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An efficient estimator using median ranked set sampling for the finite population mean in case of nonresponse

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Abstract. The classical estimator for the estimation of the finite population mean in the presence of non-response is the Hansen-Hurwitz estimator. This study first examines the use of ranked set sampling in the Hansen-Hurwitz estimator for both response and non-response groups. Subsequently, a new estimator is proposed by employing median ranked set sampling for the same estimator. The sample selection is performed using the median ranked set sampling method for both response and non-response groups. A simulation study is conducted to investigate the efficiency of the estimators under different distributions, sample sizes, and subsample proportions, considering cases with perfect ranking and imperfect ranking. The obtained results are compared with various estimators available in the literature. Under unimodal symmetric distributions such as, Laplace and Normal distributions, the estimator based on the median ranked set sampling yields more efficient results, whereas under Uniform and Exponential distributions, the estimator based on the ranked set sampling is found to be more efficient. Moreover, the efficiency of the proposed estimator has been evaluated using real-life data. The proposed estimator has been found to produce more efficient results compared to other estimators in the presence of non-response.

1. Introduction

In scientific research, obtaining accurate and reliable results requires appropriate data structures and sampling techniques. However, non-response is frequently encountered during the implementation of field research and large-scale surveys. Non-response is defined as the failure to collect data from selected sample units. This situation arises for various reasons, such as the inaccessibility of sample units, reluctance to respond, and limitations in resources, time, and personnel. Non-response is considered one of the major problems in sampling theory, as it complicates the accurate and reliable estimation of population parameters and hinders the proper representation of the population. Researchers aim to minimize non-response by increasing the number of respondents or by developing better estimators through different methods. Hansen and Hurwitz [13] introduced the subsampling method to increase the number of respondents. In this approach, a subsample is selected from the non-response units to reduce the impact of non-response Hansen and Hurwitz [13]. Simple random sampling (SRS) is one of the commonly used sampling methods, even

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in the presence of non-response. However, when non-response occurs within a highly diverse population, SRS negatively affects the population parameter estimates Rehman and Shabbir [19]. For studies addressing non-response under the SRS method, the works of Metin and Özdemir [15], Shahzad et al. [20], Yaqub and Shabbir [24], Unal and Kadılar [21], and Ahmad et al. [1] can be examined. Ranked set sampling (RSS) was introduced into the literature by McIntyre [14] as an alternative to the SRS method due to its higher efficiency in estimating the population mean. This method consists of two steps. In the first step, m sample units of size m are selected from the population using SRS, where each selected sample represents a separate set. Each set is then ranked in ascending order based on a low-precision measurement of the variable of interest. In the second step, the first unit is selected from the first set, the second unit from the second set, and the m-th unit from the m-th set. These selected units are then measured with the desired level of precision, forming a ranked set sample. In recent years, various studies have been conducted in the field of RSS. For example, Dong et al. [10] proposed a nonparametric estimator based on RSS for a two-component parallel system, while Yanglem and Khongji [23] explored the use of RSS in the second stage of a two-stage sampling design to improve regression estimators. Çetin and Koyuncu [8] , proposed a new calibration estimator for the estimation of the population mean under stratified extreme RSS. Ebegil et al. [11] proposed some shrinkage estimators using a median ranked set sample in the presence of multicollinearity. Çetin and Koyuncu [9] proposed a new robust estimator class under SRS, RSS, and median ranked set sampling (MRSS) designs. RSS has also established a significant place in the literature for addressing non-response situations. Bouza [5] proposed a new estimator for the difference in means by selecting a subsample using the RSS method under non-response. The proposed estimator was found to be more efficient than the one based on the SRS method. Bouza [6] estimated the population mean by employing RSS, Extreme RSS (ERSS), and MRSS for subsample selection under non-response. Simulation results indicated that the confidence interval values were most favorable when using RSS. Bouza-Herrera [7] conducted a comprehensive analysis of missing data and non-response using the RSS method. Additionally, the study examined the methods used to address non-response and evaluated the findings of studies conducted using the RSS approach. Al-Omari and Bouza [4] utilized the RSS method in ratio estimators to estimate the population mean in the presence of missing observations, achieving more efficient estimates. Ahmed and Shabbir [2] proposed estimators under RSS, ERSS, and MRSS methods for non-response in randomized response models. These proposed estimators were found to be more efficient than the classical estimators available in the literature. Rather et al. [18] applied the RSS method to the estimator proposed by Ünal and Kadılar [21] under non-response to estimate the population mean, obtaining highly efficient estimates. Fatima et al. [12] proposed a new regression-cum-ratio estimator using RSS in the presence of non-response. The estimator uses information from two auxiliary variables. The simulation study demonstrated that the proposed estimator provides more efficient estimates compared to Mohanty's estimator. Rehman and Shabbir [19] introduced Generalized Rao-Regression type estimators using the RSS method in the presence of non-response. The proposed estimators were examined under two situations. In the first situation, subsample selection using RSS was performed exclusively on the non-response group. In the second situation, sample selection was conducted using RSS from both the response and non-response groups. As a result, under the RSS method, a class of Generalized Rao-Regression type estimators was proposed to estimate of the population means in two different situations, and it was stated that more efficient estimators were obtained compared to the SRS method. Akın Vargeloğlu and Özdemir [22] adapted the RSS and MRSS methods to regression estimators in the presence of non-response. The findings from the simulation studies and real data applications demonstrated that the adapted regression estimator based on two auxiliary variables under the MRSS method exhibited higher efficiency compared to the ratio and single auxiliary variable regression estimators. Alamri and Aloraini [3] proposed a new median-based estimator using the MRSS to address the problem of non-response by incorporating information from two auxiliary variables. The findings from real data applications revealed that the proposed two auxiliary variable regression type estimator under the MRSS achieved higher efficiency than the conventional ratio, regression, and product type estimators.

In this study, inspired by the works of Bouza [6] and Rehman and Shabbir [19], a new estimator based on the MRSS method is proposed for estimating the population mean in the presence of non-response. Sample selection was performed from both the response and non-response groups using the MRSS method and adapted to the Hansen-Hurwitz estimator. Furthermore, no study in the literature has been found that evaluates the performance of the Hansen-Hurwitz estimator when adapted using the RSS method for both groups. This study investigates the efficiency of estimators obtained through sample selection from non-response groups and both response and non-response groups in estimating the population mean under different distributions. A simulation study was conducted to obtain the Mean Squared Error (MSE) and Relative Efficiency (RE) values of the estimators. In the second section of the study, SRS, RSS, and MRSS methods are theoretically discussed under non-response. The third section provides a detailed explanation of the simulation study. The fourth section analyzes the real dataset, while the final section presents the results and recommendations.

2. Methodology

Let $U = (u_1, u_2, u_3, \dots, u_N)$ denote a finite population of size N. A sample of size n is selected from this population using the SRS method. Among the selected n units, responses are obtained from only n_1 units in the first visit, while non-response is received from the remaining n_2 units. The population is divided into two groups: the response group (N_1), and the non-response group (N_2) where $N = N_1 + N_2$. The proportion of respondents in the population is defined as $W_1 = N_1/N$, while the proportion of non-respondents is given by $W_2 = N_2/N$. The idea of dividing the population into two groups was first introduced by Hansen and Hurwitz [13]. Non-response complicates the accurate and reliable estimation of population parameters. Therefore, the sub-sampling method was introduced in the literature Hansen and Hurwitz [13]. In this method, a sub-sample of size n_2' is re-visited from the non-response sample units, where $n_2' = n_2/k$, $k > 1$, and k is defined as the sub-sampling ratio. With the sub-sampling selection method, the new sample size is given by $n = n_1 + n_2'$. Let (Y_{ji}, X_{ji}); $j = 1, 2$, $i = 1, 2, \dots, N_j$ be the study variable and auxiliary variable for the response and non-response groups, respectively. $\bar{Y}$ is defined as the population mean to be estimated, $\bar{Y}_1$ as the population mean of the response group, and $\bar{Y}_2$ as the population mean of the non-response group. Under non-response, the unbiased estimator proposed by Hansen and Hurwitz [13] for estimating the population mean is given in equation (1).

$$\bar{\bar{y}}_{\text{SRS}} = w_1\bar{y}_1 + w_2\bar{y}_2' \tag{1}$$

Here, $w_1 = \frac{n_1}{n}$ and $w_2 = \frac{n_2}{n}$ are defined as the sampling proportions of the response and non-response units, respectively. The sample mean of the response units is $\bar{y}_1 = \frac{\sum_{i=1}^{n_1} y_{1i}}{n_1}$, while the sample mean computed from the non-response units is $\bar{y}_2' = \frac{\sum_{i=1}^{n_2'} y_{2i}}{n_2'}$. The estimator $\bar{\bar{y}}_{\text{SRS}}$ is an unbiased estimator as proposed by Hansen and Hurwitz [13]. The variance of this estimator is expressed in equation (2).

$$V(\bar{\bar{y}}_{\text{SRS}}) = \frac{\sigma_y^2}{n} + W_2\frac{(k-1)\sigma_{2y}^2}{n} \tag{2}$$

Here,

$$\sigma_{2y}^2 = \frac{1}{N_2 - 1}\sum_{i=1}^{N_2}(Y_{2i} - \bar{Y}_2)^2 \quad \text{and} \quad \sigma_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2 \tag{3}$$

2.1. Ranked Set Sampling Under Non-response

The selection of a subsample using the RSS method under non-response was addressed by Bouza [6]. In Bouza's [6] study, a subsample was selected using the RSS method solely from the non-response group to estimate the population mean. Under the RSS framework, the ranking of sampling units can be performed either through visual judgment based on the researcher's expertise or by using an auxiliary variable X that is strongly related to the study variable Y and is easier or less costly to measure.In the present study, the ranking of sampling units is carried out based on the auxiliary variable X. Under non-response, sample selection based on the RSS method is examined under two different situations. *Situation I* considers the selection of a subsample using the RSS method only from the non-response group, whereas *Situation II* considers the selection of a sample using the RSS method from both the response and non-response groups. The detailed steps of subsample selection based on the RSS method are described below.

