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Istrăţescu type contractions on new extended b-metric spaces

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Abstract. This paper investigates mappings with Istrăţescu type contractive properties within the framework of new extended b-metric spaces. Various generalized contractive operators are defined, including those of order 2, two-sided, type 2, and Ćirić-convex contractions. The study establishes several fixed point theorems for these mappings under specific conditions, such as orbital continuity or the boundedness of the function involved in the structure of the new extended b-metric space. These results extend previous findings in the literature.

1. Introduction

The concept of metric spaces has been a cornerstone of analysis and topology, providing a fundamental framework for useful means of development, such as distances or convergence. Over the years, this framework has been extended to more general spaces, in order to accommodate a broader class of problems. Among these extensions, the notion of b-metric spaces, see Bakhtin [5] or Czerwik [9], stands out as a significant outcome that allows the relaxation of the triangle inequality by incorporating a scaling factor. This has proven useful in various applications, particularly in fixed point theory. Comparison functions have been effectively applied to contraction mappings in b-metric spaces to derive common fixed points, as demonstrated by Shatanawi et al. [17]. Further on, Afshari et al. [1] introduced generalized $\alpha-\psi-$Geraghty contractive mappings in b-metric spaces, offering significant extensions to fixed-point theory and applications, such as integral equations. Miculescu and Mihail [13] made a substantial contribution to the development of contraction principles by extending Nadler's framework to encompass set-valued functions within the context of b-metric spaces, thereby enriching the understanding of multivalued mappings and their applications.

In recent years, the concept of extended b-metric spaces has emerged, offering an even more flexible framework by introducing a function of two variables that changes based on the points within the space, further generalizing the distance function. This was done by Kamran et al. in [12], defining the notion of extended b-metric spaces. To broaden the applicability of metric structures in nonlinear analysis, Samreen et al. [16] developed fixed point results within extended b-metric spaces using novel comparison functions. Shatanawi et al. [18] contributed to the study of extended b-metric spaces by proving fixed-point theorems for mappings satisfying $\alpha-\psi-$contractive conditions. Recent developments, such as those by Haokip

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et al. in [10], have focused on obtaining common fixed point results in extended b-metric spaces using (α-β)-orbital-cyclic admissible triplets. Alqahtani et al. [2] explored fixed-point results in the context of extended b-metric spaces, emphasizing cases where uniqueness is not necessarily ensured.

A novel class of spaces, known as new extended b-metric spaces, introduced by Aydi et al. in [4], allows even greater generalization by using a function of three variables instead of that depending on two variables, thus providing a more comprehensive tool for analyzing fixed points in nonlinear settings. Roy and Saha [15] further contributed to this area by investigating interpolative Caristi-type contractive mappings in new extended b-metric spaces, presenting another approach to generalize classical fixed point results in these settings.

Another recent generalization of b-metric spaces is the class of (q_1, q_2)-quasimetric spaces introduced by Arutyunov and Greshnov in [3], where the triangle inequality is relaxed to $d(x, y) \leq q_1\, d(x, z) + q_2\, d(z, y)$ for fixed q_1, $q_2 \geq 1$ and symmetry is not required in general. In contrast, the new extended b-metric spaces studied here incorporate a variable coefficient $\theta(x, y, z) \geq 1$ in $d(x, z) \leq \theta(x, y, z)[d(x, y) + d(y, z)]$, preserving symmetry and enabling point-dependent flexibility not captured by fixed coefficients. Their review surveys topological properties, covering and Lipschitz mappings, coincidence point theorems, and extensions of Banach's fixed point principle in such spaces, including multivalued variants and stability results. Our results focus on fixed point theorems for Istrăţescu-type contractions, generalizing [14] and complementing the results from [3].

Fixed point theory, a crucial area in functional analysis, with applications in various fields such as differential equations, optimization, or dynamic systems, has indeed benefited from these generalizations. The Banach-Picard-Caccioppoli contraction principle [6, 7], a foundational result in this direction, has seen numerous extensions and adaptations to fit the context of these generalized spaces. The contractive mappings defined by Istrăţescu in [11], in particular, have garnered significant attention due to their ability to capture a wide range of mappings that are not necessarily contractions in the classical sense, but still possess unique fixed points.

This paper aims to build starting from the existing body of work by investigating Istrăţescu contractions within the setting of new extended b-metric spaces. We recall several types of these contractions, including those of order 2, of order k, two-sided, type 2, and Ćirić-convex contractions, and establish fixed point theorems for each. These results are proven under conditions such as the orbital continuity of the mapping and the boundedness of the auxiliary function which is the foundation of the new extended b-metric spaces. Our findings generalize results of previous studies in this framework.

The organization of the paper is as follows. In Section 2, we review the necessary preliminaries, including definitions and examples that are relevant to our discussion. Section 3 presents our main results, where we seek to generalize the results established by Ricinschi in [14], further expanding the scope of fixed point theorems for Istrăţescu contractions within new extended b-metric spaces. In doing so, we utilize a pivotal criterion introduced by Ćirić in [8], which plays a crucial role in the proofs of the main results. We also include an example that illustrates the applicability of our results, followed by a discussion on potential future research directions.

2. Preliminary definitions and auxiliary results

The aim of this section is to review and summarize relevant concepts and results from literature to provide a thorough overview regarding the work of this paper.

