



On certain categories of non-associative cyclic hypergroups

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Abstract. Numerous generalizations of the rudimentary concept of hypergroups have been explored since its introduction by Marty. In this paper, we consider a notable category of non-associative cyclic hyperstructures called single power cyclic LA-hypergroups which are a conception of cyclic hypergroups and LA-semigroups. We enumerate single power cyclic LA-hypergroups of order two (and three) and period two. Additionally, we investigate some features of these single power cyclic LA-hypergroups. We also study various properties of cyclic LA-hypergroups and examine the differences between cyclic groups and cyclic LA-hypergroups.

1. Introduction

The foundational concept of hypergroups was given in 1934 by Marty [23]. Based on the notion of hyperoperation, it was presented as an intuitive generalization of the concept of a group. The composition of two elements is an element in a group while its a set in a hypergroup. Marty proved usefulness of hypergroups for study of algebraic functions, relational fractions and groups by demonstrating its applications. The books of Davvaz [7, 8], Corsini and Leoreanu [5] and Vougiouklis [31], provide insights into the overview of hypergroups (from theoretical reference) and their applications in different areas of mathematics including cryptography and code theory, automata theory, topology, geometry, probability theory, graphs and hypergraphs, theory of fuzzy and rough sets, binary relations, etc.

The underlying concept of LA-semigroups was introduced by M.A. Kazim and M. Naseeruddin in 1977. They have explored some key properties of LA-semigroups in [18]. LA-semihypergroups are an abstraction of LA-semigroups and semihypergroups. The idea of LA-semihypergroups was given by Hila and Dine in [14], where the writers have investigated some useful and key properties of this structure. The notions of LA-hypergroups and LA-hyperrings were given by Rehman and Yaqoob in [25], where the authors have explored some essential characterizations of LA-hyperrings through their hyperideals and hypersystems. The concepts of left and right almost-groups were introduced by Massouros and Yaqoob in [24], where the writers have concentrated on left and right almost-hypergroups and their enumerations in cases of order

2020 Mathematics Subject Classification. Primary 20N20.

Keywords. LA-hypergroups, Cyclic LA-hypergroups, Single power cyclic LA-hypergroups.

Received: 30 June 2025; Revised: 12 October 2025; Accepted: 02 January 2026

Communicated by Dijana Mosić

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2 and 3. They have also revealed interesting results by comparing outcomes of these enumerations with the corresponding ones in hypergroups. Approximations in Γ -semihypergroups and LA-semihypergroups has been studied by Yaqoob and others in [33–35].

A group generated by only one element is named as a cyclic group. De Salvo and Freni [26], Vougiouklis [32], Leoreanu [21], Karimian and Davvaz [17], have studied cyclicity in associative hyperstructures, i.e. semihypergroups and hypergroups. The significance of cyclic semihypergroups arises because of finitely generated semihypergroups and secondly due to their engrossing combinatorial connotations. Many outcomes, particularly, on single power cyclic hypergroups are given by Tahan and Davvaz in [28, 29]. All commutative order three and period two single power cyclic hypergroups have been grouped in [30]. All commutative order four and period two single power cyclic hypergroups have been grouped in [20]. Fuzzy set theory was applied to BCK-algebras and Γ -semigroups in [1, 9]. Khan et al. [19] Characterized ordered Γ -Abel-Grassmann’s groupoids.

The notion of cyclicity in non-associative hyperstructures (i.e. LA-hypergroups), was initiated by Kanwal and Yaqoob in [15], where the authors have introduced concepts of cyclic LA-hypergroups, single power and non single power cyclic LA-hypergroups and have explored some characteristics of these structures.

Cyclicity for associative structures (groups and hypergroups) is historical and established. In this research work, our prime objective is to explore and investigate cyclicity for non-associative structures, particularly for LA-hyperstructures and to study characteristics of single power cyclic LA-hypergroups. LA-hypergroups are a notable category of hyperstructures, which in general may be symmetric or associative in nature. Commutative or symmetric LA-hypergroups are indeed hypergroups, which are studied by numerous mathematicians in literature, so our key focus herewith is to investigate cyclicity for non-commutative LA-hypergroups.

This paper follows the structural layout distributed in different sections. Section 1&2 provide a brief introduction of basic definitions related to LA-hyperstructures. Section 3 presents concepts of cyclic LA-hypergroups (of finite order) and single power cyclic LA-hypergroups, proposing a new type of LA-hyperstructures for further study and investigation. Section 4 gives some properties of cyclic LA-hypergroups and examines the differences between cyclic associative structures (groups) and cyclic non-associative hyperstructures (LA-hypergroups), and deduces that some properties that exist in cyclic groups do not hold for cyclic LA-hypergroups.

Section 5 categorizes non-commutative cyclic single power LA-hypergroups of period and order two where each of its elements is a generator having period two. The number of period and order two, cyclic single power LA-hypergroups is only four, amongst which two are isomorphic and one is a total LA-hypergroup. Hence, there are just two non-isomorphic, non-commutative cyclic single power LA-hypergroups having order and period two.

Section 6 characterizes non-commutative cyclic single power LA-hypergroups of period two and order three where each of its elements is a generator having period two. We also explore some interesting characteristics of these LA-hyperstructures. Lastly, we determine that there are some isomorphic single power and non-commutative cyclic LA-hypergroups of period two and order three.

2. Preliminaries

Let $P^*(H)$ be the family of all nonempty subsets of a set H , then a map $\circ : H \times H \rightarrow P^*(H)$ is called a hyperoperation and (H, \circ) is called a hypergroupoid. If $\kappa \in H$ and Φ, Ψ are nonempty subsets of H , then we denote

$$\Phi \circ \Psi = \bigcup_{\varphi \in \Phi, \delta \in \Psi} \varphi \circ \delta, \quad \kappa \circ \Phi = \{\kappa\} \circ \Phi \quad \text{and} \quad \Phi \circ \kappa = \Phi \circ \{\kappa\}.$$

A hypergroupoid (H, \circ) is called commutative if for all elements κ and ζ of (H, \circ) , the equation $\kappa \circ \zeta = \zeta \circ \kappa$ holds. An associative hypergroupoid (H, \circ) , is called a semihypergroup. A quasihypergroup is a hypergroupoid that satisfies the reproductive law, i.e. $\kappa \circ H = H = H \circ \kappa$ for every $\kappa \in H$.

A hypergroupoid (H, \circ) is called a hypergroup if it is both a semihypergroup and a quasihypergroup.

A hypergroup (H, \circ) is cyclic if there exists $\hbar \in H$ such that $H = \hbar \cup \hbar^2 \cup \dots \cup \hbar^k \cup \dots$, where $\hbar = \{\hbar\}$ is a singleton set, and $\hbar^k = \hbar \circ \hbar \circ \dots \circ \hbar$, ($k \in \mathbb{N}$) k times, \hbar is called a generator of H . H is a cyclic hypergroup with finite period if $H = \hbar \cup \hbar^2 \cup \dots \cup \hbar^k$. Otherwise, H is called a cyclic hypergroup with infinite period. If there exists $\hbar \in H$ such that $H = \hbar \cup \hbar^2 \cup \dots \cup \hbar^k \cup \dots$ and $\hbar^{p-1} \subseteq \hbar^p$ for all $p \in \mathbb{N}_{\geq 2}$, then it is called a cyclic single power hypergroup.

Definition 2.1. [14] A hypergroupoid (H, \circ) is called an LA-semihypergroup if inverted associativity holds, i.e.

$$(\chi \circ \varsigma) \circ \zeta = (\zeta \circ \varsigma) \circ \chi \quad \forall \chi, \varsigma, \zeta \in H.$$

Definition 2.2. [25] An LA-semihypergroup is called an LA-hypergroup if for every $\chi \in H$,

$$\chi \circ H = H = H \circ \chi.$$

Example 2.3. [15] Let $H = \{p, q, r, s\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r	s
p	$\{p, q\}$	$\{q\}$	$\{r, s\}$	$\{r, s\}$
q	$\{p\}$	$\{p, q\}$	$\{r, s\}$	$\{r, s\}$
r	$\{s\}$	$\{r, s\}$	$\{p, q\}$	$\{p, q\}$
s	$\{r, s\}$	$\{r, s\}$	$\{p, q\}$	$\{p, q\}$

Table 1. Tabular form of the hyperoperation " \circ " defined in Example 2.3.

Then (H, \circ) is an LA-hypergroup.

An LA-hypergroup is called a total LA-hypergroup if $\chi \circ \varsigma = H$ for each $\chi, \varsigma \in H$.

A non-empty subset S of H is called an LA-subhypergroup if $\chi \circ \varsigma \subseteq S$ for each $\chi, \varsigma \in S$ and $\chi \circ S = S = S \circ \chi$.

An element χ of an LA-hypergroup H is called an idempotent element if $\chi \circ \chi = \chi$.

Definition 2.4. Let (H_1, \circ_1) and (H_2, \circ_2) be two LA-hypergroups. A function $\eta : (H_1, \circ_1) \rightarrow (H_2, \circ_2)$ is called a homomorphism if $\eta(\chi \circ_1 \varsigma) \subseteq \eta(\chi) \circ_2 \eta(\varsigma)$ for each $\chi, \varsigma \in H_1$ and it is called a good homomorphism if $\eta(\chi \circ_1 \varsigma) = \eta(\chi) \circ_2 \eta(\varsigma)$ for each $\chi, \varsigma \in H_1$. Two LA-hypergroups are called isomorphic if there exists a good and bijective homomorphism between them.