Situation I: Sample selection using the RSS method only from the non-response group

Step 1. A sample of size n is selected using SRS. Among these units, n_1 constitute the respondent group, from which information is collected on the study variable Y and the closely related auxiliary variable X. Furthermore, it is assumed that measuring the variable Y is more costly and difficult than measuring X.

Step 2. Among the n_2 units that did not respond during the first visit, sample units are selected using RSS and revisited. For this selection, m sets, each consisting of m units, are first obtained.

Step 3. In each set, the m sample units are ranked from smallest to largest according to the auxiliary variable X.

Step 4. The first unit from the first set, the second unit from the second set, and the mth unit from the mth set are selected and measured with respect to the variable Y.

By repeating the selection of subsamples from non-response sample units via the RSS method r'_2 times, a ranked set sample of size $n'_2 = r'_2 m$ is obtained, , denoted by $y_{2[i]j};\quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, r'_2$. The estimator obtained under RSS is defined as follows:

$$\bar{\bar{y}}_{\mathrm{RSS}(1)} = w_1 \bar{y}_1 + w_2 \bar{y}'_{2(RSS)} \tag{4}$$

where the mean of the subsample selected from the non-response units is given by:

$$\bar{y}'_{2(RSS)} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{r'_2} y_{2[i]j}}{n'_2} \tag{5}$$

The estimator $\bar{\bar{y}}_{\mathrm{RSS}(1)}$ is an unbiased estimator Bouza-Herrera [7]. The variance of this estimator is expressed as:

$$V(\bar{\bar{y}}_{\mathrm{RSS}(1)}) = \frac{\sigma_y^2}{n} + \frac{W_2(k-1)}{n}\sigma_{2y}^2 - \frac{W_2 k}{n}\left(\frac{1}{m}\sum_{i=1}^{m} \Delta_{2y[i]}^2\right) \tag{6}$$

where

$$\Delta_{2y[i]}^2 = (\mu_{2y[i]} - \bar{Y}_2)^2 \tag{7}$$

Here, $\mu_{2y[i]}$ is the mean of the i-th judgement order statistic in the non-response group, while $\bar{Y}_2$ is defined as the population mean of the non-response group.

Situation-II: Sample selection using the RSS method from both the response and non-response groups

This method differs from Situation-I in that it involves selecting samples using the RSS method from both the response and non-response groups.

Step 1. A sample of size n_1 is selected from the response group using the RSS method, and this procedure is carried out in a manner similar to the RSS scheme described in Situation-I.

Step 2. The n_2 sample units that did not respond in the first visit are revisited. By following Steps 2, 3, and 4 defined under Situation I, a subsample of size n'_2 is selected using the RSS method. The obtained estimator is given by:

$$\bar{\bar{y}}_{\text{RSS}(2)} = w_1 \bar{y}_{1(RSS)} + w_2 \bar{y}'_{2(RSS)} \tag{8}$$

where $\bar{y}_{1(RSS)}$ is the sample mean of the n_1 units selected using the RSS method, and $\bar{y}'_{2(RSS)}$ is the sample mean obtained from the n'_2 units. The estimator $\bar{\bar{y}}_{\text{RSS}(1)}$ is an unbiased estimator. The variance of the estimator is given by:

$$V(\bar{\bar{y}}_{\text{RSS}(2)}) = \frac{\sigma_y^2}{n} - \frac{1}{nm}\sum_{i=1}^{m}\Delta_{y[i]}^2 + \frac{W_2(k-1)}{n}\sigma_{2y}^2 - \frac{W_2(k-1)}{nm}\sum_{i=1}^{m}\Delta_{2y[i]}^2 \tag{9}$$

where

$$\Delta_{y[i]}^2 = (\mu_{y[i]} - \bar{Y})^2 \tag{10}$$

Here, $\mu_{y[i]}$ is the mean of the i-th judgement order statistic, while $\bar{Y}$ is defined as the population mean.

2.2. Median Ranked Set Sampling under Non-Response

The MRSS method was first introduced to the literature for estimating the population mean by Muttlak [16] in 1977. In his study, Muttlak obtained more efficient results using the MRSS method under unimodal symmetric distributions. There is limited research on the application of MRSS under non-response. Some of the important studies in this area have been conducted by Bouza [6] and Ahmed and Shabbir [2]. In these studies, only the case in which subsample selection is carried out from the non-responding group under the MRSS method is considered. In this study, two different situations are considered. Under Situation-I, subsample selection is performed solely from the non-responding group using the MRSS method, whereas under Situation-II, samples are selected from both the responding and non-responding groups using the MRSS method.

Situation I: Sample selection using the MRSS method only from the non-response group

Step 1. A sample of size n is selected using the SRS method. Among these units, n_1 belong to the responding group. For these n_1 units, information on the study variable Y and the auxiliary variable X, which is related to Y, is collected.

Step 2. Among the n_2 units that did not respond during the first visit, sample units are selected using MRSS and revisited. For this selection, m sets, each consisting of m units, are first obtained.

Step 3. In each set, the m sample units are ranked from smallest to largest according to the auxiliary variable X.

Step 4. The sample selection in MRSS varies depending on whether the number of sets, m, is odd or even.

- When m is odd, the unit corresponding to the median value, which is the $((m+1)/2)$-th unit, is selected from each set.
- When m is even, the $(m/2)$-th unit is selected from the first $m/2$ sets, while the $((m/2)+1)$-th unit is selected from the remaining $m/2$ sets.

Depending on whether m is odd or even, the selected units are measured with respect to the study variable Y. Steps 2, 3 and 4 are then iteratively applied r_2' times using the MRSS method, resulting in a sample of size $n_2' = r_2' m$.

The estimator obtained based on the MRSS method is defined as:

$$\bar{\bar{y}}_{\mathrm{MRSS(1)}} = w_1 \bar{y}_1 + w_2 \bar{y}'_{2(MRSS)} \tag{11}$$

where $\bar{y}'_{2(\mathrm{MRSS})}$ denotes the sample mean obtained from the n_2' units selected using the MRSS method. The variance of the estimator $\bar{y}_{MRSS(1)}$ is:

$$V(\bar{\bar{y}}_{\mathrm{MRSS(1)}}) = \begin{cases} \frac{\sigma_y^2}{n} + \frac{W_2(k-1)}{n}\left(\sigma^2_{2y[\frac{m+1}{2}]}\right) & \text{if } m \text{ is odd} \\ \frac{\sigma_y^2}{n} + \frac{W_2(k-1)}{2n}\left(\sigma^2_{2y[\frac{m}{2}]} + \sigma^2_{2y[\frac{m}{2}+1]}\right) & \text{if } m \text{ is even} \end{cases} \tag{12}$$

Here, $\sigma^2_{2y[\frac{m}{2}]}$ and $\sigma^2_{2y[\frac{m}{2}+1]}$ denote the variances of the $(m/2)$-th and $((m/2)+1)$-th judgment order statistics, respectively, for the non-response group. In this study, the sample selection based on the MRSS method for non-response under Situation-I has been extended to Situation-II.

Situation-II: Sample selection using the MRSS method from both the response and non-response groups

Step 1. Using the MRSS method, a sample of size n_1 is selected from the response group following the sampling procedure described under Situation-I.

Step 2. The n_2 sample units that did not respond in the first visit are revisited. By following Steps 2, 3, and 4 defined under Situation I, a subsample of size n_2' is selected using the MRSS method.