We begin by presenting the definition of a b-metric space.

Definition 2.1. [5, 9] *Let X be a nonempty set and $s \geq 1$ be a given real number. A function $d\colon X \times X \to [0, \infty)$ is called a b-metric provided that for all x, y, $z \in X$,*

- $d(x, y) = 0$ *if and only if* $x = y$,
- $d(x, y) = d(y, x)$,
- $d(x, z) \leq s[d(x, y) + d(y, z)]$.

A pair (X,d) is called a b-metric space of constant s.

We proceed with a generalization of the previously defined space, in the sense of Kamran et al. [12].

Definition 2.2. [12] *Let X be a nonempty set and $\theta\colon X\times X\to[1,\infty)$. A function $d\colon X\times X\to[0,\infty)$ is called an extended b-metric if for all x, y, $z\in X$, it satisfies:*

- $d(x,y)=0$ *if and only if* $x=y$,
- $d(x,y)=d(y,x)$,
- $d(x,z)\le\theta(x,z)[d(x,y)+d(y,z)]$.

A pair (X,d) is called an extended b-metric space.

Example 2.3. [10] *Let $X=[-1,1]$. Take*

$$d\colon X\times X\to[0,\infty),\quad d(x,y)=\begin{cases}0, \text{ if and only if } x=y,\\ \frac{1}{x^2}, \text{ if } xy=0, \text{and } x^2+y^2\neq 0,\\ \frac{1}{x^2y^2}, \text{ if } 0\neq x\neq y\neq 0.\end{cases}$$

Also, consider

$$\theta\colon X\times X\to[1,\infty),\quad \theta(x,y)=\begin{cases}\dfrac{1+x^2+y^2}{x^2+y^2}, \text{ if } x^2+y^2>0,\\ 1, \text{ if } x=y=0.\end{cases}$$

It is easy to check that d defines an extended b-metric on X.
Forwards, we present a generalization of extended b-metric spaces, due to Aydi et al. in [4].

Definition 2.4. [4] *Let X be a nonempty set and $\theta\colon X\times X\times X\to[1,\infty)$. A new extended b-metric is a function $d\colon X\times X\to[0,\infty)$ such that for all x, y, $z\in X$,*

- $d(x,y)=0$ *if and only if* $x=y$,
- $d(x,y)=d(y,x)$,
- $d(x,z)\le\theta(x,y,z)[d(x,y)+d(y,z)]$.

The pair (X,d) is then called a new extended b-metric space.

Obviously, when $\theta(x,y,z)=\theta(x,z)$, the above definition coincided with Definition 2.2. If $\theta(x,y,z)=s\ge 1$, we reach the concept of a b-metric space.
The following examples provide an assurance that new extended b-metric spaces represent a proper generalization for the class of extended b-metric spaces.

Example 2.5. [4] *Let $X=\mathbb{N}$. Take*

$$d\colon X\times X\to[0,\infty),\quad d(x,y)=\begin{cases}0, \text{ if and only if } x=y,\\ \frac{1}{x}, \text{ if } x \text{ is even and } y \text{ is odd},\\ \frac{1}{y}, \text{ if } x \text{ is odd and } y \text{ is even},\\ 1, \text{ otherwise}.\end{cases}$$

Also, consider

$$\theta\colon X \times X \times X \to [1, \infty),$$

$$\theta(x,y,z) = \begin{cases} 1, \text{ if } x = z \text{ and } y \text{ is even or odd }, \\ \frac{xz}{x+z}, \text{ if } x \neq z, x \text{ and } z \text{ are even and } y \text{ is odd}, \\ \frac{y}{2}, \text{ if } x \neq z, x \text{ and } z \text{ are odd and } y \text{ is even}, \\ \frac{3}{2}, \text{ if } x \neq z \text{ and } x, y \text{ and } z \text{ are all even or all odd}, \\ \frac{x+y(1+x)}{x(1+y)}, \text{ if } x \neq z, x \text{ and } y \text{ are even and } z \text{ is odd}, \\ \frac{z+y(z+1)}{z(y+1)}, \text{ if } x \neq z, x \text{ is odd and } y \text{ and } z \text{ are even}, \\ \frac{2+z}{1+z}, \text{ if } x \neq z, x \text{ and } y \text{ are odd and } z \text{ is even}, \\ \frac{x+1}{x}, \text{ if } x \neq z, x \text{ is even and } y \text{ and } z \text{ are odd}. \end{cases}$$

The authors have verified in [4] that, in this case, (X, d) is a new extended b-metric space.

Onwards, we will prove that (X, d) is not an extended b-metric space, neither. Suppose there is a function $\theta\colon X \times X \to [1, \infty)$ such that the inequality

$$d(x,z) \leq \theta(x,z)[d(x,y) + d(y,z)]$$

is always valid.

Let $x = 1$ and $z = 3$. Thus, the previous inequality implies that

$$\theta(1,3) \geq \frac{y}{2},$$

for any even number y, which cannot be true. Then, d is not an extended b-metric on X.

