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Statistical Dunford-Pettis operators on Banach Spaces

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Abstract. We introduce the concept of statistical Dunford-Pettis operators in this paper, which extend the classical notion of Dunford-Pettis operators into the domain of statistical convergence. These operators are defined on Banach spaces by their ability to map sequences that converge weakly in a statistical sense into sequences that converge statistically. The exploration includes the properties of these operators and their connections to other operator classes, such as statistically continuous and statistically compact operators. Furthermore, we examine the behavior of the adjoint operators associated with the statistical Dunford-Pettis operators.

1. Introduction on Dunford-Pettis Operators

This study focuses on Dunford-Pettis operators, a crucial concept in functional analysis concerning the preservation of weak convergence under linear transformations between Banach spaces. These operators have broad applications across various fields, including functional analysis, probability theory, optimization, measure theory, Riesz spaces, and partial differential equations [2, 12, 17, 19, 24]. The origin of Dunford-Pettis operators lies in the work of Dunford and Pettis [13], who demonstrated that weakly compact operators on $L_1(\mu)$ map weakly convergent sequences to norm convergent sequences. Grothendieck [18] formalized this property by defining Dunford-Pettis operators as those exhibiting this behavior. Subsequent research has expanded upon this foundation. Sanchez [25] introduced almost Dunford-Pettis operators, later refined by Wnuk [26]. Aliprantis and Burkinshaw defined weak Dunford-Pettis operators [1, 2], while Aqzzouz and Bouras investigated properties of positive weak Dunford-Pettis operators on Banach lattices [4] and introduced order Dunford-Pettis operators [5]. H'michane et al. studied weak* Dunford-Pettis operators [14, 20]. The study of Dunford-Pettis operators remains an active area of research [8, 11, 19, 24]. This paper aims to define Dunford-Pettis operators by using statistical convergence. Specifically, we introduce the concept of *statistical Dunford-Pettis operators* by employing weak statistical convergence within Banach spaces, as defined in [10].

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Weak convergence is a fundamental concept in the study of normed spaces, particularly relevant in the context of Dunford-Pettis operators. It offers a powerful and versatile tool for analyzing normed spaces and their duals, often providing advantages over the direct use of norm convergence. Let $(\mathcal{U}, \|\cdot\|)$ denote a normed space. The norm dual of $\mathcal{U}$, denoted by $\mathcal{U}'$, is defined as the space of all continuous linear functionals from $\mathcal{U}$ to $\mathbb{R}$:

$$\mathcal{U}' := \{f \mid f : \mathcal{U} \to \mathbb{R} \text{ is a continuous linear functional}\}.$$

The set of all linear operators between normed spaces $\mathcal{U}$ and $\mathcal{V}$ is denoted by $\mathcal{L}(\mathcal{U}, \mathcal{V})$. In this context, the term *operator* will be used as a shorthand for a linear operator.

Definition 1.1. *Let $(\mathcal{U}, \|\cdot\|)$ be a normed space and (u_n) be a sequence in $\mathcal{U}$. The sequence (u_n) is said to converge weakly to $u \in \mathcal{U}$, denoted by $u_n \xrightarrow{w} u$, if for every $f \in \mathcal{U}'$, we have $f(u_n) \to f(u)$ in $\mathbb{R}$.*

Definition 1.2. *An operator $P \in \mathcal{L}(\mathcal{U}, \mathcal{V})$ between Banach spaces $\mathcal{U}$ and $\mathcal{V}$ is a Dunford-Pettis operator if, for any sequence (u_n) in $\mathcal{U}$, $u_n \xrightarrow{w} u$ implies $P(u_n) \xrightarrow{\|\cdot\|_{\mathcal{V}}} P(u)$.*

It is well-established that norm convergence implies weak convergence. Consequently, every Dunford-Pettis operator is norm continuous. However, the converse is not generally true because every Banach space without the Schur property contains a weakly convergent sequence that does not converge in norm.

Dunford-Pettis operators have demonstrated considerable utility across multiple disciplines. However, to the best of our knowledge, there has been no research examining these operators within the statistical convergence. The concept of the statistical convergence was introduced by Fast in 1951 [15], which generalizes the classical convergence by emphasizing the asymptotic density of terms in a sequence, rather than their individual behavior throughout the sequence (cf. [6, 7, 21–23]). We remind the fundamental tools related to statistical convergence. Let J be a subset of the positive integers $\mathbb{N}$. The *natural density* (or *asymptotic density*) of J, denoted by $\delta(J)$, is defined as

$$\delta(J) = \lim_{n\to\infty} \frac{1}{n} |\{j \leq n : j \in J\}|,$$

provided this limit exists, where $|\cdot|$ denotes the cardinality of a set. For a detailed exposition on natural density, we refer the reader to [16]. A sequence (u_n) in a normed space $(\mathcal{U}, \|\cdot\|_{\mathcal{U}})$ is said to be *statistically convergent* to $u \in \mathcal{U}$, denoted by $u_n \xrightarrow{st} u$, if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} |\{j \leq n : \|u_j - u\|_{\mathcal{U}} \geq \varepsilon\}| = 0.$$

A sequence (u_n) in a normed space $(\mathcal{U}, \|\cdot\|_{\mathcal{U}})$ is *statistically bounded* (or *st-bounded*) if there exists a positive constant $R > 0$ such that

$$\delta(\{n \in \mathbb{N} : \|u_n\|_{\mathcal{U}} > R\}) = 0$$

(cf. [3]). In this paper, $\ell_\infty(\mathcal{U})$ and $\ell_\infty^{st}(\mathcal{U})$ denote the sets of all norm-bounded and statistically bounded sequences in $\mathcal{U}$, respectively. It is clear that $\ell_\infty(\mathcal{U}) \subseteq \ell_\infty^{st}(\mathcal{U})$ for any normed space $\mathcal{U}$, and this inclusion is strict. The reverse inclusion does not hold in general. Consider the sequence (u_n) in $\mathbb{R}$ defined by

$$u_n = \begin{cases} n, & \text{if } n \text{ is prime,} \\ \frac{n+1}{n}, & \text{if } n \text{ is not prime.} \end{cases}$$

This sequence is statistically bounded, i.e., $(u_n) \in \ell_\infty^{st}(\mathbb{R})$, but it is not norm-bounded.