The new estimator based on sample selection using the MRSS method from both response and non-response groups is defined as:

$$\bar{\bar{y}}_{\mathrm{MRSS(2)}} = w_1 \bar{y}_{1(MRSS)} + w_2 \bar{y}'_{2(MRSS)} \tag{13}$$

where $\bar{y}_{1(MRSS)}$ is the sample mean of the n_1 units selected using the MRSS method. Additionally, $n_2' = mr_2'$, $n_2 = mr_2$, $n = mr$, and $n_2' = \frac{n_2}{k}$. The estimator $\bar{\bar{y}}_{\mathrm{MRSS(2)}}$ is a biased estimator, and its variance is given as follows:

$$V\left(\bar{\bar{y}}_{\mathrm{MRSS(2)}}\right) = \begin{cases} \dfrac{\sigma^2_{y[\frac{m+1}{2}]}}{n} + \dfrac{W_2(k-1)}{n}\left(\sigma^2_{2y[\frac{m+1}{2}]}\right), & \text{if } m \text{ is odd,} \\ \dfrac{\sigma^2_{y[\frac{m}{2}]} + \sigma^2_{y[\frac{m}{2}+1]}}{2n} + \dfrac{W_2(k-1)}{2n}\left(\sigma^2_{2y[\frac{m}{2}]} + \sigma^2_{2y[\frac{m}{2}+1]}\right), & \text{if } m \text{ is even.} \end{cases} \tag{14}$$

3. Simulation Study

In this section, a simulation study has been conducted to compare the efficiency of the estimator proposed under the MRSS method with those obtained from the SRS and RSS methods. The efficiency of the estimators derived from the SRS, RSS, and MRSS methods under *Situation-I* and *Situation-II* has been examined under the Normal (0,1), Laplace (0,0.5), Uniform (0,1), and Exponential (1) distributions. The study variable, Y_i, has been obtained in relation to the auxiliary variable X_i using the formula $Y_i = \rho_{yx} X_i + e_j \sqrt{1-\rho_{yx}^2}$. Here, e_j is generated from the corresponding distributions, and ρ_{yx} is defined as the correlation coefficient between the variables X and Y. The cases with perfect and imperfect rankings are considered with $\rho_{yx} = 0.50$, $\rho_{yx} = 0.90$, and $\rho_{yx} = 1$; where the perfect ranking condition is represented by $\rho_{yx} = 1$, and the imperfect ranking conditions are represented by $\rho_{yx} = 0.50$ and $\rho_{yx} = 0.90$. A dataset with a sample size of $N = 1000$ has been generated under different distributions. Based on the studies from the literature, the proportion of non-response in the population has been set at $W_2 = 0.40$, meaning the last 40% of the population represents non-response units. Additionally, in this study, the values of $k = 2,4$ and $m = 3,4,6$ are considered. Furthermore, to enable comparisons across different non-response scenarios while maintaining the same sample size, the cycle sizes (r_2) are set as $r_2 : 4,6,8$ for $k = 2$, and $r_2 : 2,3,4$ for $k = 4$. Thus, the efficiency of the estimators has been analyzed for various sample sizes, correlation values, and non-response situations. The MSE and RE values used to evaluate the performance of the estimators were obtained through 10.000 iterations. The estimators obtained under *Situation-I* are defined as $\bar{\bar{y}}_{\text{SRS}}$, $\bar{\bar{y}}_{\text{RSS(1)}}$, and $\bar{\bar{y}}_{\text{MRSS(1)}}$, while the estimators obtained under *Situation-II* are defined as $\bar{\bar{y}}_{\text{RSS(2)}}$ and $\bar{\bar{y}}_{\text{MRSS(2)}}$.

The formulas used for calculating the MSE and RE values are given in equations 15 and 16.

$$MSE(\bar{\bar{y}}_{*(i)}) = \frac{1}{10000}\sum_{j=1}^{10000}\left(\bar{\bar{y}}_{*(i)} - \mu_{\bar{\bar{y}}_{*(i)}}\right)^2 \tag{15}$$

$$RE(\bar{\bar{y}}_{*(i)}) = \frac{MSE(\bar{\bar{y}}_{\text{SRS}})}{MSE(\bar{\bar{y}}_{*(i)})}, \quad i = 1,2 \tag{16}$$

Here, * represents RSS and MRSS. The results of the simulation study conducted under Normal(0,1), Laplace(0,0.5), Uniform(0,1), and Exponential(1) distributions are presented in Tables 1-8.

Table 1: MSE and RE values for the Laplace (0, 0.5) distribution ($k = 2$)

		$k=2$																	
		MSE									RE								
		$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$			$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$		
m		$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$
	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0069	0.0064	0.0061	0.0072	0.0066	0.0063	0.0072	0.0065	0.0062	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS}(1)}$	0.0065	0.0060	0.0057	0.0062	0.0056	0.0051	0.0060	0.0053	0.0048	1.0573	1.0601	1.0741	1.1615	1.1714	1.2335	1.2004	1.2178	1.2916
3	$\bar{\bar{y}}_{\mathrm{MRSS}(1)}$	0.0064	0.0060	0.0056	0.0056	0.0050	0.0045	0.0054	0.0046	0.0041	1.0789	1.0682	1.0934	1.2799	1.3094	1.3888	1.3332	1.4038	1.5076
	$\bar{\bar{y}}_{\mathrm{RSS}(2)}$	0.0063	0.0058	0.0054	0.0048	0.0044	0.0042	0.0042	0.0039	0.0037	1.0926	1.1059	1.1299	1.5003	1.4862	1.5170	1.7109	1.6734	1.6907
	$\bar{\bar{y}}_{\mathrm{MRSS}(2)}$	0.0055	0.0052	0.0049	0.0034	0.0032	0.0030	0.0026	0.0024	0.0023	1.2440	1.2336	1.2345	2.1260	2.0765	2.0861	2.7749	2.7017	2.6675
	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0065	0.0061	0.0056	0.0068	0.0063	0.0059	0.0067	0.0062	0.0058	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS}(1)}$	0.0061	0.0056	0.0051	0.0055	0.0048	0.0043	0.0051	0.0045	0.0039	1.0594	1.0977	1.1043	1.2420	1.3032	1.3813	1.3207	1.3770	1.4922
4	$\bar{\bar{y}}_{\mathrm{MRSS}(1)}$	0.0060	0.0055	0.0050	0.0050	0.0044	0.0038	0.0046	0.0039	0.0033	1.0798	1.1131	1.1358	1.3553	1.4438	1.5690	1.4684	1.5899	1.7873
	$\bar{\bar{y}}_{\mathrm{RSS}(2)}$	0.0058	0.0053	0.0049	0.0043	0.0039	0.0035	0.0034	0.0032	0.0029	1.1174	1.1524	1.1432	1.5706	1.6075	1.6806	1.9684	1.9631	1.9966
	$\bar{\bar{y}}_{\mathrm{MRSS}(2)}$	0.0054	0.0050	0.0046	0.0030	0.0028	0.0026	0.0021	0.0019	0.0018	1.2000	1.2241	1.2373	2.2618	2.2695	2.2878	3.1713	3.2265	3.1573
	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0061	0.0055	0.0050	0.0063	0.0057	0.0052	0.0062	0.0056	0.0050	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS}(1)}$	0.0054	0.0048	0.0043	0.0044	0.0038	0.0033	0.0040	0.0033	0.0028	1.1336	1.1309	1.1450	1.4269	1.5192	1.5828	1.5444	1.8974	1.7985
6	$\bar{\bar{y}}_{\mathrm{MRSS}(1)}$	0.0057	0.0051	0.0045	0.0041	0.0035	0.0029	0.0035	0.0027	0.0022	1.0728	1.0653	1.0924	1.5238	1.6389	1.7601	1.7876	2.2719	2.2453
	$\bar{\bar{y}}_{\mathrm{RSS}(2)}$	0.0052	0.0047	0.0042	0.0032	0.0029	0.0026	0.0025	0.0022	0.0019	1.1812	1.1748	1.1889	1.9480	1.9764	2.0009	2.5032	2.8450	2.6110
	$\bar{\bar{y}}_{\mathrm{MRSS}(2)}$	0.0050	0.0046	0.0042	0.0026	0.0024	0.0022	0.0016	0.0014	0.0013	1.2103	1.1944	1.1737	2.4044	2.4053	2.3210	3.9879	4.4180	3.8444

Table 2: MSE and RE values for the Laplace (0, 0.5) distribution ($k = 4$)