Example 2.6. [15] *Let $X = (0, \infty)$. Take*

$$d\colon X \times X \to [0, \infty), \quad d(x,y) = \begin{cases} 0, \text{ if and only if } x = y, \\ e^{-x}, \text{ if } x \text{ is rational and } y \text{ is irrational}, \\ e^{-y}, \text{ if } x \text{ is irrational and } y \text{ is rational}, \\ 1, \text{ otherwise}. \end{cases}$$

Also, consider

$$\theta\colon X \times X \times X \to [1, \infty),$$

$$\theta(x,y,z) = \begin{cases} \frac{e^{x+z}}{e^x + e^z}, \text{ if } x \neq z, x \text{ and } z \text{ are rational and } y \text{ is irrational}, \\ \frac{e^y}{2}, \text{ if } x \neq z, x \text{ and } z \text{ are irrational and } y \text{ is rational}, \\ e^{y-x}, \text{ if } x \text{ and } y \text{ are rational and } z \text{ is irrational}, \\ e^{y-z}, \text{ if } x \text{ and } z \text{ are rational and } y \text{ is irrational}, \\ l(\geq 1), \text{ otherwise }. \end{cases}$$

The authors have verified in [15] that, in this case, (X, d) is a new extended b-metric space and stated that it is not an extended b-metric space.

Onwards, we will justify this statement. Let $x = \sqrt{2}$ and $z = \sqrt{3}$. Now, the inequality

$$d(x,z) \leq \theta(x,z)[d(x,y) + d(y,z)]$$

yields to

$$\theta(\sqrt{2}, \sqrt{3}) \geq \frac{e^y}{2}.$$

for any rational number y, which is false. Thus, d is not an extended b-metric on X.

The notions of Cauchy and convergent sequences in new extended b-metric spaces are defined as follows:

Definition 2.7. [4] *Let (X, d) be a new extended b-metric space, and let $\{x_n\}_n$ be a sequence in X.*

- *$\{x_n\}_n$ is convergent to $x \in X$ if and only if $\lim_{n\to\infty} d(x_n, x) = 0$.*
- *$\{x_n\}_n$ is Cauchy if for all $\varepsilon > 0$, there exists $N_\varepsilon \in \mathbb{N}$ such that for all $n, m \geq N_\varepsilon$, $d(x_n, x_m) \leq \varepsilon$.*
- *(X, d) is said complete if every Cauchy sequence $\{x_n\}_n$ in X converges.*

For the following definitions, consider X a nonempty set and (X, d) a new extended b-metric space.

Definition 2.8. [8] *A mapping $T: X \to X$ is called orbitally continuous if $\lim_{i\to\infty} T^{n_i}x = z$ implies $\lim_{i\to\infty} T(T^{n_i}x) = Tz$, where T^n denotes the n-fold composition of T with itself.*

Definition 2.9. [11] *A mapping $T: X \to X$ is said to be a contraction of order 2 (in the sense of Istrăţescu) if there exist $a, b \in [0,1)$ with $0 < a + b < 1$ such that for all $x, y \in X$, the following inequality holds:*

$$d(T^2x, T^2y) \leq ad(Tx, Ty) + bd(x, y).$$

Definition 2.10. [11] *A mapping $T: X \to X$ is said to be a two-sided contraction (in the sense of Istrăţescu) if there exist $a_1, a_2, b_1, b_2 \in [0,1)$ with $0 < a_1 + a_2 + b_1 + b_2 < 1$ such that for all $x, y \in X$, the following inequality holds:*

$$d(T^2x, T^2y) \leq a_1 d(x, Tx) + a_2 d(Tx, T^2x) + b_1 d(y, Ty) + b_2 d(Ty, T^2y).$$

Definition 2.11. [11] *A mapping $T: X \to X$ is said to be a contraction of type 2 (in the sense of Istrăţescu) if there exist constants $c_0, c_1, a_1, a_2, b_1, b_2 \in [0,1)$ with $0 < c_0 + c_1 + a_1 + a_2 + b_1 + b_2 < 1$ such that for all $x, y \in X$, the following inequality holds:*

$$\begin{aligned} d(T^2x, T^2y) \leq\ & c_0 d(x,y) + c_1 d(Tx, Ty) + a_1 d(x, Tx) \\ & + a_2 d(Tx, T^2x) + b_1 d(y, Ty) + b_2 d(Ty, T^2y). \end{aligned}$$

Definition 2.12. [11] *A mapping $T: X \to X$ is said to be a contraction of order $k \geq 2$ (in the sense of Istrăţescu) if there exist $a_0, a_1, ..., a_{k-1} \in [0,1)$ with $0 < a_0 + a_1 + ... + a_{k-1} < 1$ such that for all $x, y \in X$, the following inequality holds:*

$$d(T^kx, T^ky) \leq a_0 d(x,y) + a_1 d(Tx, Ty) + ... + a_{k-1} d(T^{k-1}x, T^{k-1}y).$$

Definition 2.13. [14] *A mapping $T: X \to X$ is said to be a Ćirić-convex contraction if there exists $h \in (0,1)$ such that for all $x, y \in X$, the following inequality holds:*

$$\begin{aligned} d(T^2x, T^2y) \leq\ & h \max\{d(x,y), d(Tx,Ty), d(x,Tx), \\ & d(Tx, T^2x), d(y,Ty), d(Ty, T^2y)\}. \end{aligned}$$

3. Results in new extended *b*-metric spaces

We refer to $\mathbb{N}$ as the set of all nonnegative integers and for a mapping $T\colon X \to X$ and a point $x_0 \in X$, we denote by $\{x_n\}_n$ the sequence of Picard iterations based on an initial point x_0, i.e. $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$.