Definition 1.3. *[9] Let $P \in \mathcal{L}(\mathcal{U}, \mathcal{V})$. Then P is called:*

(i) statistically bounded if $P(\ell_\infty^{st}(\mathcal{U})) \subseteq \ell_\infty^{st}(\mathcal{V})$;

(ii) *statistically continuous if* $u_n \xrightarrow{\text{st}} \theta$ *in* $\mathcal{U}$ *implies* $P(u_n) \xrightarrow{\text{st}} \theta$ *in* $\mathcal{V}$*;*

(iii) *statistically compact if it maps statistically bounded sequences in* $\mathcal{U}$ *to statistically convergent sequences in* $\mathcal{V}$*;*

(iv) *weakly statistically continuous if* $u_n \xrightarrow{w_{st}} u$ *implies* $P(u_n) \xrightarrow{w_{st}} P(u)$*.*

The key concept of *weak statistical convergence* is then defined based on [10].

Definition 1.4. *A sequence* (u_n) *in a normed space* $\mathcal{U}$ *is weakly statistically convergent to u, denoted by* $u_n \xrightarrow{\text{w}_{\text{st}}} u$*, if for every* $f \in \mathcal{U}'$*, the sequence* $(f(u_n - u))$ *is statistically convergent to* θ *in* $\mathbb{R}$*.*

It is important to note that every subsequence of a weakly convergent sequence is also weakly convergent. On the other hand, statistical convergence of a sequence typically leads to the existence of a convergent subsequence whose indices possess a systematic density of one, provided the chosen density definition is appropriate. For a detailed examination of the conditions under which a sequence has an st-convergent subsequence, we refer to [23].

Remark 1.5. *A sequence that is weakly statistically convergent does not necessarily possess a weakly convergent subsequence. Specifically, let* (u_n) *be a weakly statistically convergent sequence. By definition, for every* $f \in \mathcal{U}'$*, the sequence* $f(u_n - u)$ *must be statistically convergent in* $\mathbb{R}$*. However, for any given* $f \in \mathcal{U}'$*, one can identify a subsequence of* (u_n) *that converges with respect to* f*. Importantly, if* f *is replaced by another functional* $g \in \mathcal{U}'$*, the resulting convergent subsequence may differ. Consequently, the subsequences obtained vary depending on the choice of* f*, indicating the absence of a single, universal subsequence of* (u_n) *that converges for all* $f \in \mathcal{U}'$*. Therefore, the weak statistical convergence of* (u_n) *does not ensure the existence of a single subsequence that is weakly convergent with respect to every functional in* $\mathcal{U}'$*.*

Lemma 1.6. *Let* $(\mathcal{U}, \|\cdot\|)$ *represent a normed space. The following properties related to convergence in this context are valid:*

(i) *If a sequence* (u_n) *within* $\mathcal{U}$ *converges weakly to some* $u \in \mathcal{U}$*, then the sequence is also weakly statistically convergent to u.*

(ii) *If a sequence* (u_n) *in* $\mathcal{U}$ *is statistically convergent to an element* $u \in \mathcal{U}$*, then it follows that* (u_n) *is also weakly statistically convergent to u.*

(iii) *Suppose* (u_n) *is a weakly statistically convergent sequence in* $\mathcal{U}$*, and* (u_{n_j}) *is a subsequence of* (u_n) *such that the set of indices* $\{n_j : j \in \mathbb{N}\}$ *has natural density equal to one. Then, the subsequence* (u_{n_j}) *is also weakly statistically convergent to the same limit.*

(iv) *A weakly statistically convergent sequence in* $\mathcal{U}$ *possesses a unique weak statistical limit. This uniqueness is guaranteed by the Hahn-Banach theorem.*

Remark 1.7. *As demonstrated in [10, Exam.2.2], the reverse implication of Lemma 1.6 (i) does not generally hold; that is, a sequence that is weakly statistically convergent does not necessarily converge weakly. Likewise, the converse of Lemma 1.6 (ii) fails to hold in general.*

Theorem 1.8. *Let* $(\mathcal{U}, \|\cdot\|)$ *be a normed space and* (u_n) *be a sequence in* $\mathcal{U}$*. Then* $u_n \xrightarrow{\text{w}_{\text{st}}} u$ *if and only if there exists a sequence* (v_n) *in* $\mathcal{U}$ *such that* $u_n = v_n$ *for almost all n (i.e., the set* $\{n \in \mathbb{N} : u_n \neq v_n\}$ *has natural density zero) and* $v_n \xrightarrow{\text{w}_{\text{st}}} u$*.*

Proof. Assume that $u_n \xrightarrow{\text{w}_{\text{st}}} u$ holds in $\mathcal{U}$. We need to construct a sequence (v_n) that satisfies the required properties. A simple choice suffices: define $v_n = u_n$ for all n. In this case, the set $\{n : u_n \neq v_n\}$ is an empty set, and hence it is clear that it has density zero. Moreover, it follows from $v_n = u_n$ that $v_n \xrightarrow{\text{w}_{\text{st}}} u$.

We assume the existence of a sequence (v_n) in $\mathcal{U}$ such that $u_n = v_n$ for almost all n and $v_n \xrightarrow{\text{w}_{\text{st}}} u$. To prove that $u_n \xrightarrow{\text{w}_{\text{st}}} u$, we need to verify that for any $f \in \mathcal{U}'$, the sequence $f(u_n - u)$ is statistically convergent to zero in $\mathbb{R}$. From the assumption $v_n \xrightarrow{w_{st}} u$, it follows that for any $f \in \mathcal{U}'$, the sequence $f(v_n - u)$ is statistically convergent to zero in $\mathbb{R}$. Specifically, for any $\varepsilon > 0$, we have

$$\delta(\{n : |f(v_n - u)| \geq \varepsilon\}) = 0.$$

Additionally, since $u_n = v_n$ for almost all n, we know that $\delta(A) = 0$, where $A := \{n : u_n \neq v_n\}$. Next, consider the set $B := \{n : |f(u_n - u)| \geq \varepsilon\}$. Our goal is to show that $\delta(B) = 0$. We decompose B into the union of two subsets:

$$B = (B \cap A) \cup (B \cap A^c).$$

Since A has density zero, any subset of A also has density zero, so $\delta(B \cap A) = 0$. For $n \in A^c$, we have $u_n = v_n$. Consequently, if $n \in B \cap A^c$, then

$$|f(u_n - u)| = |f(v_n - u)| \geq \varepsilon.$$

This implies that $B \cap A^c$ is a subset of $\{n : |f(v_n - u)| \geq \varepsilon\}$, which has density zero. Therefore, $\delta(B \cap A^c) = 0$. Since B is the union of two sets of density zero, it follows that B itself has density zero:

$$\delta(B) = \delta(B \cap A) + \delta(B \cap A^c) = 0 + 0 = 0.$$

Thus, $f(u_n - u)$ is statistically convergent to zero in $\mathbb{R}$ for any $f \in \mathcal{U}'$. Hence, $u_n \xrightarrow{w_{st}} u$. □