		$k=4$																
		MSE									RE							
		$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$			$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$		
m		$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$
	$\bar{\bar{y}}_{\text{SRS}}$	0.0096	0.0097	0.0097	0.0101	0.0100	0.0101	0.0101	0.0099	0.0100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\text{RSS(1)}}$	0.0089	0.0089	0.0088	0.0078	0.0077	0.0076	0.0072	0.0071	0.0069	1.0873	1.0885	1.1006	1.2941	1.3081	1.3326	1.3915	1.3957	1.4439
3	$\bar{\bar{y}}_{\text{MRSS(1)}}$	0.0086	0.0088	0.0083	0.0068	0.0065	0.0060	0.0062	0.0057	0.0053	1.1190	1.1063	1.1692	1.4691	1.5302	1.6812	1.6201	1.7439	1.8807
	$\bar{\bar{y}}_{\text{RSS(2)}}$	0.0087	0.0086	0.0089	0.0066	0.0066	0.0067	0.0057	0.0058	0.0058	1.1140	1.1314	1.0897	1.5210	1.5215	1.5006	1.7534	1.7263	1.7313
	$\bar{\bar{y}}_{\text{MRSS(2)}}$	0.0080	0.0080	0.0079	0.0047	0.0047	0.0047	0.0036	0.0036	0.0036	1.2097	1.2228	1.2359	2.1297	2.1154	2.1673	2.8269	2.7671	2.8037
	$\bar{\bar{y}}_{\text{SRS}}$	0.0096	0.0097	0.0092	0.0101	0.0101	0.0095	0.0101	0.0100	0.0094	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\text{RSS(1)}}$	0.0087	0.0085	0.0083	0.0073	0.0069	0.0064	0.0094	0.0061	0.0056	1.1017	1.1408	1.1115	1.3736	1.4619	1.4740	1.0721	1.6338	1.6808
4	$\bar{\bar{y}}_{\text{MRSS(1)}}$	0.0086	0.0084	0.0078	0.0064	0.0059	0.0052	0.0055	0.0048	0.0043	1.1212	1.1594	1.1748	1.5796	1.7230	1.8335	1.8158	2.0612	2.2084
	$\bar{\bar{y}}_{\text{RSS(2)}}$	0.0084	0.0083	0.0081	0.0059	0.0059	0.0057	0.0049	0.0049	0.0047	1.1499	1.1705	1.1323	1.7056	1.7069	1.6565	2.0405	2.0347	1.9913
	$\bar{\bar{y}}_{\text{MRSS(2)}}$	0.0079	0.0079	0.0077	0.0042	0.0043	0.0041	0.0030	0.0029	0.0029	1.2151	1.2329	1.2003	2.3935	2.3501	2.3155	3.3232	3.4520	3.2939
	$\bar{\bar{y}}_{\text{SRS}}$	0.0097	0.0090	0.0085	0.0101	0.0092	0.0087	0.0100	0.0090	0.0086	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\text{RSS(1)}}$	0.0082	0.0078	0.0071	0.0063	0.0056	0.0050	0.0054	0.0046	0.0041	1.1788	1.1586	1.1953	1.6122	1.6552	1.7653	1.8429	1.9472	2.1050
6	$\bar{\bar{y}}_{\text{MRSS(1)}}$	0.0084	0.0079	0.0074	0.0053	0.0046	0.0040	0.0041	0.0033	0.0028	1.1530	1.1454	1.1581	1.9173	2.0042	2.1636	2.4150	2.7148	3.0523
	$\bar{\bar{y}}_{\text{RSS(2)}}$	0.0082	0.0079	0.0072	0.0051	0.0048	0.0044	0.0038	0.0035	0.0033	1.1829	1.1475	1.1856	1.9598	1.9302	1.9790	2.6043	2.5858	2.5960
	$\bar{\bar{y}}_{\text{MRSS(2)}}$	0.0077	0.0074	0.0070	0.0036	0.0035	0.0034	0.0021	0.0020	0.0019	1.2559	1.2252	1.2204	2.7700	2.6347	2.6106	4.6643	4.5010	4.4716

Table 3: MSE and RE values for the Normal (0,1) distribution ($k = 2$)

		$k=2$																	
		MSE									RE								
		$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$			$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$		
m		$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$
	$\bar{\bar{y}}_{SRS}$	0.0131	0.0121	0.0113	0.0139	0.0125	0.0117	0.0141	0.0127	0.0118	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0121	0.0113	0.0104	0.0116	0.0103	0.0091	0.0116	0.0101	0.0088	1.0778	1.0694	1.0946	1.1981	1.2121	1.2787	1.2197	1.2648	1.3388
3	$\bar{\bar{y}}_{MRSS(1)}$	0.0122	0.0114	0.0107	0.0114	0.0099	0.0091	0.0112	0.0095	0.0086	1.0684	1.0668	1.0634	1.2139	1.2632	1.2880	1.2629	1.3434	1.3818
	$\bar{\bar{y}}_{RSS(2)}$	0.0113	0.0106	0.0100	0.0083	0.0076	0.0071	0.0074	0.0068	0.0061	1.1577	1.1438	1.1323	1.6744	1.6512	1.6535	1.9110	1.8812	1.9442
	$\bar{\bar{y}}_{MRSS(2)}$	0.0112	0.0104	0.0097	0.0075	0.0068	0.0065	0.0064	0.0057	0.0055	1.1680	1.1641	1.1648	1.8474	1.8141	1.8089	2.2007	2.2252	2.1475
	$\bar{\bar{y}}_{SRS}$	0.0125	0.0113	0.0105	0.0131	0.0117	0.0109	0.0134	0.0118	0.0111	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0115	0.0105	0.0095	0.0103	0.0087	0.0077	0.0098	0.0081	0.0072	1.0824	1.0841	1.1082	1.2802	1.3473	1.4026	1.3658	1.4567	1.5302
4	$\bar{\bar{y}}_{MRSS(1)}$	0.0114	0.0104	0.0097	0.0100	0.0085	0.0078	0.0095	0.0079	0.0071	1.0927	1.0939	1.0889	1.3164	1.3792	1.3946	1.4224	1.5079	1.5611
	$\bar{\bar{y}}_{RSS(2)}$	0.0108	0.0099	0.0091	0.0070	0.0064	0.0058	0.0058	0.0051	0.0047	1.1555	1.1414	1.1608	1.8648	1.8358	1.8597	2.3318	2.3188	2.3543
	$\bar{\bar{y}}_{MRSS(2)}$	0.0103	0.0097	0.0089	0.0061	0.0058	0.0055	0.0048	0.0045	0.0044	1.2100	1.1662	1.1870	2.1591	2.0326	1.9798	2.7839	2.6300	2.4901
	$\bar{\bar{y}}_{SRS}$	0.0113	0.0102	0.0095	0.0117	0.0104	0.0099	0.0118	0.0106	0.0101	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0103	0.0089	0.0081	0.0081	0.0067	0.0062	0.0075	0.0060	0.0056	1.1055	1.1402	1.1666	1.4383	1.5573	1.6068	1.5831	1.7574	1.8016
6	$\bar{\bar{y}}_{MRSS(1)}$	0.0103	0.0093	0.0086	0.0079	0.0070	0.0069	0.0071	0.0061	0.0061	1.0996	1.0921	1.0981	1.4741	1.4848	1.4495	1.6589	1.7288	1.6533
	$\bar{\bar{y}}_{RSS(2)}$	0.0095	0.0082	0.0075	0.0052	0.0046	0.0046	0.0037	0.0035	0.0037	1.1945	1.2428	1.2666	2.2491	2.2831	2.1620	3.1579	3.0232	2.7336
	$\bar{\bar{y}}_{MRSS(2)}$	0.0094	0.0084	0.0080	0.0048	0.0048	0.0052	0.0033	0.0036	0.0043	1.2117	1.2032	1.1749	2.4417	2.1893	1.8828	3.6190	2.9500	2.3556

Table 4: MSE and RE values for the Laplace (0, 0.5) distribution ($k = 4$)

m		MSE $\rho_{yx}=0.50$ $r_2=2$	$r_2=3$	$r_2=4$	MSE $\rho_{yx}=0.90$ $r_2=2$	$r_2=3$	$r_2=4$	MSE $\rho_{yx}=1.00$ $r_2=2$	$r_2=3$	$r_2=4$	RE $\rho_{yx}=0.50$ $r_2=2$	$r_2=3$	$r_2=4$	RE $\rho_{yx}=0.90$ $r_2=2$	$r_2=3$	$r_2=4$	RE $\rho_{yx}=1.00$ $r_2=2$	$r_2=3$	$r_2=4$
3	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0175	0.0180	0.0179	0.0184	0.0184	0.0184	0.0190	0.0185	0.0184	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS(1)}}$	0.0167	0.0164	0.0157	0.0147	0.0139	0.0129	0.0142	0.0131	0.0121	1.0501	1.0989	1.1417	1.2510	1.3216	1.4295	1.3407	1.4118	1.5215
	$\bar{\bar{y}}_{\mathrm{MRSS(1)}}$	0.0166	0.0163	0.0162	0.0143	0.0133	0.0128	0.0135	0.0122	0.0114	1.0539	1.1020	1.1073	1.2856	1.3851	1.4398	1.4051	1.5183	1.6153
	$\bar{\bar{y}}_{\mathrm{RSS(2)}}$	0.0157	0.0159	0.0155	0.0116	0.0113	0.0108	0.0103	0.0098	0.0094	1.1177	1.1318	1.1567	1.5912	1.6175	1.6960	1.8467	1.8927	1.9599
	$\bar{\bar{y}}_{\mathrm{MRSS(2)}}$	0.0156	0.0158	0.0153	0.0104	0.0103	0.0102	0.0087	0.0086	0.0085	1.1251	1.1391	1.1705	1.7719	1.7808	1.7951	2.1848	2.1466	2.1598
4	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0175	0.0179	0.0173	0.0183	0.0184	0.0179	0.0186	0.0184	0.0181	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS(1)}}$	0.0162	0.0158	0.0151	0.0131	0.0122	0.0114	0.0123	0.0110	0.0101	1.0835	1.1324	1.1460	1.3998	1.5098	1.5724	1.5139	1.6720	1.7970
	$\bar{\bar{y}}_{\mathrm{MRSS(1)}}$	0.0163	0.0159	0.0151	0.0129	0.0117	0.0112	0.0116	0.0102	0.0097	1.0716	1.1267	1.1476	1.4200	1.5675	1.6019	1.6013	1.7977	1.8733
	$\bar{\bar{y}}_{\mathrm{RSS(2)}}$	0.0159	0.0153	0.0147	0.0103	0.0098	0.0094	0.0081	0.0078	0.0077	1.0991	1.1713	1.1793	1.7724	1.8833	1.8960	2.2933	2.3630	2.3373
	$\bar{\bar{y}}_{\mathrm{MRSS(2)}}$	0.0153	0.0146	0.0141	0.0091	0.0088	0.0087	0.0069	0.0069	0.0070	1.1465	1.2266	1.2285	2.0073	2.0834	2.0642	2.7053	2.6732	2.5693
6	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0179	0.0169	0.0159	0.0184	0.0172	0.0165	0.0184	0.0173	0.0166	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS(1)}}$	0.0154	0.0147	0.0136	0.0109	0.0099	0.0092	0.0095	0.0084	0.0077	1.1672	1.1448	1.1668	1.6838	1.7262	1.7816	1.9318	2.0629	2.1428
	$\bar{\bar{y}}_{\mathrm{MRSS(1)}}$	0.0157	0.0149	0.0141	0.0107	0.0101	0.0096	0.0089	0.0082	0.0078	1.1408	1.1326	1.1288	1.7192	1.7074	1.7128	2.0687	2.1168	2.1237
	$\bar{\bar{y}}_{\mathrm{RSS(2)}}$	0.0145	0.0135	0.0130	0.0081	0.0075	0.0075	0.0058	0.0058	0.0058	1.2369	1.2464	1.2157	2.2807	2.2899	2.1862	3.1832	3.0047	2.8451
	$\bar{\bar{y}}_{\mathrm{MRSS(2)}}$	0.0149	0.0140	0.0135	0.0075	0.0077	0.0081	0.0050	0.0052	0.0061	1.2016	1.2075	1.1769	2.4398	2.2239	2.0440	3.7140	3.3198	2.7360

