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On the deferred Riesz summability factors

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Abstract. In this study, a summability method is introduced using the deferred Riesz method, and the result presented in [12] is re-examined through this approach. Furthermore, to demonstrate its relevance to other areas, the obtained results are applied to Fourier series generated by orthogonal systems.

1. Introduction

A sequence (x_n) is called to be $\delta-$quasi monotone, if $x_n \to 0$, $x_n > 0$ eventually, and $\Delta x_n \geq -\delta_n$, where $\Delta x_n = x_n - x_{n+1}$ and $\delta = (\delta_n)$ is a sequence of positive numbers [2]. A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in BV$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty$.

Let $\sum d_n$ be an infinite series with partial sums (s_n). The nth $(C,1)$ means of the sequences (s_n) and (nd_n) are denoted by v_n and t_n, respectively. The series $\sum d_n$ is said to be summable $|C,1|_k$, where $k \geq 1$, if [15]

$$\sum_{n=1}^{\infty} n^{k-1}|v_n - v_{n-1}|^k = \sum_{n=1}^{\infty} \frac{|t_n|^k}{n} < \infty.$$

Let (p_n) be a sequence of positive real numbers such that

$$P_n = \sum_{k=0}^{n} p_k \to \infty \ \ as \ \ n \to \infty, \ \ (P_{-i} = p_{-i} = 0, \ \ i \geq 1),$$

and let $\alpha = (\alpha_n)$ and $\beta = (\beta_n)$ be sequences of nonnegative integers satisfying $\alpha_n < \beta_n$ for $n = 1, 2, 3, ...$ and $\lim_{n\to\infty} \beta_n = \infty$ with the condition

$$P_{\alpha_n+1}^{\beta_n} = \sum_{k=\alpha_n+1}^{\beta_n} p_k \neq 0.$$

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Then the transformation

$$D_{\alpha}^{\beta}R_n = \frac{1}{P_{\alpha_n+1}^{\beta_n}} \sum_{m=\alpha_n+1}^{\beta_n} p_m s_m$$

denotes the deferred Riesz mean of the sequence (s_n) [1], [14]. The series $\sum d_n$ is said to be $|\overline{DN}, p_n, \theta_n|_k$ summability if

$$\sum_{n=1}^{\infty} \theta_n^{k-1} |D_{\alpha}^{\beta}R_n - D_{\alpha}^{\beta}R_{n-1}|^k < \infty,$$

where (θ_n) is a sequence of positive numbers. If we consider that $\alpha_n = 0$ and $\beta_n = n$, then $|\overline{DN}, p_n, \theta_n|_k$ summability reduces to $|\overline{N}, p_n, \theta_n|_k$ summability [19]. Also, considering that $\theta_n = n$ and $p_n = 1$ for all n, we obtain $|C, 1|_k$ summability.

Recently, there has been considerable research on absolute summability (see [4]-[12], [16]-[18], [20]-[22]). In [12], Bor presented a result on the $|\overline{N}, p_n, \theta_n|_k$ summability factors of infinite series. Building on this work, a result generalizing Bor's study has been derived using the $|\overline{DN}, p_n, \theta_n|_k$ summability method, which is defined based on the deferred Riesz method.

Now, we need the following Lemma for the proof of our main theorem.

Lemma 1.1. ([3]), *Let $X_n = \sum_{v=0}^{n} \frac{p_v}{P_v}$, for $n \geq 0$, and let $\lambda_n \to 0$ as $n \to \infty$. Let (p_n) be a sequence of positive numbers such that*

$$P_n = O(np_n), \quad as \quad n \to \infty. \tag{1}$$

Suppose there exists a sequence of numbers (K_n) that is $\delta-$quasi monotone with $\sum_{n=1}^{\infty} nX_n\delta_n < \infty$, $\sum_{n=1}^{\infty} K_nX_n$ is convergent, and $|\Delta\lambda_n| \leq K_n$ for all n. Then we have that

$$|\lambda_n|X_n = O(1) \quad as \quad n \to \infty, \tag{2}$$

$$nX_n|K_n| = O(1) \quad as \quad n \to \infty, \tag{3}$$

$$\sum_{n=1}^{\infty} nX_n|\Delta K_n| < \infty. \tag{4}$$

2. Main result

We can now state the main theorem.

