) = e^{-ct} V(S, A, C, Q, R)$  be the function defined on  $\mathbb{D} \times [0, \infty)$ . By applying operator  $L$  on  $W$ , we obtain

$$LW(S, A, C, Q, R, t) = e^{-ct}(-cV(S, A, C, Q, R) + LV(S, A, C, Q, R)) \leq 0.$$

Denote with  $\tau_n = \min\{t, \tau(\mathbb{D})\}$  stopping times for fixed  $t \in [0, \infty)$ . By applying Dynkin's formula we get

$$\begin{aligned} \mathbb{E}W(S(\tau_n), A(\tau_n), C(\tau_n), Q(\tau_n), R(\tau_n), \tau_n) \\ = \mathbb{E}W(S(0), A(0), C(0), Q(0), R(0), 0) + \mathbb{E} \left[ \int_0^{\tau_n} LW(S(u), A(u), C(u), Q(u), R(u), u) du \right] \end{aligned}$$

$$\begin{aligned}
&\leq \mathbb{E}W(S(0), A(0), C(0), Q(0), R(0), 0) \\
&= \mathbb{E}V(S(0), A(0), C(0), Q(0), R(0)) \\
&= \mathbb{E}V(S_0, A_0, C_0, Q_0, R_0) \\
&= V(S_0, A_0, C_0, Q_0, R_0).
\end{aligned}$$

It remains to show that  $\mathbb{P}(\tau(\mathbb{D}) < t) = 0$ . Since

$$\begin{aligned}
\mathbb{E} \left[ e^{c(t-\tau_n)} V(S(\tau_n), A(\tau_n), C(\tau_n), Q(\tau_n), R(\tau_n)) \right] &= \mathbb{E} \left[ e^{ct} W(S(\tau_n), A(\tau_n), C(\tau_n), Q(\tau_n), R(\tau_n), \tau_n) \right] \\
&\leq e^{ct} V(S_0, A_0, C_0, Q_0, R_0),
\end{aligned}$$

bearing in mind that  $\mathbb{D}_n \subset \mathbb{D}$ , we obtain

$$\begin{aligned}
0 &\leq \mathbb{P}(\tau(\mathbb{D}) < t) \leq \mathbb{P}(\tau(\mathbb{D}_n) < t) \\
&= \mathbb{P}(\tau_n < t) \\
&= \mathbb{E}(\mathbf{1}_{\{\tau_n < t\}}) \\
&\leq \mathbb{E} \left( e^{c(t-\tau(\mathbb{D}_n))} \frac{V(S(\tau(\mathbb{D}_n)), A(\tau(\mathbb{D}_n)), C(\tau(\mathbb{D}_n)), Q(\tau(\mathbb{D}_n)), R(\tau(\mathbb{D}_n)))}{\inf_{(S,A,C,Q,R) \in \mathbb{D} \setminus \mathbb{D}_n} V(S, A, C, Q, R)} \mathbf{1}_{\{\tau_n < t\}} \right) \\
&\leq e^{ct} \frac{V(S_0, A_0, C_0, Q_0, R_0)}{\inf_{(S,A,C,Q,R) \in \mathbb{D} \setminus \mathbb{D}_n} V(S, A, C, Q, R)} \\
&\leq e^{ct} \frac{V(S_0, A_0, C_0, Q_0, R_0)}{3e^n} \rightarrow 0, \quad n \rightarrow \infty,
\end{aligned}$$

for all  $(S_0, A_0, C_0, Q_0, R_0) \in \mathbb{D}_n$  and for all fixed  $t \in [0, \infty)$ . Thus,  $\mathbb{P}(\tau(\mathbb{D}) < t) = 0$  for  $(S_0, A_0, C_0, Q_0, R_0) \in \mathbb{D}_n$ , and  $t \geq 0$ . Consequently,  $\mathbb{P}(\tau(\mathbb{D}) = \infty) = 1$ .

Finally, the invariance property of the solution  $(S(t), A(t), C(t), Q(t), R(t))$ ,  $t \geq 0$  is proved.

In view of the above theorem, we see that the positively invariant set  $\mathbb{D}$  can attract every solution with initial conditions starting in its state space  $\mathbb{R}_+^5$ . Namely, every trajectory of model (3)\* with the initial conditions in  $\mathbb{R}_+^5$  eventually stays in  $\mathbb{D}$ .

II Page 4801, the line 15 from bellow. In [2] is stated „Now, if we substitute  $A = C = Q = 0$  in (9)\* we obtain” what can’t hold, since we have shown that solution is positive. Conditions (19)\* and (20)\* will be obtained in the following way.