3. Cyclic LA-hypergroups

In this section, we present some definitions (given in [15]) and examples related to cyclic non-associative hyperstructures (LA-hypergroups).

Definition 3.1. [15] An LA-hypergroup (LA-semihypergroup) (H, \circ) is cyclic if there exists $\hbar \in H$ such that

$$H = \hbar \cup \hbar^2 \cup \dots \cup \hbar^v \cup \dots \text{ where } \hbar = \{\hbar\} \text{ is singleton}$$

and

$$\begin{aligned} \hbar^v &= \hbar^{v-1} \circ \hbar = (\hbar^{v-2} \circ \hbar) \circ \hbar \\ &= ((\hbar^{v-3} \circ \hbar) \circ \hbar) \circ \hbar \\ &= (\dots((\hbar \circ \hbar) \circ \hbar) \circ \hbar) \dots \circ \hbar, \quad v \text{ times, } v \in \mathbb{N}. \end{aligned}$$

$$\text{Also, } \hbar^v \hbar = \hbar^{v+1} \neq \hbar \hbar^v.$$

\hbar is called a generator of H . If $H = \hbar \cup \hbar^2 \cup \dots \cup \hbar^v$, then H is called a cyclic LA-hypergroup (LA-semihypergroup) with finite period. If v is the minimal number for which this relation holds then it is said that \hbar has period v . Otherwise, H is called a cyclic LA-hypergroup (LA-semihypergroup) with infinite period.

Note: It is obvious from this definition that cyclicity for LA-hypergroups and hypergroups is indistinguishable but inverted associativity holds in LA-hypergroups and associativity holds in hypergroups. The condition $\hbar^v \hbar = \hbar^{v+1} \neq \hbar \hbar^v$, signifies that for LA-hypergroups exponents are added using left nvertive law only, not in conventional way (as for associative frameworks).

Example 3.2. Let $H = \{p, q, r, s\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r	s
p	$\{q, r, s\}$	$\{q, r\}$	$\{q, r\}$	$\{q, r\}$
q	$\{q, r\}$	$\{q, r\}$	$\{q, r\}$	$\{q, r\}$
r	$\{q, s\}$	$\{q, s\}$	$\{q, s\}$	$\{q, s\}$
s	$\{q, s\}$	$\{q, s\}$	$\{q, s\}$	$\{q, s\}$

Table 2. Tabular form of the hyperoperation " \circ " defined in Example 3.2.

Then (H, \circ) is a cyclic LA-semihypergroup generated by p , it can be easily seen that

$$p \in p; q, r, s \in p^2 = p \circ p \text{ and } H = p \cup p^2.$$

This example is a cyclic LA-semihypergroup but is not a semihypergroup.

Example 3.3. Let $H = \{p, q, r\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r
p	$\{q, r\}$	$\{p, r\}$	$\{p, q\}$
q	$\{p, q\}$	$\{p, q\}$	$\{q, r\}$
r	$\{p, r\}$	$\{q, r\}$	$\{p, r\}$

Table 3. Tabular form of the hyperoperation " \circ " defined in Example 3.3.

Then (H, \circ) is a cyclic LA-hypergroup generated by p, q, r . It can be easily seen that

$$\begin{aligned} p &\in p; q, r \in p^2 = p \circ p \text{ and } H = p \cup p^2, \\ q &\in q; p \in q^2 = q \circ q; r \in q^3 = q^2 \circ q \text{ and } H = q \cup q^2 \cup q^3, \\ r &\in r; p \in r^2 = r \circ r; q \in r^3 = r^2 \circ r \text{ and } H = r \cup r^2 \cup r^3. \end{aligned}$$

This example is a cyclic LA-hypergroup but is not a hypergroup.

Definition 3.4. [15] A cyclic LA-hypergroup (LA-semihypergroup) (H, \circ) is called a single power cyclic LA-hypergroup (LA-semihypergroup) if there exists $\hbar \in H$ such that $H = \hbar \cup \hbar^2 \cup \dots \cup \hbar^v \cup \dots$ and $\hbar^{\rho-1} \subseteq \hbar^\rho$ for all $\rho \in \mathbb{N}_{\geq 2} = \{2, 3, 4, 5, \dots\}$.

Example 3.5. Let $H = \{p, q, r\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r
p	$\{p, q\}$	$\{p, q\}$	$\{p, q\}$
q	$\{p, r\}$	$\{r\}$	$\{r\}$
r	$\{p, q, r\}$	$\{q, r\}$	$\{q, r\}$

Table 4. Tabular form of the hyperoperation " \circ " defined in Example 3.5.

Then (H, \circ) is a single power cyclic LA-semihypergroup generated by p . It can be easily seen that $H = p \cup p^2 \cup p^3$ and $p \subseteq p^2$.

Example 3.6. Let $H = \{p, q, r, s, t\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r	s	t
p	H	$\{r\}$	$\{r\}$	$\{r\}$	$\{r\}$
q	$\{r\}$	H	$\{r\}$	$\{r\}$	$\{r\}$
r	H	H	H	H	H
s	$\{r\}$	$\{r\}$	$\{r\}$	H	$\{r\}$
t	$\{r\}$	$\{r\}$	$\{r\}$	$\{r\}$	H

Table 5. Tabular form of the hyperoperation " \circ " defined in Example 3.6.

Then (H, \circ) is a single power cyclic LA-hypergroup generated by p, q, r, s, t . It can be easily seen that

$$\begin{aligned}
 p &\in p; q, r, s, t \in p^2 = p \circ p; H = p \cup p^2 \text{ and } p \subseteq p^2, \\
 q &\in q; p, r, s, t \in q^2 = q \circ q; H = q \cup q^2 \text{ and } q \subseteq q^2, \\
 r &\in r; p, q, s, t \in r^2 = r \circ r; H = r \cup r^2 \text{ and } r \subseteq r^2, \\
 s &\in s; p, q, r, t \in s^2 = s \circ s; H = s \cup s^2 \text{ and } s \subseteq s^2, \\
 t &\in t; p, q, r, s \in t^2 = t \circ t; H = t \cup t^2 \text{ and } t \subseteq t^2.
 \end{aligned}$$

4. Differences between cyclic LA-hypergroups and groups

In this section, we investigate some properties of cyclic LA-hypergroups and examine the differences between cyclic groups (associative structures) and cyclic LA-hypergroups (non-associative hyperstructures). We notice that some well known characteristics that hold for cyclic groups are inapplicable to cyclic LA-hypergroups.

Proposition 4.1. Let $H = \{p, q, r\}$ and define a hyperoperation \circ on H as follows:

\circ	p	q	r
p	$\{q, r\}$	$\{p\}$	$\{p\}$
q	$\{p\}$	$\{q, r\}$	$\{q, r\}$
r	$\{p\}$	$\{q\}$	$\{q, r\}$

Then (H, \circ) is a non-commutative cyclic LA-hypergroup.

Proof. It is evident from the table above that (H, \circ) is non-commutative, satisfies reproductive axiom and is cyclic generated by p , that is, $H = p \cup p^2$. We now show from the following table that (H, \circ) also satisfies left invertive law.

$$\begin{aligned}
 (p \circ p) \circ q &= \{q, r\} \circ q = \{q, r\} & (q \circ p) \circ p &= p \circ p = \{q, r\} \\
 (p \circ p) \circ r &= \{q, r\} \circ r = \{q, r\} & (r \circ p) \circ p &= p \circ p = \{q, r\} \\
 (p \circ q) \circ q &= p \circ q = p & (q \circ q) \circ p &= \{q, r\} \circ p = p \\
 (p \circ q) \circ r &= p \circ r = p & (r \circ q) \circ p &= q \circ p = p \\
 (p \circ r) \circ q &= p \circ q = p & (q \circ r) \circ p &= \{q, r\} \circ p = p \\
 (p \circ r) \circ r &= p \circ r = p & (r \circ r) \circ p &= \{q, r\} \circ p = p \\
 (q \circ p) \circ p &= p \circ p = \{q, r\} & (p \circ p) \circ q &= \{q, r\} \circ q = \{q, r\} \\
 (q \circ p) \circ r &= p \circ r = p & (r \circ p) \circ q &= p \circ q = p \\
 (q \circ q) \circ p &= \{q, r\} \circ p = p & (p \circ q) \circ q &= p \circ q = p \\
 (q \circ q) \circ r &= \{q, r\} \circ r = \{q, r\} & (r \circ q) \circ q &= q \circ q = \{q, r\} \\
 (q \circ r) \circ p &= \{q, r\} \circ p = p & (p \circ r) \circ q &= p \circ q = p \\
 (q \circ r) \circ r &= \{q, r\} \circ r = \{q, r\} & (r \circ r) \circ r &= \{q, r\} \circ r = \{q, r\} \\
 (r \circ p) \circ p &= p \circ p = \{q, r\} & (p \circ p) \circ r &= \{q, r\} \circ r = \{q, r\} \\
 (r \circ p) \circ q &= p \circ q = p & (q \circ p) \circ r &= p \circ r = p \\
 (r \circ q) \circ p &= q \circ p = p & (p \circ q) \circ r &= p \circ r = p \\
 (r \circ q) \circ q &= q \circ q = \{q, r\} & (q \circ q) \circ r &= \{q, r\} \circ r = \{q, r\} \\
 (r \circ r) \circ p &= \{q, r\} \circ p = p & (p \circ r) \circ r &= p \circ r = p \\
 (r \circ r) \circ q &= \{q, r\} \circ q = \{q, r\} & (q \circ r) \circ r &= \{q, r\} \circ r = \{q, r\}
 \end{aligned}$$