All columns: $k = 4$.

Table 5: MSE and RE values for the Uniform (0,1) distribution ($k = 2$)

		$k = 2$																	
		MSE									RE								
		$\rho_{yx} = 0.50$			$\rho_{yx} = 0.90$			$\rho_{yx} = 1.00$			$\rho_{yx} = 0.50$			$\rho_{yx} = 0.90$			$\rho_{yx} = 1.00$		
m		$r_2 = 4$	$r_2 = 6$	$r_2 = 8$	$r_2 = 4$	$r_2 = 6$	$r_2 = 8$	$r_2 = 4$	$r_2 = 6$	$r_2 = 8$	$r_2 = 4$	$r_2 = 6$	$r_2 = 8$	$r_2 = 4$	$r_2 = 6$	$r_2 = 8$	$r_2 = 4$	$r_2 = 6$	$r_2 = 8$
	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0011	0.0010	0.0010	0.0011	0.0010	0.0010	0.0011	0.0010	0.0010	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS(1)}}$	0.0010	0.0010	0.0009	0.0009	0.0008	0.0008	0.0009	0.0008	0.0007	1.0668	1.0664	1.0684	1.2018	1.2344	1.2806	1.2546	1.3099	1.3737
3	$\bar{\bar{y}}_{\mathrm{MRSS(1)}}$	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0009	0.0008	0.0008	1.0518	1.0339	1.0381	1.1727	1.1769	1.2121	1.1880	1.2203	1.2608
	$\bar{\bar{y}}_{\mathrm{RSS(2)}}$	0.0010	0.0009	0.0008	0.0007	0.0006	0.0006	0.0006	0.0005	0.0005	1.1134	1.1181	1.1469	1.6633	1.6401	1.7041	1.9938	1.9575	2.0023
	$\bar{\bar{y}}_{\mathrm{MRSS(2)}}$	0.0010	0.0009	0.0009	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	1.0945	1.0897	1.0928	1.4805	1.4658	1.4789	1.6482	1.6167	1.6408
	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0010	0.0010	0.0009	0.0011	0.0010	0.0009	0.0011	0.0010	0.0009	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS(1)}}$	0.0010	0.0009	0.0008	0.0008	0.0007	0.0006	0.0008	0.0007	0.0006	1.0750	1.0964	1.1082	1.3073	1.3803	1.4725	1.4023	1.4999	1.6271
4	$\bar{\bar{y}}_{\mathrm{MRSS(1)}}$	0.0010	0.0009	0.0008	0.0008	0.0008	0.0007	0.0008	0.0007	0.0006	1.0608	1.0533	1.0889	1.2657	1.2925	1.3908	1.3287	1.3824	1.4830
	$\bar{\bar{y}}_{\mathrm{RSS(2)}}$	0.0009	0.0008	0.0008	0.0006	0.0005	0.0005	0.0004	0.0004	0.0004	1.1075	1.1660	1.1861	1.8706	1.9001	1.9462	2.4408	2.4021	2.4013
	$\bar{\bar{y}}_{\mathrm{MRSS(2)}}$	0.0009	0.0009	0.0008	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	1.1054	1.1352	1.1366	1.7057	1.7381	1.7372	2.0106	2.0278	1.9956
	$\bar{\bar{y}}_{\mathrm{SRS}}$	0.0009	0.0008	0.0007	0.0006	0.0005	0.0005	0.0006	0.0005	0.0004	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\mathrm{RSS(1)}}$	0.0009	0.0008	0.0007	0.0006	0.0005	0.0005	0.0006	0.0005	0.0004	1.1147	1.1122	1.1493	1.5114	1.6236	1.7527	1.7021	1.9369	2.0927
6	$\bar{\bar{y}}_{\mathrm{MRSS(1)}}$	0.0009	0.0008	0.0007	0.0007	0.0006	0.0005	0.0006	0.0005	0.0004	1.0509	1.0357	1.0581	1.4125	1.5096	1.6221	1.5539	1.7184	1.8291
	$\bar{\bar{y}}_{\mathrm{RSS(2)}}$	0.0008	0.0007	0.0006	0.0004	0.0004	0.0003	0.0003	0.0003	0.0002	1.2042	1.2224	1.2238	2.3569	2.4040	2.4023	3.4480	3.5382	3.4812
	$\bar{\bar{y}}_{\mathrm{MRSS(2)}}$	0.0009	0.0008	0.0007	0.0005	0.0004	0.0004	0.0004	0.0003	0.0003	1.1136	1.0904	1.1093	2.0088	2.0401	2.0410	2.6079	2.6579	2.6223

Table 6: MSE and RE values for the Uniform (0,1) distribution ($k = 4$)

		$k = 4$																	
		MSE									RE								
		$\rho_{yx} = 0.50$			$\rho_{yx} = 0.90$			$\rho_{yx} = 1.00$			$\rho_{yx} = 0.50$			$\rho_{yx} = 0.90$			$\rho_{yx} = 1.00$		
m		$r_2 = 2$	$r_2 = 3$	$r_2 = 4$	$r_2 = 2$	$r_2 = 3$	$r_2 = 4$	$r_2 = 2$	$r_2 = 3$	$r_2 = 4$	$r_2 = 2$	$r_2 = 3$	$r_2 = 4$	$r_2 = 2$	$r_2 = 3$	$r_2 = 4$	$r_2 = 2$	$r_2 = 3$	$r_2 = 4$
	$\bar{\bar{y}}_{\text{SRS}}$	0.0015	0.0015	0.0015	0.0015	0.0016	0.0016	0.0016	0.0016	0.0016	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\text{RSS}(1)}$	0.0014	0.0014	0.0014	0.0012	0.0011	0.0011	0.0011	0.0011	0.0010	1.0659	1.0924	1.1203	1.3235	1.3842	1.4323	1.4638	1.5159	1.5779
3	$\bar{\bar{y}}_{\text{MRSS}(1)}$	0.0014	0.0015	0.0014	0.0012	0.0012	0.0012	0.0012	0.0012	0.0011	1.0587	1.0492	1.1013	1.2588	1.2738	1.3578	1.3289	1.3608	1.4054
	$\bar{\bar{y}}_{\text{RSS}(2)}$	0.0013	0.0013	0.0014	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	1.1248	1.1691	1.1210	1.6988	1.6944	1.6806	2.0570	1.9947	1.9847
	$\bar{\bar{y}}_{\text{MRSS}(2)}$	0.0014	0.0014	0.0014	0.0010	0.0010	0.0010	0.0009	0.0010	0.0010	1.0628	1.1001	1.1116	1.4740	1.4935	1.4977	1.6711	1.6345	1.6214
	$\bar{\bar{y}}_{\text{SRS}}$	0.0015	0.0015	0.0015	0.0016	0.0016	0.0015	0.0016	0.0016	0.0015	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\text{RSS}(1)}$	0.0014	0.0013	0.0013	0.0011	0.0010	0.0009	0.0010	0.0009	0.0008	1.1231	1.1473	1.1326	1.4874	1.5811	1.6261	1.6833	1.8154	1.8982
4	$\bar{\bar{y}}_{\text{MRSS}(1)}$	0.0014	0.0014	0.0013	0.0011	0.0011	0.0010	0.0011	0.0010	0.0009	1.0858	1.0875	1.1115	1.3960	1.4575	1.5225	1.5267	1.6236	1.6790
	$\bar{\bar{y}}_{\text{RSS}(2)}$	0.0013	0.0013	0.0012	0.0008	0.0008	0.0008	0.0006	0.0006	0.0006	1.1666	1.1896	1.1742	1.9367	1.9873	1.9564	2.4951	2.5247	2.4850
	$\bar{\bar{y}}_{\text{MRSS}(2)}$	0.0014	0.0013	0.0013	0.0009	0.0009	0.0009	0.0008	0.0008	0.0007	1.1091	1.1446	1.1223	1.7039	1.7891	1.7387	2.0188	2.1063	2.1154
	$\bar{\bar{y}}_{\text{SRS}}$	0.0015	0.0014	0.0013	0.0016	0.0014	0.0014	0.0016	0.0015	0.0014	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{\text{RSS}(1)}$	0.0013	0.0012	0.0011	0.0009	0.0008	0.0007	0.0008	0.0006	0.0005	1.1683	1.1595	1.1805	1.7546	1.8553	1.9797	2.1127	2.3064	2.5346
6	$\bar{\bar{y}}_{\text{MRSS}(1)}$	0.0014	0.0013	0.0012	0.0010	0.0009	0.0008	0.0009	0.0008	0.0007	1.0976	1.0921	1.0945	1.6265	1.6750	1.7678	1.8416	1.9538	2.0880
	$\bar{\bar{y}}_{\text{RSS}(2)}$	0.0013	0.0012	0.0011	0.0006	0.0006	0.0006	0.0005	0.0004	0.0004	1.2279	1.2159	1.2022	2.4198	2.3539	2.3770	3.5265	3.4463	3.4857
	$\bar{\bar{y}}_{\text{MRSS}(2)}$	0.0013	0.0013	0.0012	0.0007	0.0007	0.0007	0.0006	0.0006	0.0005	1.1533	1.1055	1.1058	2.0892	2.0415	2.0412	2.6873	2.6319	2.6194