We consider equation

$$d\tilde{R}(t) = [\kappa\tilde{A}(t) + \psi\tilde{C}(t) + \gamma\tilde{Q}(t) - k_4\tilde{R}(t)]dt + \sigma_5\tilde{R}(t)dw_5(t), \quad t \geq 0, \quad (2)$$

with initial condition  $\tilde{R}(0) = R_0$ .

Since  $\tilde{A}(t)$ ,  $\tilde{C}(t)$  i  $\tilde{Q}(t)$  are asymptotically mean square stable then there exist times  $T_1, T_2$  i  $T_3$  such that for arbitrary  $\varepsilon > 0$  holds:  $\mathbb{E}\tilde{A}^2(t) < \varepsilon$  for every  $t \geq T_1$ ,  $\mathbb{E}\tilde{C}^2(t) < \varepsilon$  for every  $t \geq T_2$  and  $\mathbb{E}\tilde{Q}^2(t) < \varepsilon$  for every  $t \geq T_3$ . Hence, there exists  $T = \max\{T_1, T_2, T_3\}$  such that for every  $\varepsilon > 0$  and  $t \geq T$  holds

$$\max\{\mathbb{E}\tilde{A}^2(t), \mathbb{E}\tilde{C}^2(t), \mathbb{E}\tilde{Q}^2(t)\} < \varepsilon. \quad (3)$$

Let us denote  $f(t) = \kappa\tilde{A}(t) + \psi\tilde{C}(t) + \gamma\tilde{Q}(t)$ ,  $t \geq T$ . Equation (2) is linear stochastic differential equation, which solution has the following form,

$$\tilde{R}(t) = e^{-\left(k_4 + \frac{\sigma_5^2}{2}\right)(t-T)} e^{\sigma_5(w_5(t) - w_5(T))} \tilde{R}_0(T) + \int_T^t e^{-\left(k_4 + \frac{\sigma_5^2}{2}\right)(t-s)} e^{\sigma_5(w_5(t) - w_5(s))} \cdot f(s) ds, \quad s \geq T.$$

By calculating the expectation  $\mathbb{E}\tilde{R}^2(t)$ , applying elementary inequality  $(a+b)^2 \leq a^2 + b^2$ , and Jensen inequality, we get

$$\mathbb{E}\tilde{R}^2(t) \leq 2e^{-2\left(k_4 + \frac{\sigma_5^2}{2}\right)(t-T)} \mathbb{E} \left[ \tilde{R}_0^2(T) e^{2\sigma_5(w_5(t) - w_5(T))} \right] + 2 \int_T^t e^{-2\left(k_4 + \frac{\sigma_5^2}{2}\right)(t-s)} \mathbb{E} \left[ e^{2\sigma_5(w_5(t) - w_5(s))} \cdot f^2(s) \right] ds, \quad t \geq T.$$

Having in mind that  $w_i = \{w_i(t), t \geq 0\}$ ,  $i = 1, 2, \dots, 5$ , are independent Brownian motions with independent increments and the definition of quadratic variation of Wiener process, yields

$$\mathbb{E}\tilde{R}^2(t) \leq 2e^{-2\left(k_4 + \frac{\sigma_5^2}{2}\right)(t-T)} \mathbb{E}\tilde{R}_0^2(T) \cdot e^{4\frac{\sigma_5^2}{2}(t-T)} + 2 \int_T^t e^{-2\left(k_4 + \frac{\sigma_5^2}{2}\right)(t-s)} e^{4\frac{\sigma_5^2}{2}(t-T)} \cdot \mathbb{E}f^2(s) ds, \quad t \geq T. \quad (4)$$

By using inequality  $(a + b + c)^2 \leq 3a^2 + 3b^2 + 3c^2$  and estimation (3), we obtain

$$f^2(s) \leq c(\varepsilon), \quad s \geq T,$$

where  $c(\varepsilon) = 3\varepsilon^2(\kappa^2 + \psi^2 + \gamma^2)$ . Therefore, we get

$$\begin{aligned} \mathbb{E}\tilde{R}^2(t) &\leq 2e^{-2\left(k_4 - \frac{\sigma_5^2}{2}\right)(t-T)} \mathbb{E}\tilde{R}_0^2(T) + 2e^{-2\left(k_4 - \frac{\sigma_5^2}{2}\right)t} \int_T^t e^{2\left(k_4 - \frac{\sigma_5^2}{2}\right)s} c(\varepsilon) ds \\ &= 2e^{-2\left(k_4 - \frac{\sigma_5^2}{2}\right)(t-T)} \mathbb{E}\tilde{R}_0^2(T) + \frac{1 - e^{-2\left(k_4 - \frac{\sigma_5^2}{2}\right)(t-T)}}{k_4 - \frac{\sigma_5^2}{2}} c(\varepsilon). \end{aligned} \quad (5)$$

Calculating the limit in (5) when  $t \rightarrow \infty$ , we conclude that

$$\lim_{t \rightarrow \infty} \mathbb{E}\tilde{R}^2(t) = 0,$$

when  $\sigma_5^2 < 2k_4$ . This implies the solution  $\tilde{R}(t), t \geq 0$ , is asymptotically mean square stable, when condition (20)\* is satisfied.