□

Theorem 4.2. Let $H = \{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\}$ be a finite set with m elements. Define a hyperoperation \circ on H as follows:

$$\hbar_\alpha \circ \hbar_\zeta = \begin{cases} H - \hbar_1 & \begin{cases} \text{when } \alpha = \zeta, \\ \text{when } \alpha = 2, 3, \dots, m \text{ and } \zeta = 3, 4, \dots, m, \end{cases} \\ \hbar_1 & \begin{cases} \text{when } \alpha = 1 \text{ and } \zeta = 2, 3, \dots, m, \\ \text{when } \alpha = 2, 3, \dots, m \text{ and } \zeta = 1, \end{cases} \\ \hbar_2 & \text{when } \alpha = 3, 4, \dots, m \text{ and } \zeta = 2. \end{cases}$$

Then (H, \circ) is a finite order cyclic LA-hypergroup generated by the element \hbar_1 ; however it is not commutative.

Proof. (H, \circ) is cyclic generated by the element \hbar_1 , that is, $H = \hbar_1 \cup \hbar_1^2 = \hbar_1 \cup H - \hbar_1$. For any choices of α and ζ it also satisfies the reproductive axiom and the left invertive law. For instance let $\alpha = 5$ and $\zeta = 2$, then

$$\begin{aligned} (\hbar_5 \circ \hbar_2) \circ \hbar_2 &= (\hbar_2 \circ \hbar_2) \circ \hbar_5 \\ \hbar_2 \circ \hbar_2 &= \{H - \hbar_1\} \circ \hbar_5 \\ H - \hbar_1 &= \{\hbar_2, \hbar_3, \dots, \hbar_m\} \circ \hbar_5 \\ H - \hbar_1 &= H - \hbar_1. \end{aligned}$$

Also

$$\begin{aligned} \hbar_5 \circ H &= H \circ \hbar_5 \\ \hbar_5 \circ \{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\} &= \{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\} \circ \hbar_5 \\ \{H - \hbar_1\} \cup \hbar_1 \cup \hbar_1 \cup \dots \cup \hbar_1 &= \hbar_1 \cup \{H - \hbar_1\} \cup \{H - \hbar_1\} \dots \cup \{H - \hbar_1\} \\ H &= H. \end{aligned}$$

But

$$\begin{aligned} \hbar_5 \circ \hbar_2 &\neq \hbar_2 \circ \hbar_5 \\ \{\hbar_2\} &\neq \{H - \hbar_1\} \end{aligned}$$

In a similar way we can prove that for any $\alpha \in \{1, 2, 3, \dots, m\}$ and $\zeta \in \{1, 2, 3, 4, \dots, m\}$, the reproductive axiom and left invertive law is satisfied.

Hence (H, \circ) is a non-commutative finite order cyclic LA-hypergroup. □

Proposition 4.1 implies that the well-known result in group theory: Every cyclic group is abelian, is not true for LA-hypergroups. We present our next Corollary.

Corollary 4.3. Not every cyclic LA-hypergroup is commutative.

Proof. Let $H = (\{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\}, \circ)$ be a finite order cyclic LA-hypergroup generated by the element \hbar_1 , then all other elements of H can be written as some power of \hbar_1 , i.e. $\hbar_2, \hbar_3, \dots, \hbar_m \in \hbar_1^k$ for some $k \in \{2, 3, 4, \dots, m\}$. For (H, \circ) to be commutative $\hbar_i \circ \hbar_j = \hbar_j \circ \hbar_i$ for every $i, j \in \{1, 2, 3, 4, \dots, m\}$, but $\hbar_i \circ \hbar_j \neq \hbar_j \circ \hbar_i$, follows from Proposition 4.1. □

Proposition 4.4. Let $H = \{p, q, r, s, t\}$ with the hyperoperation \circ on H defined as follows:

\circ	p	q	r	s	t
p	$\{p\}$	H	H	H	H
q	$H - \{q\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$
r	H	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$
s	H	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$
t	H	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$

Then (H, \circ) is an LA-hypergroup that is not cyclic.

Proof. It is clear from the above table that (H, \circ) satisfies reproduction axiom. We can easily check that (H, \circ) also satisfies left invertive law. Thus, (H, \circ) is an LA-hypergroup. Since $p^2 = p^3 = \dots = p^n = \{p\}$; $q^2 = q^3 = \dots = q^n = H - \{p\}$; $r^2 = r^3 = \dots = r^n = H - \{p\}$; $s^2 = s^3 = \dots = s^n = H - \{p\}$; $t^2 = t^3 = \dots = t^n = H - \{p\}$. It follows that H is not generated by either of its elements so (H, \circ) is not cyclic. \square

Theorem 4.5. Let $H = \{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\}$ be a finite set with prime number of elements. Define a hyperoperation \circ on H as follows:

$$\hbar_\chi \circ \hbar_\varsigma = \begin{cases} \hbar_1 & \text{when } \chi = \varsigma = 1, \\ H & \begin{cases} \text{when } \chi = 1 \text{ and } \varsigma = 2, 3, \dots, m, \\ \text{when } \chi = 3, 4, \dots, m \text{ and } \varsigma = 1, \end{cases} \\ H - \hbar_1 & \text{when } \chi = 2, 3, \dots, m \text{ and } \varsigma = 2, 3, \dots, m, \\ H - \hbar_2 & \text{when } \chi = 2 \text{ and } \varsigma = 1. \end{cases}$$

Then (H, \circ) is a prime order LA-hypergroup that is not cyclic.

Proof. (H, \circ) is not cyclic since H can not be written as a power of any \hbar_χ for $\chi \in \{1, 2, 3, 4, \dots, m\}$. For any choices of χ and ς , (H, \circ) also satisfies the reproductive axiom and the left invertive law. For instance let $\chi = 3$ and $\varsigma = 7$, then

$$\begin{aligned} (\hbar_3 \circ \hbar_7) \circ \hbar_7 &= (\hbar_7 \circ \hbar_7) \circ \hbar_3 \\ \{H - \hbar_1\} \circ \hbar_7 &= \{H - \hbar_1\} \circ \hbar_7 \\ \{H - \hbar_1\} &= \{H - \hbar_1\}. \end{aligned}$$

Also

$$\begin{aligned} \hbar_3 \circ H &= H \circ \hbar_3 \\ H \cup \{H - \hbar_1\} \cup \{H - \hbar_1\} \cup \dots \cup \{H - \hbar_1\} &= H \cup \{H - \hbar_1\} \cup \{H - \hbar_1\} \cup \dots \cup \{H - \hbar_1\} \\ H &= H. \end{aligned}$$

In a similar way we can prove that for any $\chi \in \{1, 2, 3, \dots, m\}$ and $\varsigma \in \{1, 2, 3, 4, \dots, m\}$, the reproductive axiom and left invertive law is satisfied.

Hence (H, \circ) is a prime order cyclic LA-hypergroup. \square

Proposition 2 implies that the well-known result in group theory: Every group of prime order is cyclic, does not hold for LA-hypergroups. We present our next corollary.

Corollary 4.6. Not every LA-hypergroup of prime order is cyclic.

Proof. Let $H = (\{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\}, \circ)$ be a prime order LA-hypergroup. For (H, \circ) to be cyclic, every element of H must be some power of a single element of H . But it follows from Proposition 4.4 that H is not generated by either of its elements so (H, \circ) is not cyclic. \square

Proposition 4.7. Let $H = \{p, q, r\}$ and define hyperoperations \circ and \circ_* on H as follows

\circ	p	q	r	\circ_*	p	q	r
p	$\{q\}$	$\{q\}$	$\{q\}$	p	$\{p, r\}$	$\{p, r\}$	$\{p, r\}$
q	$\{q, r\}$	$\{q, r\}$	$\{q, r\}$	q	$\{p\}$	$\{p\}$	$\{p\}$
r	$\{q\}$	$\{q\}$	$\{q\}$	r	$\{p\}$	$\{p\}$	$\{p\}$

Then (H, \circ) and (H, \circ_*) are non-commutative, single power cyclic isomorphic LA-semihypergroups. Moreover, (H, \circ) and (H, \circ_*) are not LA-hypergroups.