Theorem 2.1. *Let $X_n = \sum_{v=0}^{n} \frac{p_v}{P_v}$, for $n \geq 0$, where $(\frac{\theta_n p_n}{P_n})$ is a non-increasing sequence, and let $\lambda_n \to 0$ as $n \to \infty$. Let (p_n) be a sequence of positive numbers such that*

$$P_n = O(np_n), \quad as \quad n \to \infty. \tag{5}$$

Suppose there exists a sequence of numbers (K_n) that is $\delta-$quasi monotone with $\sum_{n=1}^{\infty} nX_n\delta_n < \infty$, $\sum_{n=1}^{\infty} K_nX_n$ is convergent, and $|\Delta\lambda_n| \leq K_n$ for all n. If the condition

$$\sum_{n=1}^{\beta_m} \theta_n^{k-1} (\frac{p_n}{P_n})^k \frac{|t_n|^k}{X_n^{k-1}} = O(X_{\beta_m}) \quad as \quad m \to \infty, \tag{6}$$

holds, then the series $\sum d_n\lambda_n$ is summable $|\overline{DN}, p_n, \theta_n|_k$, $k \geq 1$.

Proof. Let (M_n) denote the sequence of deferred $(\overline{N},p_n)$ mean of the series $\sum d_n\lambda_n$. Therefore from the definition, we write

$$M_{\beta_n}=\frac{1}{P^{\beta_n}_{\alpha_n+1}}\sum_{i=\alpha_n+1}^{\beta_n}p_i\sum_{j=\alpha_n+1}^{i}d_j\lambda_j.$$

Let $P^{\beta_n}_{\alpha_n+1}=P_{\beta_n}$. Then we get $M_{\beta_n}=\frac{1}{P_{\beta_n}}\sum_{i=\alpha_n+1}^{\beta_n}d_i\lambda_i(P_{\beta_n}-P_{i-1})$.

For $n\geq 1$, we get

$$M_{\beta_n}-M_{\beta_n-1}=\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n}\frac{P_{i-1}\lambda_i}{i}id_i.$$

Applying Abel's transformation to $M_{\beta_n}-M_{\beta_n-1}$, we have

$$M_{\beta_n}-M_{\beta_n-1}=\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}}[\sum_{i=\alpha_n+1}^{\beta_n-1}\Delta(\frac{P_{i-1}\lambda_i}{i})\sum_{r=0}^{i}rd_r+(\sum_{r=0}^{\beta_n}rd_r)\frac{P_{\beta_n-1}\lambda_{\beta_n}}{\beta_n}]$$

$$=\frac{p_{\beta_n}t_{\beta_n}\lambda_{\beta_n}(\beta_n+1)}{P_{\beta_n}\beta_n}-\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n-1}\frac{p_i\lambda_i(i+1)t_i}{i}+\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n-1}P_i\Delta\lambda_it_i\frac{i+1}{i}+\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n-1}\frac{P_i\lambda_{i+1}t_i}{i}$$

$$=M_{\beta_n,1}+M_{\beta_n,2}+M_{\beta_n,3}+M_{\beta_n,4}.$$

To finish the proof of the theorem using Minkowski's inequality, it will be sufficient to show that

$$\sum_{n=1}^{\infty}\theta_n^{k-1}|M_{\beta_n,r}|^k<\infty,\quad for\ \ r=1,2,3,4.$$