Since the solutions  $\tilde{A}(t), \tilde{C}(t), \tilde{Q}(t), \tilde{R}(t), t \geq 0$ , are asymptotically mean square stable, in a similar way, we obtain

$$\mathbb{E}\tilde{X}^2(t) \leq 2e^{-2\left(\mu - \frac{\sigma_1^2}{2}\right)(t-T)} \mathbb{E}\tilde{X}_0^2(T) + \frac{1 - e^{-2\left(\mu - \frac{\sigma_1^2}{2}\right)(t-T)}}{\mu - \frac{\sigma_1^2}{2}} c_1(\varepsilon), \quad t \geq T,$$

where  $T = \max\{T_1, T_2, T_3, T_4\}$  and  $T_4$  is a point in time such that  $\mathbb{E}\tilde{X}^2(t) < \varepsilon$  for every  $t \geq T_4$ , while  $\varepsilon > 0, T_1, T_2$  and  $T_3$  are as previously defined. The constant  $c_1(\varepsilon)$  has the form  $c_1(\varepsilon) = 4\varepsilon^2(\beta^2(\eta^2 + 1 + \zeta^2) + \omega^2)$ . Condition  $\sigma_1^2 < 2\mu$ , implies that

$$\lim_{t \rightarrow \infty} \mathbb{E}\tilde{X}^2(t) = 0,$$

from what we conclude that the solution  $\tilde{X}(t), t \geq 0$ , is asymptotically mean square stable under condition (19)\*.

**III** Page 4802, the line 15 from below: The rest of this section is completely revised due to limitation in assumption  $N = N^*$  (line 1, page 4803) and substitution  $\tilde{x} = \tilde{y} = \tilde{z} = \tilde{v} = 0$  which doesn't hold.

In addition we determine the sufficient conditions for coefficients of the system (21)\* that guarantee the stability in probability of endemic equilibrium. For that reason, we introduce new variables into system (21)\*  $u = S - S^*, x = A - A^*, y = C - C^*, z = Q - Q^*, v = R - R^*$  and we obtain stochastic system

$$\begin{aligned} du(t) &= \left[ \Pi + \omega(v(t) + R^*) - \left( \beta \frac{\eta(x(t) + A^*) + y(t) + C^* + \xi(z(t) + Q^*)}{u(t) + S^* + x(t) + A^* + y(t) + C^* + z(t) + Q^* + v(t) + R^*} + \mu \right) \right] dt + \sigma_1 u(t) dw_1(t) \\ dx(t) &= \left[ \beta \frac{\eta(x(t) + A^*) + y(t) + C^* + \xi(z(t) + Q^*)}{u(t) + S^* + x(t) + A^* + y(t) + C^* + z(t) + Q^* + v(t) + R^*} - k_1(x(t) + A^*) \right] dt + \sigma_2 x(t) dw_2(t) \\ dy(t) &= [\xi(x(t) + A^*) - k_2(y(t) + C^*)] dt + \sigma_3 y(t) dw_3(t) \\ dz(t) &= [\alpha(y(t) + C^*) - k_3(z(t) + Q^*)] dt + \sigma_4 z(t) dw_4(t) \end{aligned} \quad (6)$$



$$dv(t) = [\kappa(x(t) + A^*) + \psi(y(t) + C^*) + \gamma(z(t) + Q^*) - k_4(v(t) + R^*)] dt + \sigma_5 v(t) dw_5(t), \quad t \geq 0$$

with initial condition

$$u(0) = s_0 - S^*, \quad x(0) = a_0 - A^*, \quad y(0) = c_0 - C^*, \quad z(0) = q_0 - Q^*, \quad v(0) = r_0 - R^*. \quad (7)$$

Stability in probability for system (6) is equivalent to the stability in probability of endemic equilibrium of system (21)\*.

Before we prove stability in probability of the trivial solution of system (6), with the same motivation as in the Section 2.2\*, first of all we consider its linear form. The procedure of linearization is similar to one described in 2.2\*, whereby the functions

$$F^1(u, x, y, z, v) = \Pi + \omega(v + R^*) - \left( \beta \frac{\eta(x + A^*) + y + C^* + \xi(z + Q^*)}{u + S^* + x + A^* + y + C^* + z + Q^* + v + R^*} + \mu \right) (u + S^*),$$

$$F^2(u, x, y, z, v) = \beta \frac{\eta(x + A^*) + y + C^* + \xi(z + Q^*)}{u + S^* + x + A^* + y + C^* + z + Q^* + v + R^*} - k_1(x + A^*),$$

are nonlinear, and hence they must be linearized.