Proof. Simple computations show that (H, \circ) satisfies left invertive law. Thus, (H, \circ) is an LA-semihypergroup. Since $p \circ H = H \circ p \neq H$, it follows that (H, \circ) is not an LA-hypergroup. It is clear that (H, \circ) is a non-commutative, single power cyclic LA-semihypergroup with generator p . To prove that (H, \circ) and (H, \circ_*) are isomorphic, let $f : (H, \circ) \rightarrow (H, \circ_*)$ defined by $f(p) = q, f(q) = p, f(r) = r$. It is easy to see that f is an isomorphism. \square

Proposition 4.8. Let $H = \{p, q, r\}$ and define hyperoperations \circ and \circ_* on H as follows:

\circ	p	q	r	\circ_*	p	q	r
p	H	$\{p, q\}$	$\{p\}$	p	H	$\{r\}$	$\{r\}$
q	$\{p\}$	H	$\{p\}$	q	$\{r\}$	H	$\{r\}$
r	$\{p\}$	$\{p\}$	H	r	$\{r\}$	$\{q, r\}$	H

Then (H, \circ) and (H, \circ_*) are non-commutative, single power cyclic isomorphic H_v -groups. Moreover, (H, \circ) and (H, \circ_*) are not LA-hypergroups.

Proof. Simple computations show that (H, \circ) satisfies weak associative law. Thus, (H, \circ) is an H_v -semigroup. Since $(q \circ r) \circ r = p \neq (r \circ r) \circ q = H$, it follows that (H, \circ) is not an LA-hypergroup. It is clear that (H, \circ) is a non-commutative, single power cyclic H_v -semigroup with three generators p, q and r . To prove that (H, \circ) and (H, \circ_*) are isomorphic, let $f : (H, \circ) \rightarrow (H, \circ_*)$ defined by $f(p) = r, f(q) = q, f(r) = p$. It is easy to see that f is an isomorphism. \square

Remark 4.9. Every single power cyclic LA-hypergroup is a cyclic LA-hypergroup. But the converse needs not to be true. The following proposition illustrates this fact.

Proposition 4.10. Let $H = \{p, q, r, s, t\}$ and (H, \circ) be given by the following table:

\circ	p	q	r	s	t
p	$H - \{p\}$	$\{p\}$	$\{p\}$	$\{p\}$	$\{p\}$
q	$\{p\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$
r	$\{p\}$	$\{q\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$
s	$\{p\}$	$\{q\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$
t	$\{p\}$	$\{q\}$	$H - \{p\}$	$H - \{p\}$	$H - \{p\}$

Then (H, \circ) is a cyclic LA-hypergroup that is not single power.

Proof. It is clear from the table above that (H, \circ) is non-commutative, satisfies reproductive axiom and is cyclic (generated by p) $p \in p; q, r, s, t \in p^2 = p \circ p$ and $H = p \cup p^2$. But is not single power because $p \notin p^2 = H - \{p\} \neq H$. We can easily prove by simple computations that (H, \circ) also satisfies left invertive law. Therefore, (H, \circ) is a cyclic LA-hypergroup that is not single power. \square

Theorem 4.11. Let $H = (\{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\}, \circ)$ be given by the following table:

$$\hbar_\alpha \circ \hbar_\zeta = \begin{cases} H - \hbar_1 & \begin{cases} \text{when } \alpha = \zeta, \\ \text{when } \alpha = 2, 3, \dots, m \text{ and } \zeta = 3, 4, \dots, m \end{cases} \\ \hbar_1 & \begin{cases} \text{when } \alpha = 1 \text{ and } \zeta = 2, 3, \dots, m, \\ \text{when } \alpha = 2, 3, \dots, m \text{ and } \zeta = 1, \end{cases} \\ \hbar_2 & \text{when } \alpha = 3, 4, \dots, m \text{ and } \zeta = 2. \end{cases}$$

Then (H, \circ) is a cyclic LA-hypergroup that is not single power.

Proof. The proof follows from Proposition 4.10 \square

Example 4.12. Let $H = \{p, q, r, s\}$ and define a hyperoperation \circ on H as follows:

\circ	p	q	r	s
p	$\{q, r, s\}$	$\{p, s\}$	$\{p, s\}$	$\{p, s\}$
q	$\{p, q, r\}$	$\{p, r, s\}$	$\{q, r\}$	$\{p, s\}$
r	$\{p, s\}$	$\{q, s\}$	$\{p, q, s\}$	$\{q, r\}$
s	$\{p, s\}$	$\{q, r\}$	$\{q, r\}$	$\{p, q, r\}$

Table 6. Tabular form of the hyperoperation " \circ " defined in Example 4.12.

Then (H, \circ) is a cyclic LA-hypergroup generated by all of its elements but it is not single power.

- $p \in p; q, r, s \in p^2 = p \circ p; H = p \cup p^2$ and $p \notin p^2,$
- $q \in q; p, r, s \in q^2 = q \circ q; H = q \cup q^2$ and $q \notin q^2,$
- $r \in r; p, q, s \in r^2 = r \circ r; H = r \cup r^2$ and $r \notin r^2,$
- $s \in s; p, q, r \in s^2 = s \circ s; H = s \cup s^2$ and $s \notin s^2.$

Remark 4.13. In a group the only idempotent element is the identity element, while in an LA-hypergroup there can be more than one idempotent elements. We present our next result.

Proposition 4.14. Let $H = \{p, q, r, s\}$ with the hyperoperation \circ defined as follows:

\circ	p	q	r	s
p	p	$\{p, q\}$	$\{p, r, s\}$	$\{p, r, s\}$
q	$\{p, q\}$	q	$\{q, r, s\}$	$\{q, r, s\}$
r	$\{p, s\}$	$\{q, s\}$	$\{r, s\}$	$\{r, s\}$
s	$\{p, r, s\}$	$\{q, r, s\}$	$\{r, s\}$	s

Then (H, \circ) is a non-commutative LA-hypergroup with three idempotent elements p, q and s .

Proof. It is evident from the table above that (H, \circ) is non-commutative, satisfies reproductive axiom. Easy computations show that (H, \circ) also satisfies left invertive law. It is also evident from table that p, q and s are idempotent elements since $p^2 = p \circ p = p; q^2 = q \circ q = q$ and $s^2 = s \circ s = s. \quad \square$

Remark 4.15. An order n single power non-commutative cyclic LA-hypergroup (generated by a single element) can have at the most $n - 1$ pairwise distinct idempotent elements. We present our next result.

Lemma 4.16. Let $H = \{\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_m\}$ is a finite set having m elements. Define a hyperoperation \circ on H as follows:

$$\hbar_\alpha \circ \hbar_\zeta = \begin{cases} \hbar_\alpha & \text{when } \alpha = \zeta, \text{ but } \alpha = \zeta \neq m, \\ H & \text{when } \alpha = 1, 2, 3, \dots, m - 1 \text{ and } \zeta = 1, 2, 3, \dots, m \text{ but } \alpha \neq \zeta, \\ \{\hbar_1, \hbar_2\} & \text{when } \alpha = m \text{ and } \zeta = 1, 2, 3, \dots, m - 1. \end{cases}$$

Then (H, \circ) is a cyclic single power non-commutative LA-hypergroup generated by the element \hbar_m ; all other $(m - 1)$ idempotent elements $\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_{m-1}$ are pairwise distinct but not generators.

Proof. (H, \circ) is a cyclic single power LA-hypergroup generated by the element \hbar_m since $H = \hbar_m^2$ and $\hbar_m \subseteq \hbar_m^2$. (H, \circ) is not commutative because $\hbar_\alpha \circ \hbar_\zeta \neq \hbar_\zeta \circ \hbar_\alpha$. All elements $\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_{m-1}$ are idempotents since $\hbar_\alpha^2 = \hbar_\alpha \circ \hbar_\alpha = \hbar_\alpha$ for $\alpha = 1, 2, 3, \dots, m - 1$. The elements $\hbar_1, \hbar_2, \hbar_3, \dots, \hbar_{m-1}$ are not generators since $\hbar_\alpha^2 \neq H$ for $\alpha = 1, 2, 3, \dots, m - 1$. The relationship between the inclusion chain of powers and the number of idempotent elements is reverse, that is if an element is a generator for a cyclic single power LA-hypergroup it satisfies the inclusion chain of powers but it is not an idempotent element and vice versa. \square

Remark 4.17. Lagrange theorem does not hold for LA-hypergroups. The order of a LA-subhypergroup of a finite LA-hypergroup may not always divide order of the LA-hypergroup.

Example 4.18. Let $H = \{p, q, r, s, t\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r	s	t
p	$\{p, q\}$	$\{p, q\}$	r	$\{s, t\}$	$\{s, t\}$
q	$\{p, q\}$	$\{p, q\}$	r	$\{s, t\}$	$\{s, t\}$
r	$\{s, t\}$	$\{s, t\}$	$\{p, q\}$	r	r
s	r	r	$\{s, t\}$	$\{p, q\}$	$\{p, q\}$
t	r	r	$\{s, t\}$	$\{p, q\}$	$\{p, q\}$

Table 7. Tabular form of the hyperoperation " \circ " defined in Example 4.18.

Then (H, \circ) is an LA-hypergroup of prime order, but is not cyclic. $H \neq h \cup h^2 \cup \dots \cup h^v \cup \dots$ for any $h \in \{p, q, r, s, t\}$. $(\{p, q\}, \circ)$ is an LA-subhypergroup of H which is cyclic.