Therefore we see that

$$\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}|M_{\beta_n,1}|^k=\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}|\frac{p_{\beta_n}t_{\beta_n}\lambda_{\beta_n}(\beta_n+1)}{P_{\beta_n}\beta_n}|^k$$

$$=O(1)\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}|\lambda_{\beta_n}||\lambda_{\beta_n}|^{k-1}(\frac{p_{\beta_n}}{P_{\beta_n}})^k|t_{\beta_n}|^k$$

$$=O(1)\sum_{n=\alpha_m+1}^{\beta_m+1}|\lambda_{\beta_n}|\theta_n^{k-1}(\frac{p_{\beta_n}}{P_{\beta_n}})^k\frac{|t_{\beta_n}|^k}{X_{\beta_n}^{k-1}}.$$

Now, by applying Abel's transformation to this last sum, we obtain

$$=O(1)\sum_{n=\alpha_m+1}^{\beta_m}\Delta|\lambda_{\beta_n}|\sum_{r=1}^{n}\theta_r^{k-1}(\frac{p_{\beta_r}}{P_{\beta_r}})^k\frac{|t_{\beta_r}|^k}{X_{\beta_r}^{k-1}}+O(1)|\lambda_{\beta_m+1}|\sum_{r=0}^{\beta_m+1}\theta_r^{k-1}(\frac{p_{\beta_r}}{P_{\beta_r}})^k\frac{|t_{\beta_r}|^k}{X_{\beta_r}^{k-1}}$$

$$= O(1) \sum_{n=\alpha_m+1}^{\beta_m} |\Delta\lambda_{\beta_n}| X_{\beta_n} + O(1)|\lambda_{\beta_m+1}| X_{\beta_m+1}$$

$$= O(1) \sum_{n=\alpha_m+1}^{\beta_m} |K_{\beta_n}| X_{\beta_n} + O(1)|\lambda_{\beta_m+1}| X_{\beta_m+1} = O(1) \ \ as \ \ m \to \infty,$$

according to the conditions of the theorem and Lemma. From here, continuing with the Hölder's inequality in a similar way to $M_{\beta_n,1}$, the following result is obtained:

$$\sum_{n=\alpha_m+1}^{\beta_m+1} \theta_n^{k-1} |M_{\beta_n,2}|^k = O(1) \sum_{n=\alpha_m+1}^{\beta_m+1} \theta_n^{k-1} (\frac{p_{\beta_n}}{P_{\beta_n}})^k \frac{1}{P_{\beta_n-1}^k} (\sum_{i=\alpha_n+1}^{\beta_n-1} p_i |\lambda_i||t_i|)^k$$

$$= O(1) \sum_{n=\alpha_m+1}^{\beta_m+1} \theta_n^{k-1} (\frac{p_{\beta_n}}{P_{\beta_n}})^k \frac{1}{P_{\beta_n-1}} [\sum_{i=\alpha_n+1}^{\beta_n-1} p_i |\lambda_i|^k |t_i|^k][\frac{1}{P_{\beta_n-1}} \sum_{i=\alpha_n+1}^{\beta_n-1} p_i]^{k-1}$$

$$= O(1) \sum_{i=\alpha_m+1}^{\beta_m} p_i |\lambda_i|^{k-1} |\lambda_i||t_i|^k \sum_{n=i+1}^{\beta_n+1} (\frac{\theta_n p_{\beta_n}}{P_{\beta_n}})^{k-1} \frac{p_{\beta_n}}{P_{\beta_n} P_{\beta_n-1}}$$

$$= O(1) \sum_{i=\alpha_m+1}^{\beta_m} (\frac{\theta_i p_{\beta_i}}{P_{\beta_i}})^{k-1} p_i |t_i|^k |\lambda_i| (\frac{1}{X_i})^{k-1} \sum_{n=i+1}^{\beta_n+1} \frac{p_{\beta_n}}{P_{\beta_n} P_{\beta_n-1}}$$

$$= O(1) \sum_{i=\alpha_m+1}^{\beta_m} \theta_i^{k-1} (\frac{p_{\beta_i}}{P_{\beta_i}})^k \frac{|t_i|^k}{X_i^{k-1}} |\lambda_i| = O(1) \ \ as \ \ m \to \infty.$$