By using notation from [5]\*, we denote  $Y = \frac{k_2}{\xi} + 1 + \frac{\alpha}{k_3} + \frac{1}{k_4} \left( \frac{k_1 k_2}{\xi} + \psi \right)$ , and obtain:

$$C^* = \frac{\mathcal{R}_0 - 1}{Y} S^*, \quad A^* = \frac{k_2}{\xi} \frac{\mathcal{R}_0 - 1}{Y} S^*, \quad Q^* = \frac{\alpha}{k_3} \frac{\mathcal{R}_0 - 1}{Y} S^*, \quad R^* = \frac{1}{k_4} \left( \frac{k_1 k_2}{\xi} + \psi \right) \frac{\mathcal{R}_0 - 1}{Y} S^*.$$

Furthermore, it holds

$$N^* = S^* + A^* + C^* + Q^* + R^* = \frac{\mathcal{R}_0 - 1}{Y} S^* \left[ \frac{k_2}{\xi} + 1 + \frac{\alpha}{k_3} + \frac{1}{k_4} \left( \frac{k_1 k_2}{\xi} + \psi \right) \right] + S^* = \mathcal{R}_0 S^*.$$

Having in mind this notation, and the fact  $\lambda^* = \beta \frac{\eta A^* + C^* + \xi Q^*}{S^* + A^* + C^* + Q^* + R^*}$ , linearized system for system (6) has the form

$$\begin{aligned} d\tilde{u}(t) &= \left[ -\left( \mu + (\mathcal{R}_0 - 1) \frac{\lambda^*}{\mathcal{R}_0} \right) \tilde{u}(t) + \frac{\lambda^* - \beta \eta}{\mathcal{R}_0} \tilde{x}(t) + \frac{\lambda^* - \beta}{\mathcal{R}_0} \tilde{y}(t) + \frac{\lambda^* - \beta \zeta}{\mathcal{R}_0} \tilde{z}(t) + \left( \omega + \frac{\lambda^*}{\mathcal{R}_0} \right) \tilde{v}(t) \right] dt + \sigma_1 \tilde{u}(t) dw_1(t) \\ d\tilde{x}(t) &= \left[ \frac{\lambda^*}{\mathcal{R}_0} (\mathcal{R}_0 - 1) \tilde{u}(t) - \left( \frac{\lambda^* - \beta \eta}{\mathcal{R}_0} + k_1 \right) \tilde{x}(t) - \frac{\lambda^* - \beta}{\mathcal{R}_0} \tilde{y}(t) - \frac{\lambda^* - \beta \zeta}{\mathcal{R}_0} \tilde{z}(t) - \frac{\lambda^*}{\mathcal{R}_0} \tilde{v}(t) \right] dt + \sigma_2 \tilde{x}(t) dw_2(t) \\ d\tilde{y}(t) &= [\xi \tilde{x}(t) - k_2 \tilde{y}(t)] dt + \sigma_3 \tilde{y}(t) dw_3(t) \\ d\tilde{z}(t) &= [\alpha \tilde{y}(t) - k_3 \tilde{z}(t)] dt + \sigma_4 \tilde{z}(t) dw_4(t) \\ d\tilde{v}(t) &= [\kappa \tilde{x}(t) + \psi \tilde{y}(t) + \gamma \tilde{z}(t) - k_4 \tilde{v}(t)] dt + \sigma_5 \tilde{v}(t) dw_5(t), \quad t \geq 0 \end{aligned} \quad (8)$$

with initial condition

$$\tilde{u}(0) = s_0 - S^*, \quad \tilde{x}(0) = a_0 - A^*, \quad \tilde{y}(0) = c_0 - C^*, \quad \tilde{z}(0) = q_0 - Q^*, \quad \tilde{v}(0) = r_0 - R^*. \quad (9)$$

Now, we may proceed by considering the conditions for asymptotic mean square stability of system (8).