Henceforth, consider that X is a nonempty set and (X, d) is a new extended b-metric space.

The following results can be proved in a manner similar as in [14], for the case of new extended b-metric.

Lemma 3.1. *If T is an Istrăţescu contraction on X, then for any $x_0 \in X$ and for all $n \in \mathbb{N}$, we have that*

$$d(x_n, x_{n+1}) \leq \sqrt{(a+b)^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}.$$

Lemma 3.2. *For a two-sided contraction T on X, for any $x_0 \in X$ and any $n \in \mathbb{N}$ it holds that*

$$d(x_n, x_{n+1}) \leq \sqrt{\Big(\frac{a_1 + a_2 + b_1}{1 - b_2}\Big)^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}.$$

Lemma 3.3. *Consider that T is a contraction of type 2 on X. Then, for all $x_0 \in X$ and any $n \in \mathbb{N}$, it is true that*

$$d(x_n, x_{n+1}) \leq \sqrt{\Big(\frac{c_0 + c_1 + a_1 + a_2 + b_1}{1 - b_2}\Big)^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}.$$

Lemma 3.4. *If T is a Ćirić-convex contraction on X, the following inequality holds for any $x_0 \in X$ and for all $n \in \mathbb{N}$:*

$$d(x_n, x_{n+1}) \leq \sqrt{h^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}.$$

Lemma 3.5. *If T is a contraction of order $k \geq 2$ on X, then for all $x_0 \in X$ and $n \in \mathbb{N}$, the following inequality holds:*

$$d(x_n, x_{n+1}) \leq \sqrt[k]{(a_0 + ... + a_{k-1})^{n-k}} \max\{d(x_0, x_1), ..., d(x_{k-1}, x_k)\}.$$

Next, we present some useful results for proving fixed point theorems in the setting of new extended b-metric spaces.

Lemma 3.6. *Let (X, d) be a new extended b-metric space. If the function $\theta(\cdot, \cdot, \cdot)$ is bounded, then every convergent sequence in X has a unique limit.*

Proof. Let $\{x_n\}_n$ be a convergent sequence such that $\lim_{n\to\infty} x_n = u$ and $\lim_{n\to\infty} x_n = v$, where $u, v \in X$. Then

$$d(u, v) \leq \theta(u, x_n, v)[d(u, x_n) + d(x_n, v)].$$

As $\theta(\cdot, \cdot, \cdot)$ is bounded, by taking limit when $n \to \infty$ in the previous inequality, we obtain that $d(u, v) = 0$, thus $u = v$. □

Lemma 3.7. *Let (X, d) be a new extended b-metric space, let $T\colon X \to X$ be a mapping, and $x_0 \in X$. If there exist $\lambda \in (0, 1)$ such that $d(x_n, x_{n+1}) \leq \sqrt{\lambda^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}$ for all $n \in \mathbb{N}$, and $\beta < \frac{1}{\sqrt{\lambda}}$ and $n_0 \in \mathbb{N}$ such that the inequality*

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$ and $j \in \{n, n+1, ..., n+p-2\}$, then $\{x_n\}_n$ is a Cauchy sequence.

Proof. Set $d_n = d(x_n, x_{n+1})$ for all $n \geq 0$ and $M = \max\{d_0, d_1\}$.

Consider $n, p, j \in \mathbb{N}$ with $n \geq n_0$, $p \geq 1$ and $j \in \{n, n+1, ..., n+p-2\}$.

Will justify that $\{x_n\}_n$ is a Cauchy sequence:

If $p = 1$, then

$$d(x_n, x_{n+1}) \leq \sqrt{\lambda^{n-2}} M \to 0, \text{ when } n \to \infty,$$

and if $p \geq 2$, consider that

$$\begin{aligned}
d(x_n, x_{n+p}) &\leq \theta(x_n, x_{n+1}, x_{n+p})\Big[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+p})\Big] \\
&\leq \theta(x_n, x_{n+1}, x_{n+p})\, d_n + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\, d(x_{n+1}, x_{n+2}) \\
&\quad + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\, d(x_{n+2}, x_{n+p}) \\
&\leq \theta(x_n, x_{n+1}, x_{n+p})\, d_n + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\, d_{n+1} \\
&\quad + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\theta(x_{n+2}, x_{n+3}, x_{n+p})\, d_{n+2} \\
&\quad + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\theta(x_{n+2}, x_{n+3}, x_{n+p})\, d(x_{n+3}, x_{n+p}) \\
&\leq \cdots \\
&\leq \sum_{i=n}^{n+p-2}\left(\prod_{j=n}^{i}\theta(x_j, x_{j+1}, x_{n+p})\right)d_i + \left(\prod_{j=n}^{n+p-2}\theta(x_j, x_{j+1}, x_{n+p})\right)d_{n+p-1} \\
&\leq \sum_{i=n}^{n+p-2}\left(\prod_{j=n}^{i}\theta(x_j, x_{j+1}, x_{n+p})\right)d_i + \left(\prod_{j=n}^{n+p-2}\theta(x_j, x_{j+1}, x_{n+p})\right)\theta(x_{n+p-2}, x_{n+p-1}, x_{n+p})\, d_{n+p-1} \\
&\leq \beta\sqrt{\lambda^{n-2}}M + \beta^2\sqrt{\lambda^{n-1}}M + \cdots + \beta^p\sqrt{\lambda^{n+p-3}}M = \beta\sqrt{\lambda^{n-2}}M\Big[1 + \beta\sqrt{\lambda} + \cdots + \beta^{p-1}\sqrt{\lambda^{p-1}}\Big] \\
&= \beta\sqrt{\lambda^{n-2}}M\,\frac{1-(\beta\sqrt{\lambda})^p}{1-\beta\sqrt{\lambda}} \leq \beta\sqrt{\lambda^{n-2}}M\,\frac{1}{1-\beta\sqrt{\lambda}} \to 0, \quad \text{when } n \to \infty.
\end{aligned}$$