Corollary 1.9. *Let $(\mathcal{U}, \|\cdot\|)$ be a normed space. If a sequence (u_n) in $\mathcal{U}$ possesses a dense subsequence that is weakly statistically convergent to some $u \in \mathcal{U}$, then the sequence (u_n) itself is weakly statistically convergent to u.*

Theorem 1.10. *Let $(\mathcal{U}, \|\cdot\|)$ be a normed space, and let (u_n) be a sequence in $\mathcal{U}$ such that $u_n \xrightarrow{w_{st}} u$ for some $u \in \mathcal{U}$. If (u_{n_k}) is a subsequence of (u_n) such that*

$$\liminf_{n\to\infty} \frac{|K(n)|}{n} > 0,$$

where $K(n) = \{n_k \leq n : k \in \mathbb{N}\}$, then (u_{n_k}) is also weakly statistically convergent to u.

Proof. Suppose $u_n \xrightarrow{w_{st}} u$. This means that for every $f \in \mathcal{U}'$ and every $\varepsilon > 0$, we have

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |f(u_k) - f(u)| \geq \varepsilon\}| = 0.$$

Consider the subsequence (u_{n_k}). Let $A_\varepsilon = \{k \in \mathbb{N} : |f(u_k) - f(u)| \geq \varepsilon\}$. Since $u_n \xrightarrow{w_{st}} u$, we have $\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : k \in A_\varepsilon\}| = 0$. For any $n \in \mathbb{N}$, the set of indices in the subsequence satisfying the condition is the intersection of $K(n)$ and A_ε:

$$\{n_k \leq n : |f(u_{n_k}) - f(u)| \geq \varepsilon\} = K(n) \cap A_\varepsilon \subseteq \{k \leq n : k \in A_\varepsilon\}.$$

We can rewrite the ratio of the cardinality as follows:

$$\begin{aligned} \frac{|\{n_k \leq n : |f(u_{n_k}) - f(u)| \geq \varepsilon\}|}{|K(n)|} &= \frac{|K(n) \cap A_\varepsilon|}{n} \cdot \frac{n}{|K(n)|} \\ &\leq \frac{|\{k \leq n : |f(u_k) - f(u)| \geq \varepsilon\}|}{n} \cdot \frac{n}{|K(n)|}. \end{aligned}$$

Let $\delta = \liminf_{n\to\infty} \frac{|K(n)|}{n}$. By the hypothesis, $\delta > 0$. Thus, there exists N_0 such that for all $n > N_0$, $\frac{|K(n)|}{n} > \frac{\delta}{2}$, which implies $\frac{n}{|K(n)|} < \frac{2}{\delta}$. Since the first factor on the right-hand side converges to 0 because of $u_n \xrightarrow{w_{st}} u$, and the second factor is bounded, we obtain:

$$\limsup_{n\to\infty} \frac{|\{n_k \leq n : |f(u_{n_k}) - f(u)| \geq \varepsilon\}|}{|K(n)|} = 0.$$

Consequently, the natural density of the set $\{k \in \mathbb{N} : |f(u_{n_k}) - f(u)| \geq \varepsilon\}$ is zero. Therefore, we get $u_{n_k} \xrightarrow{w_{st}} u$. □

Corollary 1.11. *Let $(\mathcal{U}, \|\cdot\|)$ be a normed space and (u_n) be a sequence in $\mathcal{U}$. The following statements are equivalent:*

(i) $u_n \xrightarrow{w_{st}} u$.

(ii) Every subsequence (u_{n_k}) of (u_n) with $\liminf_{n\to\infty} \frac{|K(n)|}{n} > 0$*, where $K(n) = \{n_k \leq n : k \in \mathbb{N}\}$, is weakly statistically convergent to u.*

(iii) Every dense subsequence of (u_n) is weakly statistically convergent to u.

Proof. $(i) \Rightarrow (ii)$: This follows directly from Theorem 1.10.

$(ii) \Rightarrow (iii)$: Assume that the condition (ii) holds. If $\tilde{u} = (u_{n_k})$ is a dense subsequence of (u_n), then the set of indices $K(n) = \{n_k \leq n : k \in \mathbb{N}\}$ has natural density 1, i.e., $\delta(K(n)) = 1$. This implies that $\liminf_{n\to\infty} \frac{|K(n)|}{n} = 1 > 0$. Thus, by (ii), $\tilde{u}$ is weakly statistically convergent to u.

$(iii) \Rightarrow (i)$: Assume that (iii) holds. Since (u_n) is a subsequence of itself (and trivially dense in itself), it follows from (iii) that (u_n) is weakly statistically convergent to u. □

The remainder of this paper is organized as follows: Section 2 introduces the concept of statistical Dunford-Pettis operators and explores their relationships with other classes of operators. Section 3 presents the main results concerning statistical Dunford-Pettis operators.

2. Statistical Dunford-Pettis

In this section, we define the concept of statistical Dunford-Pettis operators and establish fundamental results related to these operators. The Dunford-Pettis property characterizes operators that transform weak convergence of sequences into norm convergence on Banach spaces.

Similarly, statistical Dunford-Pettis operators can be defined as follows:

Definition 2.1. *Let $P \in \mathcal{L}(\mathcal{U}, \mathcal{V})$ be an operator between Banach spaces. Then, P is called* statistical Dunford-Pettis *if $u_n \xrightarrow{w_{st}} u$ in $\mathcal{U}$ implies $P(u_n) \xrightarrow{st} P(u)$ in $\mathcal{V}$.*

Throughout this paper, $C(\mathcal{U}, \mathcal{V})$, $C_{st}(\mathcal{U}, \mathcal{V})$, $B_{st}(\mathcal{U}, \mathcal{V})$, $K_{st}(\mathcal{U}, \mathcal{V})$, and $DP_{st}(\mathcal{U}, \mathcal{V})$ represent the sets of all continuous, statistical continuous, statistical bounded, statistical compact, and statistical Dunford-Pettis operators between normed spaces $\mathcal{U}$ and $\mathcal{V}$, respectively, unless otherwise specified. We have some observation in the following remark.

Remark 2.2.