**Theorem 3.2** Let the parameters of system (8), for every initial value (9), satisfy conditions

$$\beta \zeta < \lambda^* < \beta < \beta \eta, \quad (10)$$

$$\omega(\mu + k_1 + \lambda^*) < 2(\lambda^*(k_1 + 2\mu) + \mu(k_1 + \mu)), \quad (11)$$

$$2\kappa(\mu + k_1 + \lambda^*)(2.5\alpha + \xi) < \psi\omega\lambda^* + 4k_2\kappa(\mu + k_1 + \lambda^*), \quad (12)$$

$$5\kappa\alpha(\mu + k_1 + \lambda^*) < 10\kappa k_3(\mu + k_1 + \lambda^*) + \gamma\omega\lambda^*, \quad (13)$$

$$\gamma + \psi < 2k_4, \quad (14)$$

$$\mathcal{R}_0 > \max\{1, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}\}, \quad (15)$$

$$\sigma_1^2 < 2 \left( \mu - \frac{\omega}{2} - a^* \frac{\beta(1-\zeta) + \lambda^*(3-2\mathcal{R}_0)}{2\mathcal{R}_0} \right), \quad (16)$$

$$\sigma_2^2 < 2 \left( \kappa + \mu + \delta_a - a^* \frac{\beta(1-\zeta)}{2\mathcal{R}_0} \right), \quad (17)$$

$$\sigma_3^2 < 2 \left( k_2 - \frac{\xi}{2} - \frac{a^* \frac{\beta-\lambda^*}{\mathcal{R}_0} + 2.5\alpha + b^*\psi}{2} \right), \quad (18)$$

$$\sigma_4^2 < \frac{5k_3 - 2.5\alpha - \left( a^* \frac{(\lambda^*-\beta\zeta)}{\mathcal{R}_0} + b^*\gamma \right)}{2.5}, \quad (19)$$

$$\sigma_5^2 < \frac{2b^*k_4 - \left( b^*(\gamma + \psi) + \frac{\omega}{2} + a^* \frac{\lambda^*}{2\mathcal{R}_0} \right)}{b^*}, \quad (20)$$

where  $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, a^*$  and  $b^*$  are positive constants defined as

$$\begin{aligned} \mathcal{A} &= \frac{2\beta\eta}{\mu + k_1 + 2\lambda^*}, \\ \mathcal{B} &= \frac{\beta\eta(2\mu - \omega) + (\mu + k_1)((1-\zeta)\beta + 3\lambda^*)}{(2\mu - \omega)(\mu + k_1 + \lambda^*) + 2\lambda^*(\mu + k_1)}, \\ \mathcal{C} &= \frac{2\beta\eta(\mu + \kappa + \delta_a) + \beta(1-\zeta)(\mu + k_1)}{2(\mu + k_1 + \lambda^*)(\mu + \kappa + \delta_a)}, \\ \mathcal{D} &= \frac{2\kappa(2k_2 - (2.5\alpha + \xi))\beta\eta + \psi\beta\eta\omega + (\mu + k_1)(\psi\lambda^* + 2\kappa(\beta - \lambda^*))}{2\kappa(2k_2 - (2.5\alpha + \xi))(\mu + k_1 + \lambda^*) + \psi\omega\lambda^*}, \\ \mathcal{E} &= \frac{\beta(\gamma\eta\omega - 2\kappa\zeta(\mu + k_1)) + (\mu + k_1)(\gamma + 2\kappa)\lambda^* + 2\kappa\beta\eta(5k_3 - 2.5\alpha)}{2\kappa(5k_3 - 2.5\alpha)(\mu + k_1 + \lambda^*) + \gamma\omega\lambda^*}, \\ \mathcal{F} &= \frac{\beta\eta\omega(\kappa + 2k_4 - (\gamma + \psi)) + \lambda^*(\mu + k_1)(2k_4 - (\gamma + \kappa + \psi))}{\omega(\lambda^*(2k_4 - (\gamma + \psi)) + \kappa(\mu + k_1 + \lambda^*))}, \\ a^* &= \frac{k_1 + \mu}{k_1 + \mu + \lambda^* - \frac{\beta\eta}{\mathcal{R}_0}}, \end{aligned} \quad (21)$$

$$b^* = \frac{a^* \left( \omega + \frac{\lambda^*}{\mathcal{R}_0} \right) - \omega}{2\kappa}. \quad (22)$$

Then, the trivial equilibrium of system (8) is asymptotically mean square stable.

*Proof.* In order to prove this theorem we use Lyapunov function of the form

$$V(\tilde{u}, \tilde{x}, \tilde{y}, \tilde{z}, \tilde{v}) = \frac{(\tilde{u} + \tilde{x})^2}{2} - a\tilde{u}\tilde{x} + \tilde{y}^2 + 2.5\tilde{z}^2 + b\tilde{v}^2,$$

where  $a$  and  $b$  are positive constants that will be determined in the sequel. Function  $V$  is positive for  $a \in (0, 2)$  what is guaranteed under condition (15). By applying operator  $L$  on function  $V$  we get