We can also see from this example that Lagrange theorem does not hold, as order of LA-hypergroup is 5 and order of its LA-subhypergroup is 2.

Remark 4.19. For groups, every subgroup of a cyclic group is cyclic but for LA-hypergroups this result may not always hold.

Example 4.20. Let $H = \{p, q, r, s, t\}$ with the hyperoperation \circ defined as follows

\circ	p	q	r	s	t
p	p	$\{p, q, r\}$	$\{p, q, r\}$	$\{s, t\}$	$\{s, t\}$
q	$\{p, q, r\}$	$\{q, r\}$	$\{q, r\}$	$\{s, t\}$	$\{s, t\}$
r	$\{p, q\}$	q	$\{q, r\}$	t	$\{s, t\}$
s	t	t	$\{s, t\}$	$\{p, q, r\}$	$\{p, q\}$
t	$\{s, t\}$	$\{s, t\}$	$\{s, t\}$	$\{p, q, r\}$	$\{p, q, r\}$

Table 8. Tabular form of the hyperoperation " \circ " defined in Example 4.20.

Then (H, \circ) is a cyclic LA-hypergroup generated by s and t . $(\{p, q, r\}, \circ)$ is an LA-subhypergroup which is not cyclic.

5. Non-commutative cyclic LA-hypergroups of order two

In this section, we find and classify all non-commutative single power cyclic LA-hypergroups having order and period two up to isomorphism where every element is a generator having period two. The number of cyclic single power LA-hypergroups is only four, from whom one is a total LA-hypergroup and two are isomorphic. As a result, there are only two non-isomorphic, non-commutative cyclic single power LA-hypergroups having order and period two.

Proposition 5.1. Let $H = \{p, q\}$ and define hyperoperations \circ and \diamond on H as given in the following:

\circ	p	q	\diamond	p	q
p	H	$\{p\}$	p	H	H
q	H	H	q	$\{q\}$	H

Then (H, \circ) and (H, \diamond) are cyclic non-commutative single power isomorphic H_v -groups. Additionally (H, \circ) and (H, \diamond) are not LA-hypergroups.

Proof. It can be easily shown that (H, \circ) satisfies weak associative and reproductive law, so it is an H_v -group. Since $(p \circ q) \circ q = p \circ q = p \neq (q \circ q) \circ p = H \circ p = H$, so it is not an LA-hypergroup. Besides $p \circ q \neq q \circ p$, so it is also non-commutative. In addition it is obvious that p and q both are generators of H ; So (H, \circ) is a cyclic non-commutative single power H_v -group. Now to manifest that (H, \circ) and (H, \diamond) are isomorphic H_v -groups, define a mapping $\phi : (H, \circ) \rightarrow (H, \diamond)$ by $\phi(p) = q, \phi(q) = p$. It can be easily proved that ϕ is an isomorphism. \square

Theorem 5.2. *The single power cyclic LA-hypergroups $(H = \{p, q\}, \diamond_m)$ of period two and order two given in following are the only ones.*

\diamond_1	p	q	\diamond_2	p	q	\diamond_3	p	q	\diamond_4	p	q
p	H	$\{q\}$	p	H	$\{q\}$	p	H	H	p	H	H
q	$\{p\}$	H	q	H	H	q	H	H	q	$\{p\}$	H

\diamond_5	p	q	\diamond_6	p	q	\diamond_7	p	q
p	H	H	p	H	H	p	H	$\{p\}$
q	H	$\{p\}$	q	H	$\{q\}$	q	$\{p\}$	$\{q\}$

Proof. It is obvious from the tables above that (H, \diamond_m) for $m = 1, 2, 4$ are non-commutative, satisfy reproductive axiom and that $H = p^2 = q^2$. We now show from following table that (H, \diamond_1) also satisfies left invertive law.

$$\begin{aligned}
 (p \diamond_1 p) \diamond_1 q &= H \diamond_1 q = H & (q \diamond_1 p) \diamond_1 p &= p \diamond_1 p = H \\
 (p \diamond_1 q) \diamond_1 q &= q \diamond_1 q = H & (q \diamond_1 q) \diamond_1 p &= H \diamond_1 p = H \\
 (q \diamond_1 p) \diamond_1 p &= p \diamond_1 p = H & (p \diamond_1 p) \diamond_1 q &= H \diamond_1 q = H \\
 (q \diamond_1 q) \diamond_1 p &= H \diamond_1 p = H & (p \diamond_1 q) \diamond_1 q &= q \diamond_1 q = H
 \end{aligned}$$

In the same way, we can show that (H, \diamond_m) 's for $m \in \{1, 2, 3, 4, 5, 6, 7\}$ satisfy left invertive law.

Let $H = (\{p, q\}, \circ)$ be a single power cyclic LA-hypergroup. Then generators of H can be p or q or both of them. Since H is cyclic so $H = p \cup p^2 \cup p^3 \cup \dots$. Also since H is single power so $p \subset p^2 \subset p^3 \subset \dots$. Hence there is an element of H in p^2 but not in p . It implies that $p^2 = \{p, q\} = H$.

We get by utilizing Proposition 5.1 and abolishing its possibilities that the above given single power cyclic LA-hypergroups of order and period two are the only ones. If p and q both generate H then $(H, \diamond_1), (H, \diamond_2), (H, \diamond_3), (H, \diamond_4)$ are the only possibilities. If only p (or only q) generates H then $(H, \diamond_5), (H, \diamond_6)$ and (H, \diamond_7) are the only possibilities. Now it follows that the LA-hypergroups having two generators namely $(H, \diamond_1), (H, \diamond_2), (H, \diamond_3), (H, \diamond_4)$ are not isomorphic to LA-hypergroups $(H, \diamond_5), (H, \diamond_6), (H, \diamond_7)$ each having only one generator. Also $(H, \diamond_5), (H, \diamond_6)$ and (H, \diamond_7) are commutative single power cyclic LA-hypergroups so they are hypergroups as well, while $(H, \diamond_1), (H, \diamond_2), (H, \diamond_3)$ and (H, \diamond_4) are purely non-commutative single power cyclic LA-hypergroups. (H, \diamond_3) is the total single power cyclic LA-hypergroup as well as a hypergroup. (H, \diamond_2) and (H, \diamond_4) are isomorphic non-commutative single power cyclic LA-hypergroups. Define a mapping $\phi : (H, \diamond_2) \rightarrow (H, \diamond_4)$ by $\phi(p) = q, \phi(q) = p$. It can be easily proved that ϕ is an isomorphism. (H, \diamond_1) is not isomorphic to (H, \diamond_2) and (H, \diamond_4) . To prove that (H, \diamond_1) and (H, \diamond_2) are not isomorphic LA-hypergroups, we assume that there is a bijective mapping $\phi : (H, \diamond_1) \rightarrow (H, \diamond_2)$ which is a homomorphism. Here ϕ has two options, either it is the identity map or it is given as $\phi(p) = q, \phi(q) = p$. The first choice, ϕ is the identity map is not possible, for

$$\phi(q \diamond_1 p) = \phi(p) = p \neq \phi(q) \diamond_2 \phi(p) = q \diamond_2 p = H.$$

The other choice is not possible as well, for

$$\phi(p \diamond_1 q) = \phi(q) = p \neq \phi(p) \diamond_2 \phi(q) = q \diamond_2 p = H.$$

Likewise, (H, \diamond_1) is not isomorphic to (H, \diamond_4) . So there are only two non-commutative single power cyclic LA-hypergroups of order and period two up to isomorphism, i.e. (H, \diamond_1) and (H, \diamond_2) . \square

6. Non-commutative cyclic LA-hypergroups of order three

In this section, we present some interesting characteristics of cyclic LA-hypergroups. We find non-commutative single power cyclic LA-hypergroups having order three and period two up to isomorphism, where every element is a generator having period two. We also find that there are some isomorphic non-commutative single power cyclic LA-hypergroups in this class.

Theorem 6.1. Let (H, \circ) be a cyclic LA-hypergroup generated by an element $\bar{h} \in H$. If there exists $v \in \mathbb{N}$, that satisfies $\bar{h}^{v+1} = \bar{h}^v$, then period of $H \leq v$.

Proof. We use mathematical induction to prove that $\bar{h}^{v+t} = \bar{h}^v$, for all $t \geq 0$. Evidently, $\bar{h}^{v+t} = \bar{h}^v$, for $t = 0$ and $t = 1$. Let $t = 2$, then $\bar{h}^{v+2} = (\bar{h}^{v+1}) \circ \bar{h} = \bar{h}^v \circ \bar{h} = \bar{h}^{v+1} = \bar{h}^v$. Now let $t = 3$, then $\bar{h}^{v+3} = (\bar{h}^{v+2}) \circ \bar{h} = ((\bar{h}^{v+1}) \circ \bar{h}) \circ \bar{h} = (\bar{h}^v \circ \bar{h}) \circ \bar{h} = (\bar{h}^{v+1}) \circ \bar{h} = \bar{h}^v \circ \bar{h} = \bar{h}^{v+1} = \bar{h}^v$. Hence $\bar{h}^{v+t} = \bar{h}^v$ for $t = s$ (say). For $t = s + 1$, we have $\bar{h}^{v+t} = \bar{h}^{v+s+1} = (\bar{h}^{v+s}) \circ \bar{h} = \bar{h}^v \circ \bar{h} = \bar{h}^{v+1} = \bar{h}^v$ (This result does not hold for $\bar{h}^{1+v} = \bar{h}^v$). So for $v \leq i$ we have $\bar{h}^i = \bar{h}^v$. Hence $H \subseteq \bar{h} \cup \bar{h}^2 \cup \bar{h}^3 \cup \dots \cup \bar{h}^v$. Therefore period of $H \leq v$. \square

Proposition 6.2. If H is a cyclic LA-hypergroup having a finite order, then H has a finite period.