For $r = 3$, we have that

$$\sum_{n=\alpha_m+1}^{\beta_m+1} \theta_n^{k-1} |M_{\beta_n,3}|^k = O(1) \sum_{n=\alpha_m+1}^{\beta_m+1} \theta_n^{k-1} (\frac{p_{\beta_n}}{P_{\beta_n} P_{\beta_n-1}})^k (\sum_{i=\alpha_n+1}^{\beta_n-1} p_i |K_i||t_i| i)^k$$

$$= O(1) \sum_{n=\alpha_m+1}^{\beta_m+1} \theta_n^{k-1} (\frac{p_{\beta_n}}{P_{\beta_n}})^k \frac{1}{P_{\beta_n-1}} (\sum_{i=\alpha_n+1}^{\beta_n-1} p_i |K_i|^k |t_i|^k i^k)(\frac{1}{P_{\beta_n-1}} \sum_{i=\alpha_n+1}^{\beta_n-1} p_i)^{k-1}$$

$$= O(1) \sum_{i=\alpha_m+1}^{\beta_m} p_i |t_i|^k (i|K_i|)^{k-1} i|K_i| \sum_{n=i+1}^{\beta_n+1} (\frac{\theta_n p_{\beta_n}}{P_{\beta_n}})^{k-1} \frac{p_{\beta_n}}{P_{\beta_n} P_{\beta_n-1}}$$

$$= O(1) \sum_{i=\alpha_m+1}^{\beta_m} (\frac{\theta_i p_{\beta_i}}{P_{\beta_i}})^{k-1} \frac{p_{\beta_i}}{P_{\beta_i}} (\frac{1}{X_i})^{k-1} i|K_i||t_i|^k$$

$$= O(1) \sum_{i=\alpha_m+1}^{\beta_m} \theta_i^{k-1} i|K_i| (\frac{p_{\beta_i}}{P_{\beta_i}})^k \frac{|t_i|^k}{X_i^{k-1}}.$$

Now applying Abel's transformation, we obtain

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}\Delta(i|K_i|)\sum_{r=1}^{i}\theta_r^{k-1}(\frac{p_{\beta_r}}{P_{\beta_r}})^k\frac{|t_r|^k}{X_r^{k-1}}+O(1)\beta_m|K_{\beta_m}|\sum_{i=\alpha_m+1}^{\beta_m-1}\theta_i^{k-1}(\frac{p_{\beta_i}}{P_{\beta_i}})^k\frac{|t_i|^k}{X_i^{k-1}}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}|\Delta(iK_i)|X_i+O(1)\beta_m|K_{\beta_m}|X_{\beta_m}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}iX_i\Delta|K_i|+O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}|K_i|X_i+O(1)\beta_m|K_{\beta_m}|X_{\beta_m}=O(1)\ \ as\ \ m\to\infty.$$

Finally, for $r=4$ we get

$$\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}|M_{\beta_n,4}|^k=O(1)\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}(\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}})^k(\sum_{i=\alpha_n+1}^{\beta_n-1}p_i|\lambda_{i+1}||t_i|)^k.$$

Considering Hölder's inequality, it follows from the conditions of theorem and Lemma that

$$= O(1)\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}(\frac{p_{\beta_n}}{P_{\beta_n}})^k\frac{1}{P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n-1}p_i|\lambda_{i+1}|^k|t_i|^k(\frac{1}{P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n-1}p_i)^{k-1}$$