$$\begin{aligned} LV &= -\tilde{u}^2 \left[ \left( \mu - \frac{\sigma_1^2}{2} \right) + a \frac{\lambda^*}{\mathcal{R}_0} (\mathcal{R}_0 - 1) \right] - \tilde{x}^2 \left[ \left( k_1 - \frac{\sigma_2^2}{2} \right) - a \frac{\beta\eta - \lambda^*}{\mathcal{R}_0} \right] - 2\tilde{y}^2 \left[ k_2 - \frac{\sigma_3^2}{2} \right] - 5\tilde{z}^2 \left[ k_3 - \frac{\sigma_4^2}{2} \right] - 2\tilde{v}^2 \left[ k_4 - \frac{\sigma_5^2}{2} \right] \\ &\quad - \tilde{u}\tilde{y}a \frac{\beta - \lambda^*}{\mathcal{R}_0} + \tilde{u}\tilde{z}a \frac{\lambda^* - \beta\zeta}{\mathcal{R}_0} + \tilde{u}\tilde{v} \left[ \omega + a \frac{\lambda^*}{\mathcal{R}_0} \right] + \tilde{x}\tilde{y} \left[ 2\xi + a \frac{\beta - \lambda^*}{\mathcal{R}_0} \right] - \tilde{x}\tilde{z}a \frac{\lambda^* - \beta\zeta}{\mathcal{R}_0} + \tilde{v}\tilde{x} \left[ \omega + 2kb - a \left( \omega + \frac{\lambda^*}{\mathcal{R}_0} \right) \right] \\ &\quad + 5\alpha\tilde{y}\tilde{z} + \tilde{u}\tilde{x} \left[ a \left( k_1 + \frac{\lambda^* - \beta\zeta}{\mathcal{R}_0} + \mu + \frac{\mathcal{R}_0 - 1}{\mathcal{R}_0} \lambda^* \right) - (k_1 + \mu) \right] + 2b\psi\tilde{y}\tilde{v} + 2b\gamma\tilde{z}\tilde{v}. \end{aligned}$$

In order to annul the quantities in the brackets multiplying  $\tilde{u}\tilde{x}$  and  $\tilde{v}\tilde{x}$ , we choose constants  $a$  i  $b$  to be as  $a^*$  i  $b^*$ , i.e. as in (21) and (22), respectively. Thus, we obtain

$$LV = -\tilde{u}^2 \left[ \left( \mu - \frac{\sigma_1^2}{2} \right) + a^* \frac{\lambda^*}{\mathcal{R}_0} (\mathcal{R}_0 - 1) \right] - \tilde{x}^2 \left[ \left( k_1 - \frac{\sigma_2^2}{2} \right) - a^* \frac{\beta\eta - \lambda^*}{\mathcal{R}_0} \right] - 2\tilde{y}^2 \left[ k_2 - \frac{\sigma_3^2}{2} \right] - 5\tilde{z}^2 \left[ k_3 - \frac{\sigma_4^2}{2} \right] - 2\tilde{v}^2 b^* \left[ k_4 - \frac{\sigma_5^2}{2} \right] \\ - \tilde{u}\tilde{y} a^* \frac{\beta - \lambda^*}{\mathcal{R}_0} + \tilde{x}\tilde{y} \left[ 2\xi + a^* \frac{\beta - \lambda^*}{\mathcal{R}_0} \right] + \tilde{u}\tilde{z} a^* \frac{\lambda^* - \beta\zeta}{\mathcal{R}_0} + \tilde{u}\tilde{v} \left[ \omega + a^* \frac{\lambda^*}{\mathcal{R}_0} \right] - \tilde{x}\tilde{z} a^* \frac{\lambda^* - \beta\zeta}{\mathcal{R}_0} + 5\alpha\tilde{y}\tilde{z} + 2b^*\psi\tilde{y}\tilde{v} + 2b^*\gamma\tilde{z}\tilde{v}.$$

In addition, by using elementary inequality  $\pm 2xy \leq x^2 + y^2$ , we conclude that