Proof. Let H be a cyclic LA-hypergroup with ω elements then $H = \{\bar{h}_1, \bar{h}_2, \bar{h}_3, \dots, \bar{h}_\omega\}$. Because H is cyclic, so there exists $\bar{h} \in H$ such that $H = \bar{h} \cup \bar{h}^2 \cup \bar{h}^3 \cup \dots$ and it intimates that $\bar{h}_i \in \bar{h}^j$ for some $0 \leq j \in \mathbb{N}$. Let we denote $j_i = \{\min j : \bar{h}_i \in \bar{h}^j\}$ and $v = \max\{j_1, j_2, \dots, j_\omega\} < \infty$. So, $H = \{\bar{h}_1, \bar{h}_2, \bar{h}_3, \dots, \bar{h}_\omega\} \subseteq \bar{h}^{j_1} \cup \bar{h}^{j_2} \cup \bar{h}^{j_3} \cup \dots \cup \bar{h}^{j_\omega} \subseteq \bar{h} \cup \bar{h}^2 \cup \bar{h}^3 \cup \dots \cup \bar{h}^v$. Hence period of $H \leq v$. \square

Lemma 6.3. Let H is a cyclic LA-hypergroup. Then period of $H = 1$ iff H is trivial LA-hypergroup.

Proof. If period of $H = 1$, then there exists an element $\bar{h} \in H$ such that $H = \{\bar{h}\}$, that is, H has just one element. On the other hand if H is a trivial LA-hypergroup then period of $H = 1$. \square

Proposition 6.4. If H is a cyclic single power LA-hypergroup of order $\omega \geq 2$, then $2 \leq \text{period of } H \leq \omega$.

Proof. Let H be a cyclic single power LA-hypergroup of order $\omega \geq 2$, then Proposition 6.3 propounds that H has a finite period v . So H can be written as $H = \bar{h} \cup \bar{h}^2 \cup \bar{h}^3 \cup \dots \cup \bar{h}^v$ and $\bar{h} \in \bar{h}^2 \subset \bar{h}^3 \subset \dots \subset \bar{h}^v$. The last of the two implies that there occurs at least one element $x_{i+1} \neq \bar{h} \in \bar{h}^{i+1}$ such that $x_{i+1} \notin \bar{h}^{i+1}$ for every $i = 1, 2, \dots, v - 1$. So

$$\{\bar{h}, x_2, x_3, \dots, x_v\} \subseteq H \text{ and it propounds that } v \leq \omega. \quad \square$$

Proposition 6.5. The single power cyclic LA-hypergroup generated by all of its elements does not have any LA-subhypergroups.

Proof. Let (H, \circ) be a single power cyclic LA-hypergroup generated by all of its elements then $\bar{h}^2 = \bar{h} \circ \bar{h} = H$, for every $\bar{h} \in H$. \square

Theorem 6.6. The LA-hypergroups $(H = \{p, q, r\}, \circ_k)$ of period two and order three given in the following are non-commutative, cyclic and single power.

\circ_1	p	q	r	\circ_2	p	q	r	\circ_3	p	q	r
p	H	H	$\{r\}$	p	H	H	$\{q, r\}$	p	H	H	$\{q, r\}$
q	H	H	$\{r\}$	q	H	H	$\{q, r\}$	q	$\{p\}$	H	$\{q, r\}$
r	$\{p, q\}$	$\{p, q\}$	H	r	$\{p\}$	$\{p\}$	H	r	$\{p\}$	H	H
\circ_4	p	q	r	\circ_5	p	q	r	\circ_6	p	q	r
p	H	$\{q, r\}$	$\{q, r\}$	p	H	H	$\{r\}$	p	H	$\{q, r\}$	$\{r\}$
q	H	H	H	q	H	H	$\{r\}$	q	H	H	$\{q, r\}$
r	H	H	H	r	H	H	H	r	H	H	H
\circ_7	p	q	r	\circ_8	p	q	r	\circ_9	p	q	r
p	H	$\{p, r\}$	$\{p, q\}$	p	H	$\{p, r\}$	$\{p, r\}$	p	H	$\{p, r\}$	$\{p, r\}$
q	$\{p, q\}$	H	$\{q, r\}$	q	$\{p, q\}$	H	$\{q, r\}$	q	$\{q, r\}$	H	$\{q, r\}$
r	$\{p, r\}$	$\{q, r\}$	H	r	$\{p, r\}$	$\{q, r\}$	H	r	$\{p, r\}$	$\{q, r\}$	H
\circ_{10}	p	q	r	\circ_{11}	p	q	r	\circ_{12}	p	q	r
p	H	$\{p, r\}$	$\{p, q\}$	p	H	$\{p, q\}$	$\{p, q\}$	p	H	$\{p, q\}$	$\{p, q\}$
q	$\{q, r\}$	H	$\{q, r\}$	q	$\{q, r\}$	H	$\{q, r\}$	q	$\{q, r\}$	H	$\{q, r\}$
r	$\{p, r\}$	$\{q, r\}$	H	r	$\{p, r\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H

\circ_{13}	p	q	r	\circ_{14}	p	q	r	\circ_{15}	p	q	r
p	H	$\{p, r\}$	$\{p, q\}$	p	H	$\{p, r\}$	$\{p, r\}$	p	H	$\{p, q\}$	$\{p, r\}$
q	$\{q, r\}$	H	$\{q, r\}$	q	$\{q, r\}$	H	$\{q, r\}$	q	$\{q, r\}$	H	$\{q, r\}$
r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H
\circ_{16}	p	q	r	\circ_{17}	p	q	r	\circ_{18}	p	q	r
p	H	$\{p, q\}$	$\{p, r\}$	p	H	$\{p, q\}$	$\{p, r\}$	p	H	$\{p, q\}$	$\{p, r\}$
q	$\{p, r\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{p, q\}$
r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{p, r\}$	H	r	$\{p, r\}$	$\{p, r\}$	H
\circ_{19}	p	q	r	\circ_{20}	p	q	r	\circ_{21}	p	q	r
p	H	$\{p, q\}$	$\{p, r\}$	p	H	$\{p, q\}$	$\{q, r\}$	p	H	$\{p, q\}$	$\{q, r\}$
q	$\{p, r\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{p, q\}$	q	$\{q, r\}$	H	$\{p, q\}$
r	$\{q, r\}$	$\{p, r\}$	H	r	$\{p, q\}$	$\{p, r\}$	H	r	$\{p, q\}$	$\{p, r\}$	H
\circ_{22}	p	q	r	\circ_{23}	p	q	r	\circ_{24}	p	q	r
p	H	$\{p, q\}$	$\{q, r\}$	p	H	$\{p, q\}$	$\{q, r\}$	p	H	$\{p, r\}$	$\{q, r\}$
q	$\{q, r\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{p, q\}$
r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H
\circ_{25}	p	q	r	\circ_{26}	p	q	r	\circ_{27}	p	q	r
p	H	$\{q, r\}$	$\{q, r\}$	p	H	$\{q, r\}$	$\{q, r\}$	p	H	$\{q, r\}$	$\{q, r\}$
q	$\{p, r\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{p, r\}$	q	$\{p, r\}$	H	$\{q, r\}$
r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{q, r\}$	H
\circ_{28}	p	q	r	\circ_{29}	p	q	r	\circ_{30}	p	q	r
p	H	$\{q, r\}$	$\{q, r\}$	p	H	$\{q, r\}$	$\{q, r\}$	p	H	$\{q, r\}$	$\{q, r\}$
q	$\{q, r\}$	H	$\{q, r\}$	q	$\{p, r\}$	H	$\{q, r\}$	q	$\{p, r\}$	H	$\{q, r\}$
r	$\{p, r\}$	$\{q, r\}$	H	r	$\{q, r\}$	$\{q, r\}$	H	r	$\{q, r\}$	$\{p, r\}$	H
\circ_{31}	p	q	r	\circ_{32}	p	q	r	\circ_{33}	p	q	r
p	H	$\{q, r\}$	$\{q, r\}$	p	H	$\{p, q\}$	$\{p, r\}$	p	H	$\{p, q\}$	$\{p, r\}$
q	$\{p, r\}$	H	$\{q, r\}$	q	$\{q, r\}$	H	$\{p, q\}$	q	$\{p, q\}$	H	$\{p, q\}$
r	$\{p, q\}$	$\{p, r\}$	H	r	$\{p, r\}$	$\{q, r\}$	H	r	$\{p, r\}$	$\{q, r\}$	H
\circ_{34}	p	q	r	\circ_{35}	p	q	r	\circ_{36}	p	q	r
p	H	$\{p, q\}$	$\{q, r\}$	p	H	$\{q, r\}$	$\{q, r\}$	p	H	$\{p, r\}$	$\{r\}$
q	$\{p, q\}$	H	$\{p, q\}$	q	$\{p, r\}$	H	$\{q, r\}$	q	$\{p, r\}$	H	$\{r\}$
r	$\{p, r\}$	$\{q, r\}$	H	r	$\{p, q\}$	$\{p, r\}$	H	r	$\{p, q\}$	$\{p, q\}$	H