Table 7: MSE and RE values for the Exponential (1) distribution ($k=2$)

		$k=2$																	
		MSE									RE								
		$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$			$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$		
m		$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$	$r_2=4$	$r_2=6$	$r_2=8$
	$\bar{\bar{y}}_{SRS}$	0.0139	0.0124	0.0113	0.0143	0.0133	0.0123	0.0143	0.0137	0.0127	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0135	0.0120	0.0111	0.0128	0.0114	0.0104	0.0123	0.0110	0.0101	1.0337	1.0369	1.0238	1.1218	1.1660	1.1824	1.1628	1.2448	1.2573
3	$\bar{\bar{y}}_{MRSS(1)}$	0.0125	0.0123	0.0125	0.0120	0.0128	0.0139	0.0119	0.0125	0.0136	1.1117	1.0100	0.9076	1.1939	1.0422	0.8846	1.2015	1.0943	0.9350
	$\bar{\bar{y}}_{RSS(2)}$	0.0126	0.0112	0.0104	0.0100	0.0091	0.0084	0.0090	0.0083	0.0078	1.1040	1.1119	1.0870	1.4305	1.4594	1.4600	1.5945	1.6446	1.6252
	$\bar{\bar{y}}_{MRSS(2)}$	0.0170	0.0180	0.0189	0.0282	0.0308	0.0327	0.0335	0.0349	0.0358	0.8202	0.6901	0.5976	0.5072	0.4310	0.3763	0.4271	0.3930	0.3546
	$\bar{\bar{y}}_{SRS}$	0.0129	0.0113	0.0105	0.0138	0.0123	0.0114	0.0140	0.0127	0.0119	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0122	0.0108	0.0098	0.0113	0.0098	0.0087	0.0109	0.0095	0.0083	1.0606	1.0508	1.0730	1.2218	1.2513	1.3110	1.2890	1.3392	1.4366
6	$\bar{\bar{y}}_{MRSS(1)}$	0.0121	0.0125	0.0135	0.0120	0.0136	0.0157	0.0119	0.0131	0.0146	1.0727	0.9051	0.7749	1.1499	0.9023	0.7233	1.1749	0.9665	0.8121
	$\bar{\bar{y}}_{RSS(2)}$	0.0114	0.0100	0.0094	0.0083	0.0076	0.0071	0.0071	0.0068	0.0062	1.1379	1.1342	1.1179	1.6590	1.6235	1.6152	1.9585	1.8731	1.9188
	$\bar{\bar{y}}_{MRSS(2)}$	0.0170	0.0190	0.0210	0.0290	0.0323	0.0354	0.0335	0.0352	0.0366	0.7603	0.5959	0.4979	0.4746	0.3805	0.3221	0.4179	0.3599	0.3241
	$\bar{\bar{y}}_{SRS}$	0.0113	0.0100	0.0093	0.0123	0.0108	0.0101	0.0127	0.0113	0.0103	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0107	0.0093	0.0086	0.0092	0.0078	0.0070	0.0085	0.0072	0.0062	1.0594	1.0787	1.0779	1.3397	1.3957	1.4300	1.4872	1.5735	1.6598
3	$\bar{\bar{y}}_{MRSS(1)}$	0.0143	0.0180	0.0217	0.0168	0.0230	0.0289	0.0157	0.0206	0.0250	0.7894	0.5547	0.4266	0.7325	0.4706	0.3486	0.8072	0.5493	0.4117
	$\bar{\bar{y}}_{RSS(2)}$	0.0099	0.0087	0.0081	0.0064	0.0058	0.0054	0.0052	0.0047	0.0044	1.1471	1.1488	1.1469	1.9110	1.8673	1.8676	2.4382	2.3845	2.3643
	$\bar{\bar{y}}_{MRSS(2)}$	0.0271	0.0315	0.0354	0.0514	0.0574	0.0620	0.0554	0.0585	0.0606	0.4178	0.3169	0.2619	0.2389	0.1888	0.1626	0.2290	0.1930	0.1700

Table 8: MSE and RE values for the Exponential (1) distribution ($k = 4$)

		$k=4$																	
		MSE									RE								
		$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$			$\rho_{yx}=0.50$			$\rho_{yx}=0.90$			$\rho_{yx}=1.00$		
m		$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$	$r_2=2$	$r_2=3$	$r_2=4$
	$\bar{\bar{y}}_{SRS}$	0.0181	0.0180	0.0171	0.0191	0.0196	0.0188	0.0196	0.0202	0.0198	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0175	0.0167	0.0161	0.0163	0.0156	0.0148	0.0159	0.0153	0.0145	1.0346	1.0768	1.0629	1.1703	1.2515	1.2727	1.2288	1.3264	1.3618
3	$\bar{\bar{y}}_{MRSS(1)}$	0.0162	0.0163	0.0170	0.0145	0.0153	0.0168	0.0141	0.0147	0.0158	1.1163	1.1071	1.0036	1.3161	1.2808	1.1206	1.3947	1.3786	1.2517
	$\bar{\bar{y}}_{RSS(2)}$	0.0165	0.0157	0.0158	0.0136	0.0129	0.0132	0.0124	0.0121	0.0125	1.0984	1.1446	1.0810	1.4101	1.5107	1.4269	1.5821	1.6763	1.5807
	$\bar{\bar{y}}_{MRSS(2)}$	0.0201	0.0216	0.0234	0.0305	0.0331	0.0356	0.0314	0.0321	0.0328	0.9004	0.8317	0.7296	0.6264	0.5900	0.5287	0.6245	0.6315	0.6027
	$\bar{\bar{y}}_{SRS}$	0.0181	0.0171	0.0165	0.0194	0.0188	0.0186	0.0200	0.0198	0.0196	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0165	0.0154	0.0149	0.0149	0.0136	0.0129	0.0143	0.0131	0.0123	1.0995	1.1102	1.1043	1.3001	1.3881	1.4384	1.3930	1.5081	1.5872
4	$\bar{\bar{y}}_{MRSS(1)}$	0.0161	0.0170	0.0181	0.0145	0.0165	0.0186	0.0138	0.0153	0.0170	1.1273	1.0030	0.9101	1.3395	1.1425	1.0012	1.4448	1.2947	1.1519
	$\bar{\bar{y}}_{RSS(2)}$	0.0158	0.0153	0.0143	0.0117	0.0117	0.0113	0.0105	0.0106	0.0104	1.1508	1.1156	1.1481	1.6572	1.6118	1.6405	1.9082	1.8653	1.8723
	$\bar{\bar{y}}_{MRSS(2)}$	0.0208	0.0241	0.0259	0.0312	0.0360	0.0384	0.0309	0.0327	0.0335	0.8713	0.7086	0.6359	0.6228	0.5228	0.4838	0.6471	0.6041	0.5845
	$\bar{\bar{y}}_{SRS}$	0.0171	0.0159	0.0149	0.0188	0.0179	0.0167	0.0198	0.0191	0.0175	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\bar{\bar{y}}_{RSS(1)}$	0.0154	0.0144	0.0136	0.0127	0.0114	0.0104	0.0129	0.0119	0.0105	1.1101	1.1006	1.0981	1.4829	1.5692	1.6035	1.5320	1.6018	1.6682
6	$\bar{\bar{y}}_{MRSS(1)}$	0.0184	0.0223	0.0258	0.0188	0.0251	0.0310	0.0170	0.0218	0.0264	0.9267	0.7144	0.5764	1.0029	0.7129	0.5388	1.1607	0.8737	0.6635
	$\bar{\bar{y}}_{RSS(2)}$	0.0145	0.0136	0.0128	0.0099	0.0093	0.0088	0.0084	0.0079	0.0074	1.1758	1.1664	1.1604	1.9084	1.9212	1.8952	2.3610	2.4175	2.3720
	$\bar{\bar{y}}_{MRSS(2)}$	0.0316	0.0360	0.0391	0.0538	0.0595	0.0638	0.0514	0.0539	0.0557	0.5402	0.4419	0.3803	0.3503	0.3009	0.2619	0.3843	0.3539	0.3147