$$LV \leq -\tilde{u}^2 \left[ \mu - \frac{\omega}{2} + a^* \left( \frac{\lambda^*}{\mathcal{R}_0} (\mathcal{R}_0 - 1) - \frac{\beta - \lambda^*}{2\mathcal{R}_0} - \frac{\lambda^* - \beta\zeta}{2\mathcal{R}_0} - \frac{\lambda^*}{2\mathcal{R}_0} \right) - \frac{\sigma_1^2}{2} \right] \\ - \tilde{x}^2 \left[ k_1 - \xi - \frac{a^*}{2} \left( \frac{\lambda^* - \beta\zeta}{\mathcal{R}_0} + \frac{\beta - \lambda^*}{\mathcal{R}_0} \right) - \frac{\sigma_2^2}{2} \right] \\ - \tilde{y}^2 \left[ 2 \left( k_2 - \frac{\xi}{2} \right) - \left( a^* \frac{\beta - \lambda^*}{\mathcal{R}_0} + 2.5\alpha + b^*\psi \right) - \sigma_3^2 \right] \\ - \tilde{z}^2 \left[ 5 \left( k_3 - \frac{\alpha}{2} \right) - \left( a^* \frac{(\lambda^* - \beta\zeta)}{\mathcal{R}_0} + b^*\gamma \right) - 2.5\sigma_4^2 \right] \\ - \tilde{v}^2 \left[ 2b^* \left( k_4 - \frac{\gamma + \psi}{2} \right) - \left( \frac{\omega}{2} + a^* \frac{\lambda^*}{2\mathcal{R}_0} \right) - b^*\sigma_5^2 \right] \\ = -\tilde{u}^2 \left[ \mu - \frac{\omega}{2} - a^* \frac{\beta(1 - \zeta) + \lambda^*(3 - 2\mathcal{R}_0)}{2\mathcal{R}_0} - \frac{\sigma_1^2}{2} \right] \\ - \tilde{x}^2 \left[ \kappa + \mu + \delta_a - a^* \frac{\beta(1 - \zeta)}{2\mathcal{R}_0} - \frac{\sigma_2^2}{2} \right] \\ - \tilde{y}^2 \left[ 2 \left( k_2 - \frac{\xi}{2} \right) - \left( a^* \frac{\beta - \lambda^*}{\mathcal{R}_0} + 2.5\alpha + b^*\psi \right) - \sigma_3^2 \right] \\ - \tilde{z}^2 \left[ 5 \left( k_3 - \frac{\alpha}{2} \right) - \left( a^* \frac{(\lambda^* - \beta\zeta)}{\mathcal{R}_0} + b^*\gamma \right) - 2.5\sigma_4^2 \right] \\ - \tilde{v}^2 \left[ 2b^* \left( k_4 - \frac{\gamma + \psi}{2} \right) - \left( \frac{\omega}{2} + a^* \frac{\lambda^*}{2\mathcal{R}_0} \right) - b^*\sigma_5^2 \right].$$

Bearing in mind conditions (10)–(20), all the terms in brackets multiplying  $\tilde{u}^2$ ,  $\tilde{x}^2$ ,  $\tilde{y}^2$ ,  $\tilde{z}^2$  and  $\tilde{v}^2$  are positive, which completes the proof.

Consequently, the Theorem 3.3\* becomes,

**Theorem 3.3** Assume that the parameters of system (8) satisfy all conditions from Theorem 3.2. Then the endemic equilibrium  $E^*$  of system (21)\* is stable in probability.

**IV** Page 4807, line 5 from bellow until 6th line from bellow on page 4808: Since the Theorem 3.3\* is corrected into Theorem 3.3, the corresponding numerical simulation is also changed.

Let the model parameters of system (21)\* be:

$$\Pi = 1, \gamma = 0.01, \kappa = 0.05, \omega = 0.01, \mu = 0.0077, \xi = 0.0001, \alpha = 0.005, \psi = 0.0001, \\ \delta_a = 0.00003, \delta_c = 0.00002, \delta_q = 0.0000269, \eta = 1.5, \zeta = 0.1, \beta = 0.5703. \quad (23)$$

For such choice of parameters the reproduction number has the value  $\mathcal{R}_0 = 9.128 > 1$ , what guarantees the existence of the endemic equilibrium  $E^*$  of system (21)\*.

We use initial value:

$$s_0 = 600, a_0 = 20, c_0 = 60, q_0 = 12, r_0 = 10. \quad (24)$$

According to the conditions (16)–(20) of Theorem 3.3 we set

$$\sigma_1^2 = 0.00936, \sigma_2^2 = 0.0001569, \sigma_3^2 = 0.000688414, \sigma_4^2 = 0.000988279, \sigma_5^2 = 0.0000679. \quad (25)$$

It is easy to check out that all the conditions of Theorem 3.3 hold and thus, we expect that the solution of model will tend to endemic equilibrium  $E^* = (10.1468, 35.0085, 0.3398, 2.00677, 106.339)$ , which means that hepatitis C will persist in population. This situation in population is presented on the Figures (1)–(3).

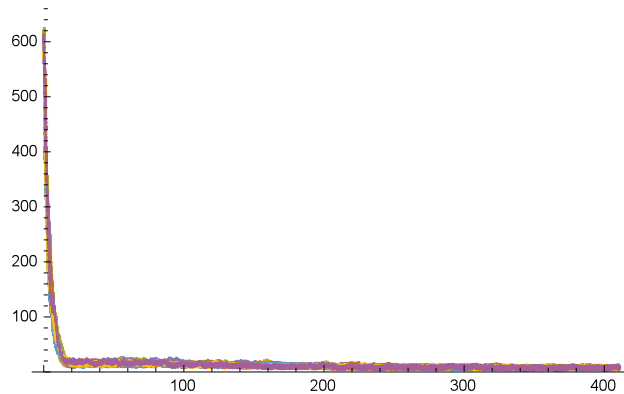


Figure 1: Stochastic trajectories (10) for susceptible individuals  $S(t)$  of model (21)\* with initial conditions (24), parameter values (23) and (25).