Proof. It is obvious from the tables above that (H, \circ_k) for $k \in \{1, 2, 3, \dots, 36\}$ are non-commutative, satisfy reproductive axiom and that $H = p^2 = q^2 = r^2$. We now show from the following table that (H, \circ_5) also

satisfies left invertive law.

$$\begin{array}{ll}
 (p \circ_5 p) \circ_5 q = H \circ_5 q = H & (q \circ_5 p) \circ_5 p = H \circ_5 p = H \\
 (p \circ_5 p) \circ_5 r = H \circ_5 r = H & (r \circ_5 p) \circ_5 p = H \circ_5 p = H \\
 (p \circ_5 q) \circ_5 q = H \circ_5 q = H & (q \circ_5 q) \circ_5 p = H \circ_5 p = H \\
 (p \circ_5 q) \circ_5 r = H \circ_5 r = H & (r \circ_5 q) \circ_5 p = H \circ_5 p = H \\
 (p \circ_5 r) \circ_5 q = r \circ_5 q = H & (q \circ_5 r) \circ_5 p = r \circ_5 p = H \\
 (p \circ_5 r) \circ_5 r = r \circ_5 r = H & (r \circ_5 r) \circ_5 p = H \circ_5 p = H \\
 (q \circ_5 p) \circ_5 p = H \circ_5 p = H & (p \circ_5 p) \circ_5 q = H \circ_5 q = H \\
 (q \circ_5 p) \circ_5 r = H \circ_5 r = H & (r \circ_5 p) \circ_5 q = H \circ_5 q = H \\
 (q \circ_5 q) \circ_5 p = H \circ_5 p = H & (p \circ_5 q) \circ_5 q = H \circ_5 q = H \\
 (q \circ_5 q) \circ_5 r = H \circ_5 r = H & (r \circ_5 q) \circ_5 q = H \circ_5 q = H \\
 (q \circ_5 r) \circ_5 p = r \circ_5 p = H & (p \circ_5 r) \circ_5 q = r \circ_5 q = H \\
 (q \circ_5 r) \circ_5 r = r \circ_5 r = H & (r \circ_5 r) \circ_5 q = H \circ_5 q = H \\
 (r \circ_5 p) \circ_5 p = H \circ_5 p = H & (p \circ_5 p) \circ_5 r = H \circ_5 r = H \\
 (r \circ_5 p) \circ_5 q = H \circ_5 q = H & (q \circ_5 p) \circ_5 r = H \circ_5 r = H \\
 (r \circ_5 q) \circ_5 p = H \circ_5 p = H & (p \circ_5 q) \circ_5 r = H \circ_5 r = H \\
 (r \circ_5 q) \circ_5 q = H \circ_5 q = H & (q \circ_5 q) \circ_5 r = H \circ_5 r = H \\
 (r \circ_5 r) \circ_5 p = H \circ_5 p = H & (p \circ_5 r) \circ_5 r = r \circ_5 r = H \\
 (r \circ_5 r) \circ_5 q = H \circ_5 q = H & (q \circ_5 r) \circ_5 r = r \circ_5 r = H
 \end{array}$$

In the same way, we can show that (H, \circ_k) for $k \in \{1, 2, \dots, 40\}$ satisfy left invertive law. \square

Theorem 6.7. Let $(H = \{p, q, r\}, \circ)$ be an order three and period two, single power, non-commutative cyclic LA-hypergroup such that each of its elements is a generator having period two. Also, let

$\Omega(H)$	$= \{\{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
A	$= \{\{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
B	$= \{\{p\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
C	$= \{\{p\}, \{q\}, \{p, q\}, \{p, r\}, \{q, r\}, H\}.$

Then all possible outcomes for elements of (H, \circ) are given in the following table.

1	$p \circ q$ and $p \circ r$ is each element of A only,
2	$q \circ p$ and $q \circ r$ is each element of B only,
3	$r \circ p$ and $r \circ q$ is each element of C only.

Proof. 1. Since $p \circ q$ is a subset of H , it implies that $p \circ q$ can be any element of $\Omega(H)$. If $p \circ q = \{p\}$, then

$$(p \circ q) \circ q = p \circ q = \{p\} \neq (q \circ q) \circ p = H \circ p = H \Rightarrow p \circ q \neq \{p\}.$$

$p \circ r$ being a subset of H , can be any element of $\Omega(H)$. If $p \circ r = \{p\}$, then

$$(p \circ r) \circ r = p \circ r = \{p\} \neq (r \circ r) \circ p = H \circ p = H \Rightarrow p \circ r \neq \{p\}.$$

2. $q \circ p$ being a subset of H , can be any element of $\Omega(H)$. If $q \circ p = \{q\}$, then

$$(q \circ p) \circ p = q \circ p = \{q\} \neq (p \circ p) \circ q = H \circ q = H \Rightarrow q \circ p \neq q.$$

$q \circ r$ being a subset of H , can be any element of $\Omega(H)$. If $q \circ r = \{q\}$, then

$$(q \circ r) \circ r = q \circ r = \{q\} \neq (r \circ r) \circ q = H \circ q = H \Rightarrow q \circ r \neq q.$$

3. $r \circ p$ being a subset of H , can be any element of $\Omega(H)$. If $r \circ p = \{r\}$, then

$$(r \circ p) \circ p = r \circ p = \{r\} \neq (p \circ p) \circ r = H \circ r = H \Rightarrow r \circ p \neq r.$$

$r \circ q$ being a subset of H , can be any element of $\Omega(H)$. If $r \circ q = \{r\}$, then

$$(r \circ q) \circ q = r \circ q = \{r\} \neq (q \circ q) \circ r = H \circ r = H \Rightarrow r \circ q \neq r.$$

□

Theorem 6.8. Let $(H = \{p, q, r\}, \circ)$ be an order three and period two, single power, non-commutative cyclic LA-hypergroup such that each of its elements is a generator having period two. Also, let

$\Omega(H)$	$= \{\{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
A	$= \{\{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
B	$= \{\{p\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
C	$= \{\{p\}, \{q\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
D	$= \{\{q\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
E	$= \{\{r\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
F	$= \{\{p\}, \{p, q\}, \{p, r\}, \{q, r\}, H\},$
G	$= \{\{p\}, \{p, q\}, \{p, r\}, H\},$
I	$= \{\{q\}, \{p, q\}, \{q, r\}, H\},$
J	$= \{\{r\}, \{p, r\}, \{q, r\}, H\}.$

Then all possible resultant outcomes for elements of (H, \circ) are given in the following table.

	If	then
1	$p \circ q \in A$ or $r \circ p \in C$ or $q \circ p \in B - \{p\}$	$p \circ r \in A$
2	$p \circ r \in A$ or $r \circ p \in D$ or $q \circ p \in B$	$p \circ q \in A$
3	$p \circ q \in A$ or $q \circ r \in B$ or $r \circ q \in F$	$q \circ p \in B$
4	$p \circ q \in E$ or $q \circ p \in B$ or $r \circ q \in C$	$q \circ r \in B$
5	$p \circ r \in A$ or $q \circ r \in F$ or $r \circ q \in C$	$r \circ p \in C$
6	$p \circ r \in D$ or $q \circ r \in B$ or $r \circ q \in C$	$r \circ q \in C$
7	$p \circ q \in A$	$r \circ p \in D$
8	$p \circ q \in D$	$r \circ q \in D$
9	$r \circ q \in C$	$p \circ r \in D$
10	$r \circ q \in D$	$p \circ q \in D$
11	$p \circ r = \{q\}$	$r \circ p \in E$
12	$p \circ r \in E$	$q \circ p \in E$ or $q \circ r \in E$
13	$q \circ r \in B$	$p \circ q \in E$
14	$q \circ r \in E$	$p \circ r \in E$
15	$q \circ p \in G$	$r \circ p \in F$
16	$q \circ p \in B$	$r \circ q \in F$
17	$r \circ p \in G$	$q \circ p \in F$
18	$r \circ p \in C$	$q \circ r \in F$
19	$p \circ r = \{p, q\}$	$q \circ r \in J$
20	$q \circ p = \{q, r\}$	$r \circ p \in G$
21	$q \circ r = \{p, q\}$	$p \circ r \in J$
22	$r \circ p = \{p\}$	$p \circ q = H$
23	$r \circ p = \{q, r\}$	$q \circ p \in G$
24	$r \circ q = \{p, r\}$	$p \circ q \in I$

Proof. Proof follows from theorem 6.7. □

In the following propositions, we show that there are some isomorphic single power and non-commutative cyclic LA-hypergroups in Theorem 6.6.

Proposition 6.9. (H, \circ_7) and (H, \circ_{32}) are isomorphic single power cyclic LA-hypergroups.