This completes the proof. □

Lemma 3.8. *Let (X, d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous mapping. Assuming that $\theta(\cdot,\cdot,\cdot)$ is bounded, if the Picard iterations sequence based on an initial point $x_0 \in X$ is a Cauchy sequence, then T has a fixed point.*

Proof. If $\{x_n\}_n$ is a Cauchy sequence, by completeness of (X, d), $\{x_n\}_n$ converges to some $u \in X$, that is,

$$\lim_{n\to\infty} d(x_n, u) = 0.$$

Since T is orbitally continuous, the fact that $\lim_{n\to\infty} x_n = u$ implies that $\lim_{n\to\infty} Tx_n = Tu$, then $\lim_{n\to\infty} x_{n+1} = Tu$, thus $\lim_{n\to\infty} x_n = Tu$. By Lemma 3.6, the sequence $\{x_n\}_n$ has a unique limit, therefore $Tu = u$. □

Henceforth, we present fixed point results for the previously introduced contractive mappings.

Theorem 3.9. *Let (X, d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of order 2. If there exist $x_0 \in X$, $\beta < \frac{1}{\sqrt{a+b}}$, and $n_0 \in \mathbb{N}$ such that the inequality*

$$\prod_{i=n}^{j}\theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$ and $j \in \{n, n+1, ..., n+p-2\}$, then T has a unique fixed point.

Proof. Let $x_0 \in X$. Note that if there exists $n_0 \in \mathbb{N}$ so that $x_{n_0} = x_{n_0+1}$, then $d(x_{n_0}, x_{n_0+1}) = 0$, thus x_{n_0} is a fixed point of T, therefore we can assume, without loss of generality, that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$.

Because of Lemma 3.1, it follows that

$$d(x_n, x_{n+1}) \leq \sqrt{(a+b)^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\},$$

for any $n \in \mathbb{N}$.

Set $\lambda = a + b$. By using Lemma 3.7 , we conclude that the sequence $\{x_n\}_n$ is Cauchy. Now, employing Lemma 3.8, we get that T has a fixed point $u \in X$.

Next, assume that there exists $v \in X$ such that $v \neq u$ and $Tv = v$. Then,

$$\begin{aligned} d(u, v) &= d(T^2u, T^2v) \leq ad(Tu, Tv) + bd(u, v) = ad(u, v) + bd(u, v) \\ &= (a+b)d(u, v) < d(u, v), \end{aligned}$$

which is a contradiction.

Consequently, u is the only fixed point of T. □

Onwards, we present an application to support our results.

Example 3.10. *Let (X, d) be defined as in Example 2.6 [15], with $l = 1.3$. Define*

$$T\colon X \to X, \quad T(x) = \begin{cases} \frac{1}{2}, & \text{if } x \text{ is rational}, \\ 2, & \text{if } x \text{ is irrational}. \end{cases}$$

Note that for any $x, y \in X$, we have that $d(T^2x, T^2y) = 0$, thus T is an Istrăţescu contraction of order 2 for $a = b = \frac{1}{4}$. Set $\lambda = \frac{1}{2}$, and $\beta = \frac{6}{5}$. Note that $\beta < \frac{1}{\sqrt{a+b}} = \sqrt{2}$.

To check that there exists $n_0 \in \mathbb{N}$ such that

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, ..., n+p-2\}$, it is enough to show that there exists $n_0 \in \mathbb{N}$ so that $\theta(x_j, x_{j+1}, x_{n+p}) \leq \beta$ holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, ..., n+p-2\}$. If we set $n_0 = 1$, note that the desired inequality is always valid.

Therefore, the mapping T satisfies all the conditions from Theorem 3.9. Thus, by employing any of the previous mentioned results, we get that T has a unique fixed point, which is $u = \frac{1}{2}$.

Theorem 3.11. *Let (X, d) be a complete new extended b-metric space, and let $T\colon X \to X$ be an orbitally continuous two-sided contraction. Presume there exist $x_0 \in X$, $\beta < \sqrt{\frac{1-b_2}{a_1+a_2+b_1}}$, and $n_0 \in \mathbb{N}$ such that the inequality*

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, ..., n+p-2\}$. Then, T has a unique fixed point.

Proof. Let $x_0 \in X$. Notice that if there exists $n_0 \in \mathbb{N}$ such that $x_{n_0} = x_{n_0+1}$, then x_{n_0} is a fixed point of T, thus we assume, without loss of generality, that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$.

According to Lemma 3.2, we have that

$$d(x_n, x_{n+1}) \leq \sqrt{\left(\frac{a_1 + a_2 + b_1}{1 - b_2}\right)^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}$$

for all $n \in \mathbb{N}$.