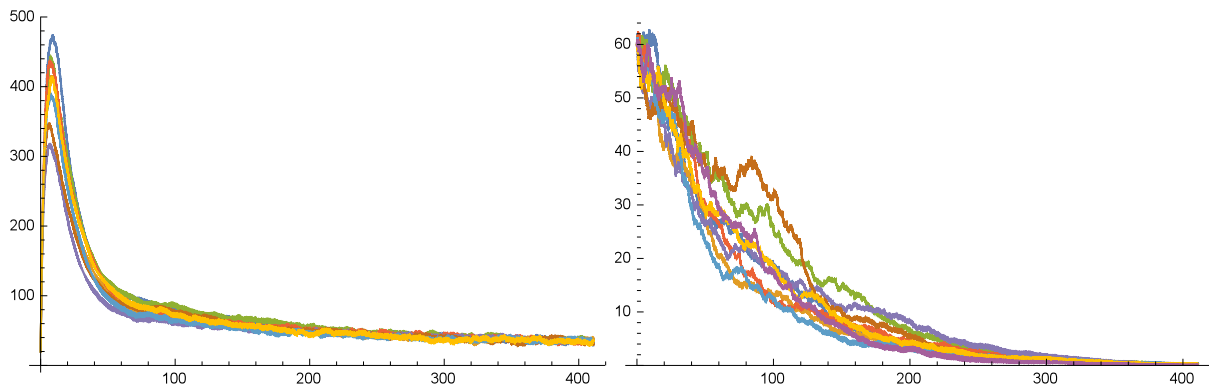


Figure 2: Stochastic trajectories (10) for acute  $A(t)$  (left) and chronic individuals  $C(t)$  (right) of model (21)\* with initial conditions (24), parameter values (23) and (25).

Based on the example, the disease persists in the population because infected individuals are still present in the population, acutely and chronically infected (Figure 2), as well as those infected who are in quarantine (Figure 3, left). Also, from Figure 1 it can be concluded that there is a decrease in the number of susceptible individuals, while the number of individuals recovered from the disease is increasing (Figure 3, right).

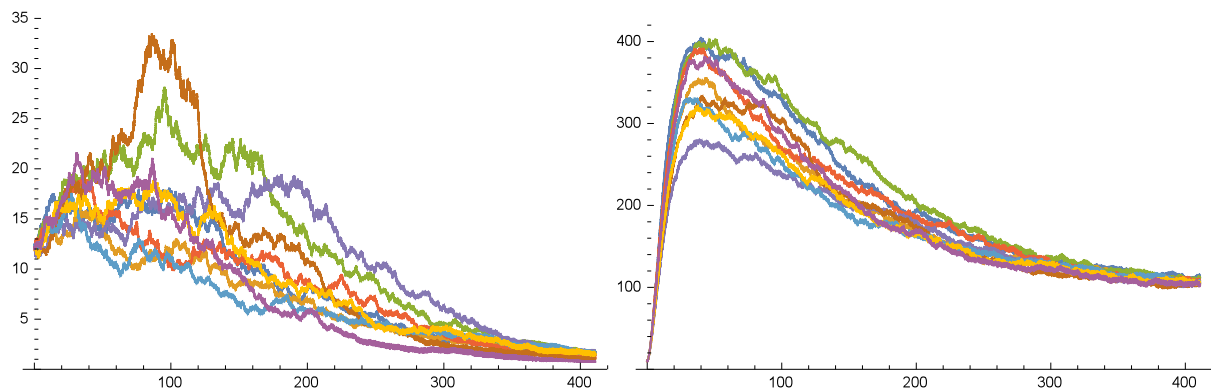


Figure 3: Stochastic trajectories (10) for isolated  $Q(t)$  (left) and recovered individuals  $R(t)$  (right) of model (21)\* with initial conditions (24), parameter values (23) and (25).

## References

- [1] H. Schurz, K. Tosun, Stochastic asymptotic stability of SIR model with variable diffusion rates, *Journal of Dynamics and Differential Equations*, 27 (1) (2014) 69–82.
- [2] Vuk Vujović, Marija Krstić, Stability of stochastic model for Hepatitis C with an isolation stage, *Filomat*, 34 (14), 2020, 4795–4809.