(i) A Dunford-Pettis operator does not have to be a statistical Dunford-Pettis operator because a statistically weakly convergent sequence may not have a weakly convergent subsequence; see Remark 1.5. Furthermore, the reverse implication is also not guaranteed in general.

(ii) It follows from Eberlein-Šmulian theorem that a compact operator with a reflexive Banach space domain is a statistical Dunford-Pettis operator. However, the reverse is not necessarily true.

(iii) There is no direct connection between $K_{st}(\mathcal{U}, \mathcal{V})$ and $DP_{st}(\mathcal{U}, \mathcal{V})$.

(iv) If $\dim(\mathcal{U}) < \infty$, then we have $DP_{st}(\mathcal{U}, \mathcal{V}) = C_{st}(\mathcal{U}, \mathcal{V})$ because weak statistical convergence implies statistical convergence in finite dimensions; see [10, Thm.2.3(iii)].

Example 2.3. *Consider the Banach spaces $\mathcal{U} := \ell_1$ and $\mathcal{V} := \ell_\infty$, and let $P : \ell_1 \to \ell_\infty$ be the* canonical embedding *operator, defined as:*

$$P(w) := (s_k(w))_{k=1}^{\infty}$$

for each $w := (w_n) \in \ell_1$, where $s_k(w) = \sum_{i=1}^{k} w_i$. Here, $P(w)$ is the sequence of partial sums of the sequence w. We will show that P is a statistical Dunford-Pettis operator. Take a sequence $(u_n) := (u_1, \cdots, u_n, \cdots)$ denoted by $u_n := (u_n^1, u_n^2, \cdots) \in \ell_1$ for each n such that it is weakly statistically convergent to $z := (z_1, z_2, \cdots) \in \ell_1$. That is, for *every $f \in \ell_1'$ (where $\ell_1' = \ell_\infty$), the sequence $f(u_n)$ is statistically convergent to $f(z)$ in $\mathbb{R}$. On the other hand, to prove that $P(u_n)$ is statistically norm-convergent to $P(z)$, we need to show the existence of the following limit;*

$$\lim_{n\to\infty} \frac{1}{n} |\{m \leq n : \|P(u_m) - P(z)\| \geq \varepsilon\}| = 0$$

for all $\varepsilon > 0$. For a fixed m, the norm of the difference between $P(u_m)$ and $P(z)$ is

$$\|P(u_m) - P(z)\| = \sup_k |s_k(u_m) - s_k(z)|.$$

Since $s_k(u_m) = \sum_{i=1}^k u_m^i$ and $s_k(z) = \sum_{i=1}^k z_i$, then we have

$$s_k(u_m) - s_k(z) = \sum_{i=1}^k u_m^i - \sum_{i=1}^k z_i = \sum_{i=1}^k (u_m^i - z_i).$$

Thus, we get $|s_k(u_m) - s_k(z)| = \left|\sum_{i=1}^k (u_m^i - z_i)\right|$. On the other hand, since (u_n) is weakly statistically convergent to z, for any $f \in \ell_1'$ we have

$$\lim_{n\to\infty} \frac{1}{n} \left|\{m \leq n : |f(u_m) - f(z)| \geq \varepsilon\}\right| = 0.$$

In particular, this holds for the specific linear functionals $f_k(w) := \sum_{i=1}^k w_i$ for all $w \in \ell_1$. Thus, for each k, the sequence $(s_k(u_n))$ converges to $s_k(z)$ statistically. It means that we have

$$\lim_{n\to\infty} \frac{1}{n} |\{m \leq n : |s_k(u_m) - s_k(z)| \geq \varepsilon\}| = 0.$$

Therefore, we obtain that $P(u_n)$ converges to $P(z)$ in ℓ_∞ statistically.

Proposition 2.4. *For any Banach spaces $\mathcal{U}$ and $\mathcal{V}$, the inclusion $DP_{st}(\mathcal{U},\mathcal{V}) \subseteq C_{st}(\mathcal{U},\mathcal{V})$ holds.*

Proof. Let $P \in DP_{st}(\mathcal{U},\mathcal{V})$ be an arbitrary element and consider any arbitrary sequence (u_n) in $\mathcal{U}$ such that $u_n \xrightarrow{st} u$ holds. It follows from Lemma 1.6 that we have $u_n \xrightarrow{w_{st}} u$ in $\mathcal{U}$ because statistical convergence implies weak statistical convergence. Now, since $P \in DP_{st}(\mathcal{U},\mathcal{V})$, the weak statistical convergence $u_n \xrightarrow{w_{st}} u$ in $\mathcal{U}$ implies $P(u_n) \xrightarrow{st} P(u)$ in $\mathcal{V}$. This fact completes the proof. □

By applying Proposition 2.4 and [9, Thm.3.4], we get the following result.

Corollary 2.5. *The inclusion $DP_{st}(\mathcal{U},\mathcal{V}) \subseteq B_{st}(\mathcal{U},\mathcal{V})$ holds for any Banach spaces $\mathcal{U}$ and $\mathcal{V}$.*

Theorem 2.6. *$DP_{st}(\mathcal{U},\mathcal{V})$ is a linear space.*

Proof. To start with, it is evident that $DP_{st}(\mathcal{U},\mathcal{V})$ is a subset of $\mathcal{L}(\mathcal{U},\mathcal{V})$ which is a vector space of all linear operators from $\mathcal{U}$ to $\mathcal{V}$. Now, suppose that P and R belong to the set $DP_{st}(\mathcal{U},\mathcal{V})$ and (u_n) is a sequence in $\mathcal{U}$ such that $u_n \xrightarrow{w_{st}} 0$. Then, for any $\varepsilon > 0$, we have

$$\delta(\{k \leq n : \|Pu_k\| > \frac{\varepsilon}{2}\}) = 0 \quad \text{and} \quad \delta(\{k \leq n : \|Ru_k\| > \frac{\varepsilon}{2}\}) = 0.$$

On the other hand, the inequality $\|(P+R)u_k\| = \|Pu_k + Ru_k\| \leq \|Pu_k\| + \|Ru_k\|$ gives the following inclusion

$$\{k \leq n : \|(P+R)u_k\| > \varepsilon\} \subseteq \{k \leq n : \|Pu_k\| > \frac{\varepsilon}{2}\} \cup \{k \leq n : \|Ru_k\| > \frac{\varepsilon}{2}\}.$$

Thus, the monotonicity of the natural density implies $\delta(\{k \leq n : \|(P+R)u_k\| > \varepsilon\}) = 0$, indicating $P + R \in DP_{st}(X,Y)$.