By using Lemma 3.7 with $\lambda = \frac{a_1+a_2+b_1}{1-b_2}$, we get that $\{x_n\}_n$ is a Cauchy sequence. Also, Lemma 3.8 provides that T has a fixed point $u \in X$.

Assume that there exists $v \in X$ such that $Tv = v$. Then,

$$\begin{aligned} d(u,v) &= d(T^2u, T^2v) \\ &\leq a_1 d(u,Tu) + a_2 d(Tu, T^2u) + b_1 d(v,Tv) + b_2 d(Tv, T^2v) = 0, \end{aligned}$$

then $d(u,v) = 0$, thus $u = v$.

Consequently, u is the only fixed point of T. □

The next result represents a generalization of Theorems 3.9 and 3.11.

Theorem 3.12. *Let (X,d) be a complete new extended b-metric space, and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of type 2. Suppose that there exist $x_0 \in X$, $\beta < \sqrt{\frac{1-b_2}{c_0+c_1+a_1+a_2+b_1}}$, and $n_0 \in \mathbb{N}$ such that the inequality*

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$ and $j \in \{n, n+1, ..., n+p-2\}$. Then, T has a unique fixed point.

Proof. Let $x_0 \in X$. Remark that if there exists $n_0 \in \mathbb{N}$ so that $x_{n_0} = x_{n_0+1}$, then it implies the fact that x_{n_0} is a fixed point of T, therefore we can assume, without loss of generality, that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$.

Due to Lemma 3.3, it is true that

$$d(x_n, x_{n+1}) \leq \sqrt{\left(\frac{c_0 + c_1 + a_1 + a_2 + b_1}{1 - b_2}\right)^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}$$

for any $n \in \mathbb{N}$.

To prove that T admits a fixed point, by using Lemma 3.7 with $\lambda = \frac{c_0+c_1+a_1+a_2+b_1}{1-b_2}$, we conclude that the sequence $\{x_n\}_n$ is Cauchy. Also, by making use of Lemma 3.8, we get that T has a fixed point $u \in X$.

Now, assume that there exists $v \in X$ such that $v \neq u$ and $Tv = v$. Then,

$$\begin{aligned} d(u,v) &= d(T^2u, T^2v) \\ &\leq c_0 d(u,v) + c_1 d(Tu,Tv) + a_1 d(u,Tu) \\ &\quad + a_2 d(Tu, T^2u) + b_1 d(v,Tv) + b_2 d(Tv, T^2v) \\ &= (c_0 + c_1) d(u,v) < d(u,v), \end{aligned}$$

which is a contradiction.

Consequently, u is the only fixed point of T. □

Theorem 3.13. *Let (X,d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous Ćirić-convex contraction. Suppose there exist $x_0 \in X$, $\beta < \frac{1}{\sqrt{h}}$, and $n_0 \in \mathbb{N}$ such that the inequality*

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$ and $j \in \{n, n+1, ..., n+p-2\}$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$. If there exists $n_0 \in \mathbb{N}$ so that $x_{n_0} = x_{n_0+1}$, then x_{n_0} is a fixed point of T.

Without loss of generality, we may assume that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$.

Due to Lemma 3.4, we obtain that

$$d(x_n, x_{n+1}) \leq \sqrt{h^{n-2}} \max\{d(x_0, x_1), d(x_1, x_2)\}$$

for all $n \in \mathbb{N}$.

Now, by using Lemma 3.7, $\{x_n\}_n$ is a Cauchy sequence. Now, employing Lemma 3.8, T has a fixed point $u \in X$.

Assume that there exists $v \in X$ such that $v \neq u$ and $Tv = v$. Then,

$$\begin{aligned} d(u, v) &= d(T^2u, T^2v) \\ &\leq h \max\{d(u, v), d(Tu, Tv), d(u, Tu), d(Tu, T^2u), \\ &\qquad d(v, Tv), d(Tv, T^2v)\} \\ &= hd(u, v) < d(u, v), \end{aligned}$$

a contradiction.

Consequently, u is the only fixed point of T. □

To substantiate the following theorem, it is necessary to initially establish the subsequent result.

Lemma 3.14. *Let (X, d) be a new extended b-metric space, $T: X \to X$ be an Istrăţescu contraction of order k and a point $x_0 \in X$. If there exist $\lambda \in (0, 1)$ and an integer $k \geq 2$ such that*

$$d(x_n, x_{n+1}) \leq \sqrt[k]{\lambda^{n-k}} \max\{d(x_0, x_1), ..., d(x_{k-1}, x_k)\}$$

for all $n \in \mathbb{N}$, and there exist $\beta < \frac{1}{\sqrt[k]{\lambda}}$, and $n_0 \in \mathbb{N}$ such that the inequality

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$ and $j \in \{n, n+1, ..., n+p-2\}$, then $\{x_n\}_n$ is a Cauchy sequence.

Proof. Set $d_n = d(x_n, x_{n+1})$ for all $n \geq 0$ and $M = \max\{d_0, d_1, ..., d_{k-1}\}$.

Consider $n, p, j \in \mathbb{N}$ with $n \geq n_0$, $p \geq 1$ and $j \in \{n, n+1, ..., n+p-2\}$.

Obviously, $\beta \sqrt[k]{\lambda} < 1$.