Proof. Define $f : (H, \circ_7) \rightarrow (H, \circ_{32})$ by $f(p) = q, f(q) = p, f(r) = r$. Since (H, \circ_7) and (H, \circ_{32}) are isomorphic cyclic LA-hypergroups, it is sufficient to check the following:

$$\begin{array}{ll}
 f(p \circ_7 p) = f(H) = H & f(p) \circ_{32} f(p) = q \circ_{32} q = H \\
 f(p \circ_7 q) = f(\{p, r\}) = \{q, r\} & f(p) \circ_{32} f(q) = q \circ_{32} p = \{q, r\} \\
 f(p \circ_7 r) = f(\{p, q\}) = \{p, q\} & f(p) \circ_{32} f(r) = q \circ_{32} r = \{p, q\} \\
 f(q \circ_7 p) = f(\{p, q\}) = \{p, q\} & f(q) \circ_{32} f(p) = p \circ_{32} q = \{p, q\} \\
 f(q \circ_7 q) = f(H) = H & f(q) \circ_{32} f(y_H) = p \circ_{32} p = H \\
 f(q \circ_7 r) = f(\{q, r\}) = \{p, r\} & f(q) \circ_{32} f(r) = p \circ_{32} r = \{p, r\} \\
 f(r \circ_7 p) = f(\{p, r\}) = \{q, r\} & f(r) \circ_{32} f(p) = r \circ_{32} q = \{q, r\} \\
 f(r \circ_7 q) = f(\{q, r\}) = \{p, r\} & f(r) \circ_{32} f(q) = r \circ_{32} p = \{p, r\} \\
 f(r \circ_7 r) = f(H) = H & f(r) \circ_{32} f(r) = r \circ_{32} r = H
 \end{array}$$

□

Proposition 6.10. (H, \circ_8) and (H, \circ_{33}) are isomorphic single power cyclic LA-hypergroups.

Proof. Define $f : (H, \circ_8) \rightarrow (H, \circ_{33})$ by $f(p) = q, f(q) = r, f(r) = p$. Since (H, \circ_8) and (H, \circ_{33}) are isomorphic cyclic LA-hypergroups, it is sufficient to check the following:

$$\begin{array}{ll}
 f(p \circ_8 p) = f(H) = H & f(p) \circ_{33} f(p) = q \circ_{33} q = H \\
 f(p \circ_8 q) = f(\{p, r\}) = \{p, q\} & f(p) \circ_{33} f(q) = q \circ_{33} r = \{p, q\} \\
 f(p \circ_8 r) = f(\{p, r\}) = \{p, q\} & f(p) \circ_{33} f(r) = q \circ_{33} p = \{p, q\} \\
 f(q \circ_8 p) = f(\{p, q\}) = \{q, r\} & f(q) \circ_{33} f(p) = r \circ_{33} q = \{q, r\} \\
 f(q \circ_8 q) = f(H) = H & f(q) \circ_{33} f(q) = r \circ_{33} r = H \\
 f(q \circ_8 r) = f(\{q, r\}) = \{p, r\} & f(q) \circ_{33} f(r) = r \circ_{33} p = \{p, r\} \\
 f(r \circ_8 p) = f(\{p, r\}) = \{p, q\} & f(r) \circ_{33} f(p) = p \circ_{33} q = \{p, q\} \\
 f(r \circ_8 q) = f(\{q, r\}) = \{p, r\} & f(r) \circ_{33} f(q) = p \circ_{33} r = \{p, r\} \\
 f(r \circ_8 r) = f(H) = H & f(r) \circ_{33} f(r) = p \circ_{33} p = H
 \end{array}$$

□

Proposition 6.11. (H, \circ_{15}) and (H, \circ_{34}) are isomorphic single power cyclic LA-hypergroups.

Proof. Define $f : (H, \circ_{15}) \rightarrow (H, \circ_{34})$ by $f(p) = r, f(q) = q, f(r) = p$. Since (H, \circ_{15}) and (H, \circ_{34}) are isomorphic cyclic LA-hypergroups, it is sufficient to check the following:

$$\begin{array}{ll}
 f(p \circ_{15} p) = f(H) = H & f(p) \circ_{34} f(p) = r \circ_{34} r = H \\
 f(p \circ_{15} q) = f(\{p, q\}) = \{q, r\} & f(p) \circ_{34} f(q) = r \circ_{34} q = \{q, r\} \\
 f(p \circ_{15} r) = f(\{p, r\}) = \{p, r\} & f(p) \circ_{34} f(r) = r \circ_{34} p = \{p, r\} \\
 f(q \circ_{15} p) = f(\{q, r\}) = \{p, q\} & f(q) \circ_{34} f(p) = q \circ_{34} r = \{p, q\} \\
 f(q \circ_{15} q) = f(H) = H & f(q) \circ_{34} f(q) = q \circ_{34} q = H \\
 f(q \circ_{15} r) = f(\{q, r\}) = \{p, q\} & f(q) \circ_{34} f(r) = q \circ_{34} p = \{p, q\} \\
 f(r \circ_{15} p) = f(\{p, q\}) = \{q, r\} & f(r) \circ_{34} f(p) = p \circ_{34} r = \{p, r\} \\
 f(r \circ_{15} q) = f(\{q, r\}) = \{p, q\} & f(r) \circ_{34} f(q) = p \circ_{34} q = \{p, q\} \\
 f(r \circ_{15} r) = f(H) = H & f(r) \circ_{34} f(r) = p \circ_{34} p = H
 \end{array}$$

□

Proposition 6.12. (H, \circ_{25}) and (H, \circ_{35}) are isomorphic single power cyclic LA-hypergroups.

Proof. Define $f : (H, \circ_{25}) \rightarrow (H, \circ_{35})$ by $f(p) = p, f(q) = r, f(r) = q$. Since (H, \circ_{25}) and (H, \circ_{35}) are isomorphic cyclic LA-hypergroups, it is sufficient to check the following:

$$\begin{array}{ll}
 f(p \circ_{25} p) = f(H) = H & f(p) \circ_{35} f(p) = p \circ_{35} x_H = H \\
 f(p \circ_{25} q) = f(\{q, r\}) = \{q, r\} & f(p) \circ_{35} f(q) = p \circ_{35} r = \{q, r\} \\
 f(p \circ_{25} r) = f(\{q, r\}) = \{q, r\} & f(p) \circ_{35} f(r) = p \circ_{35} q = \{q, r\} \\
 f(q \circ_{25} p) = f(\{p, r\}) = \{p, q\} & f(q) \circ_{35} f(p) = r \circ_{35} p = \{p, q\} \\
 f(q \circ_{25} q) = f(H) = H & f(q) \circ_{35} f(q) = r \circ_{35} r = H \\
 f(q \circ_{25} r) = f(\{p, q\}) = \{p, r\} & f(q) \circ_{35} f(r) = r \circ_{35} q = \{p, r\} \\
 f(r \circ_{25} p) = f(\{p, q\}) = \{p, r\} & f(r) \circ_{35} f(p) = q \circ_{35} p = \{p, r\} \\
 f(r \circ_{25} q) = f(\{q, r\}) = \{q, r\} & f(r) \circ_{35} f(q) = q \circ_{35} r = \{q, r\} \\
 f(r \circ_{25} r) = f(H) = H & f(r) \circ_{35} f(r) = q \circ_{35} q = H
 \end{array}$$

□

7. Conclusion

A number of researchers have explored characteristics and features of hypergroups since its origination. In this paper, we have explored an important class of LA-hypergroups namely single power and non-commutative cyclic LA-hypergroups, that is a larger class as compared to single power commutative cyclic LA-hypergroups, which are indeed cyclic hypergroups. We have identified some attributes of associative structures (groups) that do not hold for non-associative hyperstructures (LA-hypergroups). We have further inspected single power and non-commutative cyclic LA-hypergroups of order two and three where each element is a generator having period two. As a future task, firstly, it can be provocative to expand our results to cyclic LA-hypergroups by determining all order three and four cyclic LA-hypergroups, secondly some properties and characteristics related to their order and period can also be explored. Additionally, it can be interesting to investigate the connections and utility of our prime outcomes by embodying ideas from n-valued groups, neutral elements of a hypergroup, superhyper groups, neutrosophic hyperstructures, superhyperstructures, superhyper magma, neutrosophic superhyperstructures, neutrosophic superhypermodal framework, knot theory, hyperknot theory, superhyperknot theory, superhyperuncertain sets and other interconnected domains [2–4, 6, 10–13, 16, 22, 27, 36]. Furthermore, it can be overwhelming to explore all these results and characteristics for non-single power cyclic LA-hypergroups and explore their behavior, attributes and characteristics.

Author Contributions: S.S. Kanwal wrote the main manuscript text. N. Abughazalah and N. Yaqoob supplied the main results.

Data Availability: No data has been used in this study.

Conflict of Interest: All the authors in the paper have no conflict of interest.

Ethical Approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Informed Consent: Informed consent was obtained from all individual participants included in the study.

Funding: This research was funded by the Princess Nourah bint Abdulrahman University Researchers Supporting Project Number (PNURSP2026R87), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Acknowledgements: This research was funded by the Princess Nourah bint Abdulrahman University Researchers Supporting Project Number (PNURSP2026R87), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

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