Next, take any scalar $\alpha \in \mathbb{R}$. Then, we have $\|(\alpha P)(u_k)\| = \|\alpha P(u_k)\| = |\alpha|\,\|Pu_k\|$. If $\alpha = 0$, then it is clear that $(\alpha P)(u_k) = \theta_{\mathcal{V}}$, where $\theta_{\mathcal{V}}$ is the zero vector in $\mathcal{V}$. Now, assume that $\alpha \neq 0$. We obtain

$$\{k \leq n : \|(\alpha P)(u_k)\| > \varepsilon\} = \{k \leq n : \|Pu_k\| > \frac{\varepsilon}{|\alpha|}\}.$$

Hence, $\delta(\{k \leq n : \|(\alpha P)u_k\| > \varepsilon\}) = 0$ holds for both cases. This implies that αP belongs to the set $DP_{st}(\mathcal{U}, \mathcal{V})$. □

Theorem 2.7. *Let $\mathcal{U}, \mathcal{V}$ and $\mathcal{W}$ be Banach spaces and $P \in DP_{st}(\mathcal{U}, \mathcal{V})$. Then, $R \circ P \in DP_{st}(\mathcal{U}, \mathcal{W})$ holds for every $R \in C_{st}(\mathcal{V}, \mathcal{W})$.*

Proof. Consider an arbitrary sequence (u_n) in $\mathcal{U}$ such that $u_n \xrightarrow{w_{st}} \theta$. Since $P(u_n) \xrightarrow{st} \theta$ in $\mathcal{V}$, and R is statistically continuous, it follows that the sequence $(R \circ P)(u_n) = R(P(u_n))$ also converges statistically to zero in $\mathcal{W}$. This conclusion is supported by the linearity of both R and P. Therefore, $R \circ P$ is an element of the space $DP_{st}(\mathcal{U}, \mathcal{W})$. □

It is clear that Theorem 2.7 is applied directly whenever the operator R is continuous from $\mathcal{V}$ to $\mathcal{W}$. Moreover, we observe the following result.

Proposition 2.8. *Let $\mathcal{U}$ be a Banach space, and suppose that $P \in DP_{st}(\mathcal{U})$. Then, $P^n \in DP_{st}(\mathcal{U})$ holds for any $n \in \mathbb{N}$.*

We note that a Banach space is characterized by the Schur property if all weakly convergent sequences are norm convergent. Nevertheless, w_{st}-convergence in a Schur space does not necessarily entail st-convergence. Therefore, we propose the following definition.

Definition 2.9. *A Banach space $\mathcal{U}$ is said to possess the* statistical Schur property *if every w_{st}-convergent sequence is statistical norm convergent, i.e., $u_n \xrightarrow{w_{st}} u$ implies that $u_n \xrightarrow{st} u$ in $\mathcal{U}$.*

Question 2.10. *Is there an example of a Banach space that possesses the statistical Schur property but lacks the classical Schur property?*

By considering the definition of the statistical Schur property, we observe the following result.

Proposition 2.11. *$DP_{st}(\mathcal{U}, \mathcal{V}) = C_{st}(\mathcal{U}, \mathcal{V})$ holds for Banach spaces $\mathcal{U}$ and $\mathcal{V}$ whenever $\mathcal{U}$ has the statistical Schur property.*

Proof. We have the inclusion $DP_{st}(\mathcal{U}, \mathcal{V}) \subseteq C_{st}(\mathcal{U}, \mathcal{V})$ by Proposition2.4. For the reverse part, let $P \in C_{st}(\mathcal{U}, \mathcal{V})$ and take any sequence (u_n) in $\mathcal{U}$ satisfying $u_n \xrightarrow{w_{st}} u$. Since $\mathcal{U}$ has the statistical Schur property, $u_n \xrightarrow{st} u$. Because of our assumption on P, we get $P(u_n) \xrightarrow{st} P(u)$ which is the desired one. □

Theorem 2.12. *The Banach space ℓ_1 possesses the statistical Schur property.*

Proof. Let $(u_n) = (u_1^k, \cdots, u_n^k, \cdots) \xrightarrow{w_{st}} w := (w_1, w_2, \ldots)$ be a sequence in ℓ_1. That is, for any sequence $v := (v_k) \in \ell_\infty$, we have

$$< v, u_n >=< (v_k), (u_n^k) >= \sum_{k=1}^{\infty} v_k u_n^k \xrightarrow[n]{st} \sum_{k=1}^{\infty} v_k w_k =< v, w >$$

holds in $\mathbb{R}$ as $n \to \infty$. Now, consider the sequence $v = (e_k)$ in ℓ_∞, where $e_k = (0, 0, \ldots, 1, 0, 0, \ldots)$ with 1 in the k-th position and 0 elsewhere. Then, we obtain that

$$< v, u_n >=< (e_k), (u_n^k) >= u_n^k \xrightarrow[n]{st} < e_k, w >= w_k$$

is true as $n \to \infty$. It shows that weak statistical convergence implies statistical pointwise convergence, i.e., $u_n^k \xrightarrow[n]{st} w_k$ for each fixed k. Our aim is to show that

$$\|u_n - w\| = \sum_{k=1}^{\infty} |u_n^k - w_k| \xrightarrow[n]{st} 0$$

is satisfied in $\mathbb{R}$. For any $\varepsilon > 0$, we can choose N such that $\sum_{k=N+1}^{\infty} |w_k| < \frac{\varepsilon}{2}$ because of $w \in \ell_1$. Now, for each $1 \leq k \leq N$, we know that $u_n^k \xrightarrow[n]{st} w_k$ as $n \to \infty$, and hence there exists $N_k \in \mathbb{N}$ such that $\delta(\{n \in \mathbb{N} : |u_n^k - w_k| \geq \frac{\varepsilon}{2N_k}\}) = 0$. Let take $N_0 := \max\{N, N_1, N_2, \ldots, N_N\}$. Then, for $n > N_0$, we have

$$\|u_n - w\| = \sum_{k=1}^{\infty} |u_n^k - w_k| = \sum_{k=1}^{N} |u_n^k - w_k| + \sum_{k=N+1}^{\infty} |u_n^k - w_k|.$$

On the other hand, the second term of the last equality can be bounded as;

$$\sum_{k=N+1}^{\infty} |u_n^k - w_k| \leq \sum_{k=N+1}^{\infty} |u_n^k| + \sum_{k=N+1}^{\infty} |w_k|.$$