Table 1 presents the MSE and RE values of the estimators under the Laplace distribution for $k = 2$. It has been observed that as the number of sets (m) increases, the RE values of the estimators generally increase, particularly when $\rho_{yx} = 0.90$ and 1.00. The highest RE values for all estimators are obtained in the perfect ranking ($\rho_{yx} = 1$). Additionally, for $m = 3, 4$, the RE values increase as the number of repetitions within the same set size increases. The most efficient estimation of the population mean is achieved using the $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator when $m = 6$ and $r_2 = 6$. In the case of imperfect ranking, the most efficient estimates are obtained when $\rho_{yx} = 0.90$. When comparing the estimators obtained under Situation-I, the $\bar{\bar{y}}_{\text{MRSS(1)}}$ estimator is found to be more efficient than the $\bar{\bar{y}}_{\text{RSS(1)}}$ estimator in terms of RE values.

Table 2 examines the case where $k = 4$ under the same distribution, yielding results similar to those in Table 1. The highest RE value for estimating the population mean is achieved using the $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator when $m = 6$ and $r_2 = 2$. When Tables 1 and 2 are considered together, it is observed that as the value of k increases, the MSE values increase across all situations due to the higher level of non-response. This outcome is expected as an increase in k leads to more missing responses. Moreover, as k increases, the RE values of the estimators generally improve. The increase in RE values is primarily due to a greater rise in the MSE values of the $\bar{\bar{y}}_{\text{SRS}}$ estimator compared to other estimators.

Table 3 presents the results obtained under the Normal distribution when $k = 2$. It is observed that as the values of m and ρ_{yx} increase, the RE values of the estimators generally improve. The highest RE value is achieved in the perfect ranking when $m = 6$ and $r_2 = 4$, using the $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator. The estimators obtained under Situation-II are generally found to be more efficient than those obtained under Situation-I.

Table 4 examines the performance of the estimators under the Normal distribution when $k = 4$, revealing similar results to those in Table 3. The highest RE value is obtained in the perfect ranking case when $m = 6$ and $r_2 = 2$, using the $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator. As the value of k increases, the MSE values of all estimators also increase. In the imperfect ranking case, an increase in k and set size generally leads to higher RE values.

In Table 5, the results obtained under the Uniform distribution when $k = 2$ are examined. It is observed that as m increases, particularly when $\rho_{yx} = 0.90$ and 1.00, the RE values of the estimators tend to increase. The highest RE value is obtained in the perfect ranking case when $m = 6$ and $r_2 = 6$, using the $\bar{\bar{y}}_{\text{RSS(2)}}$ estimator. Table 6 presents the RE values of the estimators for the same distribution when $k = 4$. As m and ρ_{yx} increase, a rise in the RE values of the estimators is observed. The highest RE value is obtained using the $\bar{\bar{y}}_{\text{RSS(2)}}$ estimator when $m = 6$ and $r_2 = 2$, followed by the $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator, with the second-highest RE value. As k increases, an expected rise in the MSE values of all estimators is observed. Moreover, when the MSE values of $\bar{\bar{y}}_{\text{SRS}}$ estimator increase relatively more, the RE values also tend to increase. For all values of k, the estimators obtained under Situations-II are found to be more efficient than those under Situations-I at high correlation levels.

In Table 7, the results obtained for $k = 2$ under the Exponential distribution have been examined. The highest RE value is achieved using the $\bar{\bar{y}}_{\text{RSS(2)}}$ estimator in the perfect ranking case, with $m = 6$ and $r_2 = 4$. However, the $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator is not found to be efficient. When comparing the estimators obtained under Situation-I, the $\bar{\bar{y}}_{\text{RSS(1)}}$ estimator is generally found to be more efficient than the $\bar{\bar{y}}_{\text{MRSS(1)}}$ estimator.

Table 8 evaluates the case $k = 4$ under the Exponential distribution. The highest RE value is obtained using the $\bar{\bar{y}}_{\text{RSS(2)}}$ estimator in the perfect ranking case, with $m = 6$ and $r_2 = 4$. For all values of k, an increase in m and ρ_{yx} leads to higher RE values for the estimators obtained under the RSS.

The simulation study demonstrates that under unimodal symmetric distributions, such as the Laplace and Normal distributions, the proposed $\bar{\bar{y}}_{\text{MRSS(2)}}$ estimator exhibits higher efficiency. In contrast, for the Uniform and Exponential distributions, the $\bar{\bar{y}}_{\text{RSS(2)}}$ estimator is found to be more efficient. Furthermore, it is observed that the correlation coefficient, the value of k, and the sample sizes significantly impact the RE values of the estimators. When different set sizes are analyzed, better results are obtained when $m = 6$. The findings related to the estimators are presented in Figure 1 for unimodal symmetric distributions with $m = 6$, while Figure 2 illustrates the results for the other distributions.

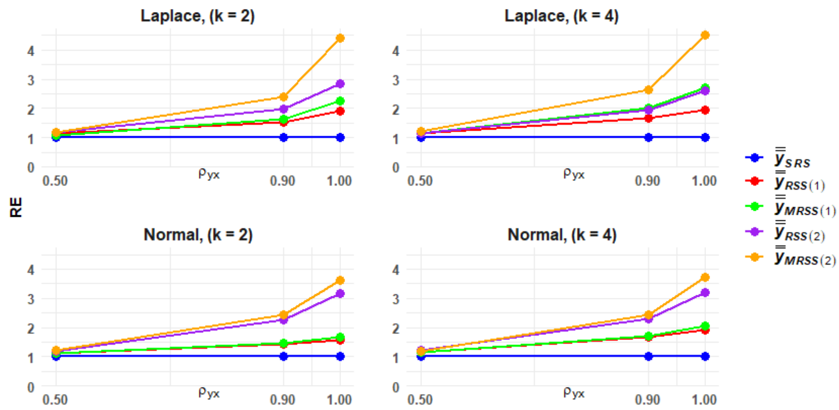

Figure 1: RE Values of Estimators Under Unimodal Symmetric Distributions

In Figure 1, it is observed that the proposed $\bar{\bar{y}}_{\text{MRSS}(2)}$ estimator yields more efficient results under symmetric distributions. In the case of perfect ranking under the Laplace distribution, the difference in RE values between the $\bar{\bar{y}}_{\text{RSS}(2)}$ and $\bar{\bar{y}}_{\text{MRSS}(2)}$ estimators is found to be greater than that observed under the Normal distribution. For both distributions, the estimators obtained under Situation-II are observed to be more efficient. When examining the estimators obtained under Situation-I, it is noted that the difference in RE values between the $\bar{\bar{y}}_{\text{RSS}(1)}$ and $\bar{\bar{y}}_{\text{MRSS}(1)}$ estimators is relatively small in the Normal distribution, whereas this difference becomes more pronounced in the Laplace distribution. Additionally, an increase in ρ_{yx} leads to a corresponding increase in the RE values of the estimators.

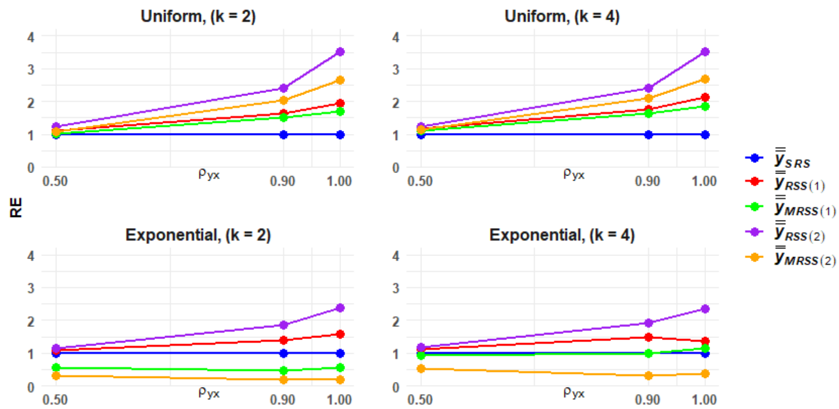

Figure 2: RE Values of Estimators Under Exponential and Uniform Distributions

In Figure 2, the $\bar{\bar{y}}_{\text{RSS}(2)}$ estimator is observed to be more efficient for the Uniform distribution and the Exponential distribution, which is asymmetric. In the case of the Uniform distribution, the second most efficient estimator after $\bar{\bar{y}}_{\text{RSS}(2)}$ is identified as $\bar{\bar{y}}_{\text{MRSS}(2)}$, whereas for the Exponential distribution, the $\bar{\bar{y}}_{\text{RSS}(1)}$ estimator is found to be more efficient.