The following estimations will justify that $\{x_n\}_n$ is a Cauchy sequence:

If $p = 1$, then

$$d(x_n, x_{n+1}) \leq \sqrt[k]{(a_0 + ... + a_{k-1})^{n-k}} M \to 0, \text{ when } n \to \infty.$$

For $p \geq 2$, we get that

$$\begin{aligned}
d(x_n, x_{n+p}) &\leq \theta(x_n, x_{n+1}, x_{n+p})\Big[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+p})\Big] \\
&\leq \theta(x_n, x_{n+1}, x_{n+p})\, d_n + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\, d(x_{n+1}, x_{n+2}) \\
&\quad + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\, d(x_{n+2}, x_{n+p}) \\
&\leq \theta(x_n, x_{n+1}, x_{n+p})\, d_n + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\, d_{n+1} \\
&\quad + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\theta(x_{n+2}, x_{n+3}, x_{n+p})\, d_{n+2} \\
&\quad + \theta(x_n, x_{n+1}, x_{n+p})\theta(x_{n+1}, x_{n+2}, x_{n+p})\theta(x_{n+2}, x_{n+3}, x_{n+p})\, d(x_{n+3}, x_{n+p}) \\
&\leq \cdots \\
&\leq \sum_{i=n}^{n+p-2}\left(\prod_{j=n}^{i}\theta(x_j, x_{j+1}, x_{n+p})\right)d_i + \left(\prod_{j=n}^{n+p-2}\theta(x_j, x_{j+1}, x_{n+p})\right)d_{n+p-1} \\
&\leq \sum_{i=n}^{n+p-2}\left(\prod_{j=n}^{i}\theta(x_j, x_{j+1}, x_{n+p})\right)d_i + \left(\prod_{j=n}^{n+p-2}\theta(x_j, x_{j+1}, x_{n+p})\right)\theta(x_{n+p-2}, x_{n+p-1}, x_{n+p})\, d_{n+p-1} \\
&\leq \beta\sqrt[k]{\lambda^{n-k}}M + \beta^2\sqrt[k]{\lambda^{n-k+1}}M + \cdots + \beta^p\sqrt[k]{\lambda^{n-k+p-1}}M = \beta\sqrt[k]{\lambda^{n-k}}M\Big[1 + \beta\sqrt[k]{\lambda} + \cdots + \beta^{p-1}\sqrt[k]{\lambda^{p-1}}\Big] \\
&= \beta\sqrt[k]{\lambda^{n-k}}M\,\frac{1-(\beta\sqrt[k]{\lambda})^p}{1-\beta\sqrt[k]{\lambda}} \leq \beta\sqrt[k]{\lambda^{n-k}}M\,\frac{1}{1-\beta\sqrt[k]{\lambda}} \to 0, \quad \text{when } n \to \infty.
\end{aligned}$$

This completes the proof. □

The following result is an extension of Theorem 3.9.

Theorem 3.15. *Let (X, d) be a complete new extended b-metric space, and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of order $k \geq 2$. If there exist $x_0 \in X$, $\beta < \frac{1}{\sqrt[k]{a_0+a_1+\ldots+a_{k-1}}}$, and $n_0 \in \mathbb{N}$ such that the inequality*

$$\prod_{i=n}^{j}\theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, ..., n+p-2\}$, then T has a unique fixed point.

Proof. Let $x_0 \in X$. If there exists $n_0 \in \mathbb{N}$ so that $x_{n_0} = x_{n_0+1}$, then x_{n_0} is a fixed point of T.

Without loss of generality, we may assume that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$.

Due to Lemma 3.5, we obtain that

$$d(x_n, x_{n+1}) \leq \sqrt[k]{(a_0 + ... + a_{k-1})^{n-k}}\max\{d(x_0, x_1), ..., d(x_{k-1}, x_k)\}$$

for all $n \in \mathbb{N}$.

By using Lemma 3.14 for $\lambda = a_0 + a_1 + ... + a_{k-1}$, we obtain that $\{x_n\}_n$ is a Cauchy sequence. Now, employing Lemma 3.8, T has a fixed point $u \in X$.

Assume that there exists $v \in X$ such that $v \neq u$ and $Tv = v$. Then,

$$\begin{aligned}
d(u, v) &= d(T^k u, T^k v) \\
&\leq a_0\, d(u, v) + a_1\, d(Tu, Tv) + \ldots + a_{k-1}\, d(T^{k-1}u, T^{k-1}v) \\
&= a_0\, d(u, v) + a_1\, d(u, v) + \ldots + a_{k-1}\, d(u, v) \\
&= (a_0 + a_1 + \ldots + a_{k-1})d(u, v) < d(u, v),
\end{aligned}$$

a contradiction.

Consequently, u is the only fixed point of T. □

By setting $b = 0$ in Theorem 3.9 or $c_0 = a_1 = a_2 = b_1 = b_2 = 0$ in Theorem 3.12, we obtain the following extension of the contraction principle in the framework of new extended b-metric spaces.

Corollary 3.16. *Let (X, d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous mapping. If there exists $k \in (0, 1)$ such that the inequality*

$$d(T^2x, T^2y) \leq kd(Tx, Ty)$$

holds for any x, $y \in X$, and if there exist $x_0 \in X$, $\beta < \frac{1}{\sqrt{k}}$, and $n_0 \in \mathbb{N}$ such that the inequality

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, \dots, n+p-2\}$, then T has a unique fixed point.