Since $u_n \in \ell_1$ for all n, the tail $\sum_{k=N+1}^{\infty} |u_n^k|$ can be made arbitrarily small by choosing a sufficiently large N. Thus, for $n > N_0$, we have:

$$\|u_n - w\| \leq \sum_{k=1}^{N} |u_n^k - w_k| + \varepsilon.$$

Now, consider the set $A = \{n \in \mathbb{N} : \|u_n - w\| \geq \varepsilon\}$. If $n \in A$ and $n > N_0$, then we have $\sum_{k=1}^{N} |u_n^k - w_k| \geq \frac{\varepsilon}{2}$. This implies that for at least one $k \in \{1, 2, \ldots, N\}$, we have $|u_n^k - w_k| \geq \frac{\varepsilon}{2N}$. Therefore, we get

$$A \subseteq \bigcup_{k=1}^{N} \{n \in \mathbb{N} : |u_n^k - w_k| \geq \frac{\varepsilon}{2N}\}.$$

Since the union of finitely many sets of density zero has density zero, we conclude that $\delta(A) = 0$. □

3. Main Results

In the next result, we show that statistical Dunford-Pettis operators are continuous in the following sense.

Proposition 3.1. *Statistical Dunford-Pettis operators preserve weak statistical convergence.*

Proof. Assume $P \in DP_{st}(\mathcal{U}, \mathcal{V})$ for Banach spaces $\mathcal{U}$ and $\mathcal{V}$, and $u_n \xrightarrow{w_{st}} u$ in $\mathcal{U}$. Since P is a statistical Dunford-Pettis operator, we have $P(u_n) \xrightarrow{st} P(u)$ in $\mathcal{V}$. By Lemma 1.6 (ii), it follows that $P(u_n)$ is weakly statistically convergent to $P(u)$. Therefore, we conclude that every statistical Dunford-Pettis operator is weakly statistically continuous. □

Recall from [9, Thm.4.4] that a statistical compact operator is also a statistical continuous operator. In this context, we present the following result concerning weak statistical convergence.

Theorem 3.2. *If all st-compact operators from a Banach space $\mathcal{U}$ into a Banach space $\mathcal{V}$ are norm continuous, then every statistically compact operator from $\mathcal{U}$ into $\mathcal{V}$ is weakly statistically continuous.*

Proof. Let $P : \mathcal{U} \to \mathcal{V}$ be an *st*-compact operator, and let (u_n) be a sequence in $\mathcal{U}$ such that $u_n \xrightarrow{w_{st}} u$. By [10, Cor. 2.9], the sequence (u_n) is *st*-bounded in $\mathcal{U}$. Since P is *st*-compact, the sequence $(P(u_n))$ converges statistically to some $v \in \mathcal{V}$. Moreover, Lemma 1.6 (ii) implies that $(P(u_n))$ is weak statistical convergent to v.

It is enough to show that $v = P(u)$. To see this, let $f \in \mathcal{V}'$ be arbitrary. As $P(u_n) \xrightarrow{w_{st}} v$, we have $f(P(u_n)) \xrightarrow{st} f(v)$ in $\mathbb{R}$. Define $g : \mathcal{U} \to \mathbb{R}$ by $g(z) = f(P(z))$ for all $z \in \mathcal{U}$. The linearity of P and f implies the linearity of g. Furthermore, the continuity of P (assumed in the theorem statement) and the continuity of f ensure the continuity of g. Hence, $g \in \mathcal{U}'$. Since $u_n \xrightarrow{w_{st}} u$ and $g \in \mathcal{U}'$, we obtain $g(u_n) = f(P(u_n)) \xrightarrow{st} g(u) = f(P(u))$. Thus, for any $f \in \mathcal{V}'$, we have both $f(P(u_n)) \xrightarrow{st} f(v)$ and $f(P(u_n)) \xrightarrow{st} f(P(u))$. By the uniqueness of the statistical limit, we conclude that $f(v) = f(P(u))$ for all $f \in \mathcal{V}'$. Finally, the Hahn-Banach theorem implies that $v = P(u)$. This establishes that $P(u_n) \xrightarrow{w_{st}} P(u)$, as desired. □

Theorem 3.3. *If every st-compact operator from a Banach space $\mathcal{U}$ into a Banach space $\mathcal{V}$ is norm continuous, then the set of statistical compact operators from $\mathcal{U}$ into $\mathcal{V}$ is contained within the class of statistical Dunford–Pettis operators.*

Proof. Assume that $P : \mathcal{U} \to \mathcal{V}$ is a statistical compact operator. We aim to demonstrate that P is a statistical Dunford-Pettis operator. Let (u_n) be a sequence in $\mathcal{U}$ such that $u_n \xrightarrow{w_{st}} u$. By [10, Cor. 2.9], if a sequence in a normed space converges weakly statistically, then it is necessarily statistically bounded. Hence, (u_n) is a statistically bounded sequence in $\mathcal{U}$. Since P is a statistical compact operator, $P(u_n) \xrightarrow{st} v$ for some $v \in \mathcal{V}$. Following the argument in the proof of Theorem 3.2, we can establish that $v = P(u)$. Therefore, we conclude that $P(u_n) \xrightarrow{st} P(u)$, as desired. □

Recall that a bounded operator between Banach spaces is said to be *weakly compact* if the image under any bounded subset is relatively weakly compact.

Theorem 3.4. *Let $P : \mathcal{U} \to \mathcal{V}$ be an operator between two Banach spaces. If P is a statistical Dunford-Pettis operator, then the composition PR is st-compact for each statistical compact operator $R : \mathcal{W} \to \mathcal{U}$, where $\mathcal{W}$ is an arbitrary Banach space.*

Proof. Let P be a statistical Dunford-Pettis operator and $R : \mathcal{W} \to \mathcal{U}$ be a statistical compact operator. Suppose that (u_n) is an *st*-bounded sequence in $\mathcal{W}$. Since R is statistically compact, the sequence $(R(u_n))$ converges statistically to some $u \in \mathcal{U}$. This implies that $(R(u_n))$ also converges weakly statistically to u because statistical convergence implies weak statistical convergence by Lemma 1.6. By the statistical Dunford-Pettis property of P, $(P(R(u_n)))$ converges statistically to $P(u)$ in $\mathcal{V}$. Therefore, the operator PR takes *st*-bounded sequences in $\mathcal{W}$ and maps them to statistically convergent sequences in $\mathcal{V}$, thereby making it a statistical compact operator. □

Recall that the adjoint operator of a bounded linear operator $P : \mathcal{U} \to \mathcal{V}$ between normed spaces is the map $P' : \mathcal{V}' \to \mathcal{U}'$ defined by $(P'f)(u) := f(Pu)$ for all $f \in \mathcal{V}'$. Furthermore, the canonical map from a normed space $\mathcal{U}$ to its second dual $\mathcal{U}''$ is often referred to as the *canonical embedding* or *canonical isometry*. This map, denoted by $\varphi : \mathcal{U} \to \mathcal{U}''$, is defined by

$$\varphi(u)(f) = f(u)$$

for all $u \in \mathcal{U}$ and $f \in \mathcal{U}'$.