$$= O(1)\sum_{n=\alpha_m+1}^{\beta_m+1}\theta_n^{k-1}(\frac{p_{\beta_n}}{P_{\beta_n}})^k\frac{1}{P_{\beta_n-1}}\sum_{i=\alpha_n+1}^{\beta_n-1}p_i|\lambda_{i+1}|^k|t_i|^k$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m}p_i|\lambda_{i+1}|^{k-1}|\lambda_{i+1}||t_i|^k\sum_{n=i+1}^{\beta_n+1}(\frac{\theta_n p_{\beta_n}}{P_{\beta_n}})^{k-1}\frac{p_{\beta_n}}{P_{\beta_n}P_{\beta_n-1}}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m}\theta_i^{k-1}(\frac{p_{\beta_i}}{P_{\beta_i}})^k\frac{p_i}{P_{\beta_i}}\frac{P_{\beta_i}}{p_{\beta_i}}|\lambda_{i+1}|^k\frac{|t_i|^k}{X_i^{k-1}}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m}\theta_i^{k-1}(\frac{p_{\beta_i}}{P_{\beta_i}})^k|\lambda_{i+1}|\frac{|t_i|^k}{X_i^{k-1}}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}\Delta|\lambda_{i+1}|\sum_{r=1}^{i}\theta_r^{k-1}\frac{|t_r|^k}{\beta_r X_r^{k-1}}+O(1)|\lambda_{\beta_m+1}|\sum_{r=1}^{\beta_m}\theta_r^{k-1}\frac{|t_r|^k}{\beta_r X_r^{k-1}}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}|\Delta\lambda_{i+1}|X_{i+1}+O(1)|\lambda_{\beta_m+1}|X_{\beta_m+1}$$

$$= O(1)\sum_{i=\alpha_m+1}^{\beta_m-1}|K_{i+1}|X_{i+1}+O(1)|\lambda_{\beta_m+1}|X_{\beta_m+1}=O(1)\ \ as\ \ m\to\infty.$$

Therefore the proof is completed. □

3. An application to Fourier series with respect to orthogonal systems

Let $\{\phi_k\}_{k=0}^{\infty}$ be a system of real functions defined on the interval $[a,b]$. If

$$\int_a^b \phi_k(t)\phi_m(t)dt = 0 \quad (m \neq k,\ m,k = 0,1,2,...),$$

then the system $\{\phi_k\}_{k=0}^{\infty}$ is said to be orthogonal on the interval $[a,b]$.

We know that the basic trigonometric system $\{1, cost, sint, ..., cosnt, sinnt, ...\}$ is orthogonal on any interval of length 2π.

Let $f(t)$ be a function defined on the interval $[a,b]$. The Fourier series of $f(t)$ with respect to the orthogonal system $\{\phi_k\}_{k=0}^{\infty}$ is defined as

$$f(t) \sim c_0\phi_0(t) + c_1\phi_1(t) + ... + c_n\phi_n(t) + ... = \sum_{k=0}^{\infty} c_k\phi_k(t) := \sum_{k=0}^{\infty} B_k(t),$$

where

$$c_n = \frac{\int_a^b f(t)\phi_n(t)dt}{\int_a^b \phi_n^2(t)dt}, \quad n = 0,1,2,... .$$

When it comes to orthogonal systems, square integrable functions are particularly preminent. In this case, unlike the ordinary convergence, the convergence in the mean is used. Because the ordinary convergence of a Fourier series to a function may not always be possible even if the system is complete. However, in the case of convergence in the mean, this convergence is always possible if the system is complete.

From this point of view, the main theorem can be specified in a Fourier series given with respect to the orthogonal system.

In [13], we know that if $\psi(t) = \frac{1}{t}\int_0^t \frac{f(t+u)-f(t-u)}{2}du$ belongs to the class $BV(0,\pi)$, then $\sigma_n(t) = O(1)$, where $\sigma_n(t)$ is Cesàro mean of the sequence $(nB_n(t))$. Therefore we write the following results.

Theorem 3.1. *Let (p_n) be a positive sequence with $P_n = O(np_n)$ as $n \to \infty$. Assume that $X = \sum_{v=0}^{n} \frac{p_v}{P_v}$, the sequence $(\frac{\theta_n p_n}{P_n})$ is a non-increasing sequence and, $\lambda_n \to 0$ as $n \to \infty$. Moreover, suppose that the conditions of Theorem 2.1 are satisfied for X_n and λ_n. If $\psi(t) \in BV(0,\pi)$ and the conditions of Theorem 2.1 are satisfied, then the series $\sum_{k=0}^{\infty} B_n(t)\lambda_n$ is summable $|\overline{DN}, p_n, \theta_n|_k$, $k \geq 1$.*

Remark 3.2. Under the conditions of Theorem 3.1, if we take the basic trigonometric system, then the result holds for the trigonometric Fourier series.