If we pick $a = 0$ in Theorem 3.9 or $c_1 = a_1 = a_2 = b_1 = b_2 = 0$ in Theorem 3.12, we get the following result.

Corollary 3.17. *Let (X, d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous mapping. If there exists $k \in (0, 1)$ such that the inequality*

$$d(T^2x, T^2y) \leq kd(x, y)$$

holds for any x, $y \in X$, and suppose there exist $x_0 \in X$, $\beta < \frac{1}{\sqrt{k}}$, and $n_0 \in \mathbb{N}$ such that the inequality

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, \dots, n+p-2\}$. Then, T has a fixed point, which is unique.

By choosing $a_1 = b_1 = 0$ and $a_2 = b_2$ in Theorem 3.11 or $c_0 = c_1 = a_1 = b_1$ and $a_2 = b_2$ in Theorem 3.12, we obtain a power contraction version of the Kannan fixed point theorem in the setting of new extended b-metric space.

Corollary 3.18. *Let (X, d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous mapping. If there exists $k \in (0, \frac{1}{2})$ such that the inequality*

$$d(T^2x, T^2y) \leq k[d(Tx, T^2x) + d(Ty, T^2y)]$$

holds for any x, $y \in X$, and if there exist $x_0 \in X$, $\beta < \sqrt{\frac{1-k}{k}}$, and $n_0 \in \mathbb{N}$ such that the inequality

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, \dots, n+p-2\}$, then T has a unique fixed point, provided that $\{\alpha(v, x_n)\}_n$ and $\{\alpha(x_n, v)\}_n$ are bounded sequences for all $v \in X$.

By taking $c_0 = a_1 = b_1 = 0$ and $a_2 = b_2$ in Theorem 3.12, we get the following power contraction version of the Reich fixed point theorem for new extended b-metric spaces.

Corollary 3.19. *Let (X, d) be a complete new extended b-metric space and let $T\colon X \to X$ be an orbitally continuous mapping. Suppose there exist a, $b \in [0, 1)$ with $0 < a + 2b < 1$ such that the inequality*

$$d(T^2x, T^2y) \leq ad(Tx, Ty) + b[d(Tx, T^2x) + d(Ty, T^2y)]$$

holds for any $x, y \in X$. Also, presume there exist $x_0 \in X$, $\beta < \sqrt{\frac{1-b}{a+b}}$, and $n_0 \in \mathbb{N}$ such that the inequality

$$\prod_{i=n}^{j} \theta(x_i, x_{i+1}, x_{n+p}) \leq \beta^{j-n+1}$$

holds for all $n \geq n_0$, $p \geq 2$, and $j \in \{n, n+1, \ldots, n+p-2\}$. Then, T has a unique fixed point.

When $\theta(x, y, z) = \theta(x, z)$, we obtain the main results from [14].

By taking $\theta(\cdot,\cdot,\cdot) \equiv s \geq 1$, we obtain the following analogous results for Theorems 3.9, 3.11, 3.12, 3.13 and 3.15 in the setting of b-metric spaces.

Corollary 3.20. *Let (X, d) be a complete b-metric space and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of order 2. Suppose that the following inequality holds:*

$$s < \frac{1}{\sqrt{a+b}}.$$

Then T has a unique fixed point.

Corollary 3.21. *Let (X, d) be a complete b-metric space and let $T\colon X \to X$ be an orbitally continuous two-sided convex contraction. If the inequality*

$$s < \sqrt{\frac{1-b_2}{a_1+a_2+b_1}}$$

holds, then T has a fixed point which is unique.

Corollary 3.22. *Let (X, d) be a complete b-metric space and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of type 2. Suppose that the following inequality is true:*

$$s < \sqrt{\frac{1-b_2}{c_0+c_1+a_1+a_2+b_1}}.$$

Then, T has a unique fixed point.

Corollary 3.23. *Let (X, d) be a complete b-metric space and let $T\colon X \to X$ be an orbitally continuous Ćirić-convex contraction. Presume that the inequality*

$$s < \frac{1}{\sqrt{h}}$$

is satisfied. Then, T has a unique fixed point.

Corollary 3.24. *Let (X, d) be a complete extended b-metric space and let k be an integer such that $k \geq 2$, and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of order k. Suppose that the inequality*

$$s < \frac{1}{\sqrt[k]{a_0+a_1+\ldots+a_{k-1}}}$$

is valid. Then, T has a unique fixed point.

By taking $s = 1$, we obtain the main results from [11] and the analogue of Theorem 3.13 in the setting of metric spaces, stated next.

Corollary 3.25. *Let (X, d) be a complete metric space and let $T\colon X \to X$ be a Ćirić-convex contraction. If T is orbitally continuous, then T has a unique fixed point.*

4. Conclusions and further development

In this paper, fixed point theory has been generalized to the class of new extended b-metric spaces by analyzing various types of contractions in the sense of Istrăţescu. We proved the existence and uniqueness of fixed points under certain conditions, broadening the scope of traditional metric space results.

Future research could focus on extending these results to more generalized types of metric spaces or exploring diverse classes of contractions. Additionally, examining the robustness of the fixed point theorems under varying conditions or relaxing some of the assumptions made in this study could offer new insights and potential applications.

Conflicts of interest

The author declares that he has no competing interest.

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