Theorem 3.5. *Let $\varphi : \mathcal{U} \to \mathcal{U}''$ be a surjective canonical embedding for a normed space $\mathcal{U}$ and (f_n) be a sequence in $\mathcal{U}'$. Then, we have $f_n \xrightarrow{w_{st}} f$ in $\mathcal{U}'$ iff $\varphi(u)(f_n) \xrightarrow{st} \varphi(u)(f)$ holds for all $u \in \mathcal{U}$.*

Proof. Suppose that $f_n \xrightarrow{w_{st}} f$ in $\mathcal{U}'$. This means that the sequence $F(f_n)$ is statistically convergent to $F(f)$ for all $F \in \mathcal{U}''$. By the definition of the canonical embedding φ, we have $\varphi(u) \in \mathcal{U}''$ for each $u \in \mathcal{U}$. Therefore, we have

$$\varphi(u)(f_n) \xrightarrow{st} \varphi(u)(f)$$

for all $u \in \mathcal{U}$.

We assume now that $\varphi(u)(f_n) \xrightarrow{\text{st}} \varphi(u)(f)$ for all $u \in \mathcal{U}$. This means that for each $u \in \mathcal{U}$ and any $\varepsilon > 0$, we have

$$\delta(\{k \leq n : |\varphi(u)(f_k) - \varphi(u)(f)| \geq \varepsilon\}) = 0.$$

By considering the surjectivity of the canonical embedding φ, every $F \in \mathcal{U}''$ is of the form $F = \varphi(u)$ for some $u \in \mathcal{U}$. This means that for any $F \in \mathcal{U}''$, there exists $u \in \mathcal{U}$ such that $F(f) = \varphi(u)(f) = f(u)$ is satisfied for all $f \in \mathcal{U}'$. Thus, for any $F \in \mathcal{U}''$, we have $F(f_n - f) = f_n(u) - f(u) \xrightarrow{\text{st}} 0$ in $\mathbb{R}$. Therefore, we conclude that $f_n \xrightarrow{\text{w}_{\text{st}}} f$ in $\mathcal{U}'$. □

Theorem 3.6. *Let $P \in DP_{st}(\mathcal{U}, \mathcal{V})$ be norm continuous and $P' : \mathcal{V}' \to \mathcal{U}'$ be the adjoint operator of P. Then, the following statements hold:*

(i) $P'(g)$ is statistical Dunford-Pettis operator for every $g \in \mathcal{V}'$;

(ii) P' is a statistical Dunford-Pettis operator if $f_n \xrightarrow{\text{w}_{\text{st}}} f$ in $\mathcal{V}'$ implies $f_n(v) \xrightarrow{\text{st}} f(v)$ for all $v \in \mathcal{V}$.

Proof. (i) Let $g \in \mathcal{V}'$ be arbitrary. We aim to show that $P'(g)$ is a statistical Dunford-Pettis operator. Consider a sequence (u_n) in $\mathcal{U}$ such that $u_n \xrightarrow{\text{w}_{\text{st}}} u$. This implies that for any $f \in \mathcal{U}'$, $f(u_n) \xrightarrow{\text{st}} f(u)$ holds. Now, examine the sequence $(P'(g)(u_n))$. By using the definition of the adjoint operator, we obtain

$$P'(g)(u_n) = g(P(u_n)).$$

Since P is a statistical Dunford-Pettis operator, we know that $P(u_n) \xrightarrow{\text{st}} P(u)$ in $\mathcal{V}$. As g is a continuous linear functional on $\mathcal{V}$ and from [9] we have $C(\mathcal{V}, \mathbb{R}) \subseteq C_{st}(\mathcal{V}, \mathbb{R})$, it follows that $g(P(u_n)) \xrightarrow{\text{st}} g(P(u))$. Hence, $P'(g)$ is a statistical Dunford-Pettis operator.

(ii) To prove that P' is a statistical Dunford-Pettis operator, let (f_n) be a sequence in $\mathcal{V}'$ such that $f_n \xrightarrow{\text{w}_{\text{st}}} f$ in $\mathcal{V}'$. This implies that $f_n(v) \xrightarrow{\text{st}} f(v)$ for all $v \in \mathcal{V}$. Now, consider $P'(f_n)(u) = f_n(P(u))$ for any $u \in \mathcal{U}$. Since $P(u) \in \mathcal{V}$ for each $u \in \mathcal{U}$, it follows that $f_n(P(u)) \xrightarrow{\text{st}} f(P(u))$. Thus, $P'(f_n) \xrightarrow{\text{st}} P'(f)$ in $\mathcal{U}'$. Therefore, P' is a statistical Dunford-Pettis operator. □

Definition 3.7. *Let $P \in \mathcal{L}(\mathcal{U}, \mathcal{V})$ and (P_n) be a sequence in $DP_{st}(\mathcal{U}, \mathcal{V})$ for Banach spaces $\mathcal{U}$ and $\mathcal{V}$. Then (P_n) is said to be statistically convergent to P if $P_n(u) \xrightarrow{\text{st}} P(u)$ holds in $\mathcal{V}$ for all $u \in \mathcal{U}$.*

Theorem 3.8. *If a sequence (P_n) in $DP_{st}(\mathcal{U}, \mathcal{V})$ converges statistically to $P \in \mathcal{L}(\mathcal{U}, \mathcal{V})$, then P is a statistical Dunford-Pettis operator.*