Furthermore, for all distributions, the most efficient results are obtained under Situation-II, which is based on sample selection from both groups.

4. Real Data

In this section, we analyze the abalone data collected by Nash et al.[17] The real data set includes the shucked weight of the abalone (as a study variable) and the diameter of the abalone (as an auxiliary variable). The motile nature of abalones poses challenges to the sampling process and may lead to non-response. The objective of this study is to estimate the population mean of the shucked weight in the presence of non-response. The calculated values are $\mu_X = 0.407$, $\mu_Y = 0.359$, $\sigma^2_X = 0.0098$, $\sigma^2_Y = 0.0492$, and $\rho_{xy} = 0.89$. In this study, the ranking process was performed using the auxiliary variable X (the diameter of the abalone).

Table 9: RE values of the real data set

m	4		6	
r_2	7	8	7	8
$\bar{\bar{y}}_{\text{SRS}}$	1.0000	1.0000	1.0000	1.0000
$\bar{\bar{y}}_{\text{RSS(1)}}$	1.5953	1.4730	1.7328	1.7655
$\bar{\bar{y}}_{\text{MRSS(1)}}$	1.7626	1.7573	2.0056	2.3230
$\bar{\bar{y}}_{\text{RSS(2)}}$	1.9069	2.1519	2.0592	2.2169
$\bar{\bar{y}}_{\text{MRSS(2)}}$	2.1525	2.4820	3.0106	3.3886

As shown in Table 9, the RE values of the proposed estimator are significantly higher than those of the other estimators.The estimators obtained under Situation II were found to be more efficient than those obtained under Situation I for the values $m = 4, r_2 = 7, 8$ and $m = 6, r_2 = 7$.

5. Results

Non-response is a significant issue affecting parameter estimation in scientific research. In this study, the estimator $\bar{\bar{y}}_{\text{MRSS(2)}}$, based on the MRSS method, is proposed for estimating the population mean under non-response. This estimator is obtained using an MRSS sample selected from both response and non-response groups. Additionally, the performance of the estimator $\bar{\bar{y}}_{\text{RSS(2)}}$ under different cases is examined. This study aims to develop new estimators that can provide better estimates of the population mean under non-response and evaluate their efficiency under different distributions while comparing them with existing estimators in the literature.

MSE and RE values of the estimators under Laplace, Normal, Exponential, and Uniform distributions were obtained through a simulation study. The results indicate that, for unimodal symmetric distributions such as Laplace and Normal, the estimator $\bar{\bar{y}}_{\text{MRSS(2)}}$ provides the highest RE values. However, for the Uniform distribution and the asymmetric Exponential distribution, the estimator $\bar{\bar{y}}_{\text{RSS(2)}}$ was found to be more efficient. Furthermore, the estimators were evaluated based on different correlation values, set sizes, sample sizes, and k values under perfect ranking and imperfect ranking cases. Since an increase in k leads to higher non-response rates, an increase in MSE values was observed across all distributions, which is an expected outcome. In cases where k increased, the estimator $\bar{\bar{y}}_{\text{SRS}}$ obtained under SRS exhibited a greater increase in MSE values compared to other estimators, leading to an increase in some RE values. Additionally, the efficiency values of the estimators obtained under Situation-II were generally found to be better than those under Situation-I.

Future studies may explore Situation-II, which involves sample selection from response and non-response groups, under different sampling designs. Furthermore, the proposed estimators can be employed to develop more efficient ratio, regression, and exponential-type estimators in the presence of non-response.

References

[1] S. Ahmad, S. Hussain, M. Aamir, F. Khan, M.N. Alshahrani, M. Alqawba, *Estimation of finite population mean using dual auxiliary variable for non-response using simple random sampling*, AIMS Math. **7**(3) (2022), doi:10.3934/math.2022256.

[2] S. Ahmed, J. Shabbir, *Ranked set sampling with scrambled response model to subsample non-respondents*, Braz. J. Probab. Stat. **31**(1) (2017), 179–193.

[3] O.A. Alamri, B. Aloraini, *Regression estimator based on median for population mean estimation under non-response analysis*, Alexandria Engineering Journal **123** (2025), 138–144, doi:10.1016/j.aej.2025.02.054.

[4] A.I. Al-Omari, C.N. Bouza, *Ratio estimators of the population mean with missing values using ranked set sampling*, Environmetrics **26**(2) (2015), doi:10.1002/env.2286.

[5] C.N. Bouza, *Estimation of the mean in ranked set sampling with non-responses*, Springer-Verlag, 2002.

[6] C.N. Bouza, *Ranked set sampling procedures for the estimation of the population mean under non-responses: A comparison*, *Statist. Papers* **31** (2010).

[7] C.N. Bouza-Herrera, *Handling missing data in ranked set sampling*, Springer, 2013.

[8] A.E. Cetin, N. Koyuncu, *Calibration estimator of population mean in stratified extreme ranked set sampling with simulation study*, Filomat **38**(2) (2024), 599–608.

[9] A.E. Cetin, N. Koyuncu, *New robust class of estimators for population mean under different sampling designs*, J. Comput. Appl. Math. **441** (2024).

[10] X. Dong, X. Fan, L. Zhang, *Non-parametric estimation of reliability for parallel system under ranked set sampling*, Commun. Statist. Theory Methods (2025), doi:10.1080/03610926.2024.2447823.

[11] M. Ebegil, Y.A. Özdemir, F. Gökpınar, *Some shrinkage estimators based on median ranked set sampling*, J. Appl. Statist. **48**(13–15) (2021), 2473–2498.

[12] M. Fatima, S.H. Shahbaz, M. Hanif, M.Q. Shahbaz, *A modified regression-cum-ratio estimator for finite population mean in presence of nonresponse using ranked set sampling*, AIMS Math. **7**(4) (2022), 6478–6488, doi:10.3934/math.2022361.

[13] M.H. Hansen, W.N. Hurwitz, *The problem of non-response in sample surveys*, J. Amer. Statist. Assoc. **41**(236) (1946), doi:10.1080/01621459.1946.10501894.

[14] G.A. McIntyre, *A method for unbiased selective sampling, using ranked sets*, Amer. Statist. **59**(3) (2005), doi:10.1198/000313005X54180.

[15] C.B. Metin, Y.A. Özdemir, *Nonresponse error and examining the effects on the estimation of population mean: Two subpopulation study*, Süleyman Demirel University Journal of Natural and Applied Sciences **18**(2) (2014), 54–63.

[16] H.A. Muttlak, *Median ranked set sampling*, Journal of Applied Statistical Sciences **6**(4) (1997), 245–255.

[17] W.J. Nash et al., *The population biology of abalone (Haliotis species) in Tasmania*, Sea Fisheries Division Technical Report **48** (1994).

[18] K.U. Rather, E.G. Koçyiğit, C. Ünal, M.I. Jeelani, *New exponential ratio estimator in ranked set sampling*, Pak. J. Stat. Oper. Res. **18**(2) (2022), doi:10.18187/pjsor.v18i2.3921.

[19] S.A. Rehman, J. Shabbir, *An efficient class of estimators for finite population mean in the presence of nonresponse under ranked set sampling*, PLoS ONE **17**(12) (2022), doi:10.1371/journal.pone.0277232.

[20] U. Shahzad et al., *Estimation of population variance using quartiles in absence and presence of non-response*, Gazi University Journal of Science **30**(2) (2017).

[21] C. Unal, C. Kadilar, *Exponential ratio type estimator in the presence of non-response*, Romanian Statistical Review **83**(3) (2021), 371–394.

[22] A. Akın Vargeloğlu, Y.A. Özdemir, *Regression estimator based on two auxiliary variables under ranked set sampling and median ranked set sampling in the presence of non-response*, Journal of Defense Sciences, **21**(2) (2025), doi:10.17134/khosbd.1758912.

[23] W. Yanglem, P. Khongji, *Ranked set sampling based regression estimators in two-stage sampling design*, J. Stat. Theory Appl. **23** (2024), 442–455, doi:10.1007/s44199-024-00092-w.

[24] M. Yaqub, J. Shabbir, *Estimation of population distribution function in the presence of non-response*, Hacet. J. Math. Statist. **47**(2) (2018), doi:10.15672/HJMS.201613118919.