Remark 3.3. If we take $\alpha_n = 0$ and $\beta_n = n$ for all n, then the summability method $|\overline{DN}, p_n, \theta_n|_k$ turns into $|\overline{N}, p_n, \theta_n|_k$. Therefore, the results obtained here coincide with the results in [12].

References

[1] R. P. Agnew, *On deferred Cesàro means*, Ann. Math. **33** (1932), 413–421 .

[2] R.P. Boas Jr., *Quasi-positive sequences and trigonometric series*, Proc. Lond. Math. Soc. **14A** (1965), 38–46 .

[3] H. Bor, *On quasi-monotone sequences and their applications*, Bull. Aust. Math. Soc. **43** (1991), 187–192.
[4] H. Bor, *An application of quasi-monotone sequences to infinite series and Fourier series*, Anal. Math. Phys. **8** (2018), 77–83.
[5] H. Bor, *On absolute summability of factored infinite series and trigonometric Fourier series*, Results Math. **73** (2018), 116.
[6] H. Bor, *On absolute Riesz summability factors of infinite series and their application to Fourier series*, Georgian Math. J. **26** (2019), 361–366.
[7] H. Bor, *Certain new factor theorems for infinite series and trigonometric Fourier series*, Quaest. Math. **43** (2020), 441–448.
[8] H. Bor, *On an application of power increasing sequences*, Trans. A. Razmadze Math. Inst. **174** (2020), 265–269.
[9] H. Bor, *A new note on factored infinite series and trigonometric Fourier series*, C. R. Math. Acad. Sci. Paris **359** (2021), 323–328.
[10] H. Bor, *Factored infinite series and Fourier series involving almost increasing sequences*, Bull. Sci. Math. **169**, (102990) (2021).
[11] H. Bor, *A new application of a wider class of power increasing sequences*, Numer. Funct. Anal. Optim. **42** (2021), 712–720.
[12] H. Bor, *Absolute weighted arithmetic mean summability of factored infinite series and Fourier series*, Bull. Sci. Math. **176** (2022), 103–116.
[13] K. K. Chen, *Functions of bounded variation and Cesàro means of Fourier series*, Acad. Sin. Sci. Rec. **1** (1954), 283–289 .
[14] U. Değer, M. Küçükaslan, *A generalization of deferred Cesáro means and some of their applications*, J. Ineq. Appl. **2015** (2015), 1–16.
[15] T. M. Flett, *On an extension of absolute summability and some theorems of Littlewood and Paley*, Proc. Lond. Math. Soc. **7** (1957), 113–141.
[16] H. S. Özarslan, *Local properties of generalized absolute matrix summability of factored Fourier series*, Southeast Asian Bull. Math. **43** (2019), 263–272.
[17] H. S. Özarslan, *On the localization of factored Fourier series*, J. Comput. Anal. Appl. **29** (2021), 344–354.
[18] H. S. Özarslan, *A study on local properties of Fourier series*, Bol. Soc. Parana. Mat. **39** (2021), 201–211.
[19] W. T. Sulaiman, *On some summability factors of infinite series*, Proc. Amer. Math. Soc. **115** (1992),313–317.
[20] Ş. Yildiz, *On the absolute matrix summability factors of Fourier series*, Math. Notes **103** (2018), 297–303.
[21] Ş. Yildiz, *On the generalizations of some factors theorems for infinite series and Fourier series*, Filomat **33** (2019), 4343–4351.
[22] Ş. Yildiz, *Matrix application of power increasing sequences to infinite series and Fourier series*, Ukr. Math. J. **72** (2020), 730–740.