Proof. Let (u_m) be any sequence in $\mathcal{U}$ such that $u_m \xrightarrow{\text{w}_{\text{st}}} u$ for some $u \in \mathcal{U}$. Since each P_n is a statistical Dunford-Pettis operator, we have $P_n(u_m) \xrightarrow{\text{st}} P_n(u)$ in $\mathcal{V}$ for each n. If we consider the following inequality:

$$\|P(u_m) - P(u)\| \leq \|P_n(u_m) - P(u_m)\| + \|P_n(u_m) - P_n(u)\| + \|P_n(u) - P(u)\|$$

then, we have

$$\begin{aligned} \{m \leq n : \|P(u_m) - P(u)\| > \varepsilon\} \quad \subseteq \quad & \{k \leq n : \|P_k(u_m) - P(u_m)\| > \frac{\varepsilon}{3}\} \\ & \cup \{m \leq k : \|P_n(u_m) - P_n(u)\| > \frac{\varepsilon}{3}\} \\ & \cup \{k \leq n : \|P_k(u) - P(u)\| > \frac{\varepsilon}{3}\}. \end{aligned}$$

Since $P_n(u) \xrightarrow{\text{st}} P(u)$ for all $u \in \mathcal{U}$, we have $\delta\Big(\{k \leq n : \|P_k(u_m) - P(u_m)\| > \frac{\varepsilon}{3}\}\Big) = 0$ and $\delta\Big(\{k \leq n : \|P_k(u) - P(u)\| > \frac{\varepsilon}{3}\}\Big) = 0$. Furthermore, since P_n is a statistical Dunford-Pettis operator for each n, we also have $\delta\Big(\{m \leq k : \|P_n(u_m) - P_n(u)\| > \frac{\varepsilon}{3}\}\Big) = 0$. As a result, we conclude that $\delta\Big(\{m \leq n : \|P(u_m) - P(u)\| > \varepsilon\}\Big) = 0$, which implies that P is a statistical Dunford-Pettis operator. □

Although the concept of a Cauchy sequence is well-defined in normed spaces, the definition of a statistically Cauchy sequence does not have a universally accepted standard. To provide clarity, we introduce the most commonly used definition of statistical Cauchy sequences in this paper.

Definition 3.9. *[16] A sequence (u_n) is called a* statistically Cauchy sequence *in a normed space $\mathcal{U}$ if, for every $\varepsilon > 0$, there exists a number $n_\varepsilon \in \mathbb{N}$ such that*

$$\lim_{n\to\infty} \frac{1}{n} \left|\{k < n : \|u_k - u_{n_\varepsilon}\| \geq \varepsilon\}\right| = 0.$$

Definition 3.10. *[10] A sequence (u_n) in a normed space $\mathcal{U}$ is called weakly statistically Cauchy if the sequence $(f(u_n))$ is a statistically Cauchy sequence for every $f \in \mathcal{U}'$.*

Theorem 3.11. *Let $P \in \mathcal{L}(\mathcal{U}, \mathcal{V})$ be an operator between Banach spaces such that $\mathcal{U}$ is reflexive. Then, the following statements hold.*

(i) If P is a statistical Dunford-Pettis, then it maps weakly statistically Cauchy sequences to statistically Cauchy sequences.

(ii) If P is a weakly statistically continuous operator and it maps weakly statistically Cauchy sequences to statistically Cauchy sequences, then it is a statistical Dunford-Pettis operator.

Proof. (*i*) Let $P \in DP_{st}(\mathcal{U}, \mathcal{V})$ and assume that (u_n) is a weakly statistically Cauchy sequence in $\mathcal{U}$. By [10, Thm.2.7], since $\mathcal{U}$ is a reflexive Banach space, it follows that (u_n) is weakly statistically convergent in $\mathcal{U}$. Furthermore, because P is a statistical Dunford-Pettis operator, $P(u_n)$ is statistically convergent. Consequently, [9, Thm.5.2] ensures that $P(u_n)$ is also a statistically Cauchy sequence.

(*ii*) Assume that $P(u_n)$ is a statistically Cauchy sequence in $\mathcal{V}$ whenever (u_n) is weakly statistically Cauchy in $\mathcal{U}$. Consider an arbitrary sequence (u_n) in $\mathcal{U}$ that is weakly statistically convergent to u, i.e., $u_n \xrightarrow{w_{st}} u$. Consequently, (u_n) is also a weakly statistically Cauchy sequence in $\mathcal{U}$ [10, p.5]. Therefore, it follows that $P(u_n)$ is a statistically Cauchy sequence in $\mathcal{V}$. Since $\mathcal{V}$ is a Banach space, [9, Thm.5.4] implies that $P(u_n)$ is statistically convergent to some element $v \in \mathcal{V}$. Additionally, because P is a weakly statistically continuous operator, we have $P(u_n) \xrightarrow{w_{st}} P(u)$ in $\mathcal{V}$. Thus, we conclude that $P(u) = v$. As a result, P is shown to be a statistical Dunford-Pettis operator. □

Corollary 3.12. *Let $\mathcal{U}$ be a reflexive Banach space. If $P \in DP_{st}(\mathcal{U}, \mathcal{V})$, then P maps weakly Cauchy sequences to statistically Cauchy sequences.*

Theorem 3.13. *Let $\mathcal{U}$ and $\mathcal{V}$ be Banach spaces such that ℓ_1 does not embed into $\mathcal{U}$. Then every statistical Dunford-Pettis operator from $\mathcal{U}$ to $\mathcal{V}$ is compact.*

Proof. Let $P : \mathcal{U} \to \mathcal{V}$ be a statistical Dunford-Pettis operator. Take a bounded subset B of $\mathcal{U}$. Consider a sequence (v_n) in $P(B)$, and so there exists a sequence (u_n) in B such that $v_n = P(u_n)$ for all n. It follows from the Rosenthal ℓ_1-Theorem that (u_n) has a weakly Cauchy subsequence denoted by (u_{n_k}) as ℓ_1 does not embed into $\mathcal{U}$. Since (u_{n_k}) is weakly Cauchy, it follows that (u_{n_k}) is weakly statistically convergent. This is because for any $f \in \mathcal{U}'$, the sequence $(f(u_{n_k}))$ is a Cauchy sequence in $\mathbb{R}$, and every Cauchy sequence in $\mathbb{R}$ is statistically convergent. Since P is a statistical Dunford-Pettis operator, we have $P(u_{n_k}) \xrightarrow{st} P(u)$ in $\mathcal{V}$ for some $u \in \mathcal{U}$. By the definition of statistical convergence, there exists a subsequence of $(P(u_{n_k}))$, which we denote as $(P(u_{n_{k_j}}))$, that converges in norm to $P(u)$. Thus, we have shown that for any sequence (v_n) in $P(B)$, there exists a subsequence $(v_{n_{k_j}}) = (P(u_{n_{k_j}}))$ that converges in norm. This implies that $P(B)$ is relatively compact in $\mathcal{V}$. Therefore, P is a compact operator. □

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