



Approximation results for sublinear operators via statistical convergence in modular spaces

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Abstract. We establish a Korovkin-type theorem for sublinear operators in the framework of modular spaces by using statistical convergence and provide an illustrative example demonstrating its applicability.

1. Introduction

The Korovkin theorem [15] holds a fundamental position in approximation theory and has led to numerous generalizations. Among these, sublinear extensions of this theorem are particularly significant. Sublinear operators introduce a level of complexity beyond that of linear operators, as they do not satisfy the principles of superposition and homogeneity which are key properties of their linear counterparts. In the past few years, considerable research has been devoted to examination of approximation aspects of sublinear operators, with notable contributions including [1], [2], [3], [10], [11], [12], [13] and [17].

In paralel, modular spaces have emerged as a powerful generalization of vector spaces, achieved by reducing the strictness of specific linear constraints. This flexibility renders them particularly suitable for addressing a broad spectrum of problems in functional analysis. Modular spaces play a crucial role in the context of Orlicz spaces, Musielak-Orlicz spaces, and other function spaces where classical norm-based approaches are inadequate. The development of modular convergence has significantly contributed to the improvement of both theoretical and applied mathematics [19], [20], [21]. We have given a Korovkin-type theorem for sublinear operators on modular spaces [18].

In this study, we formulate an approximation theorem of Korovkin-type for sublinear operators within the setting of modular spaces and provide a representative example to illustrate its validity.

The natural density of $K \subset \mathbb{N}$ is defined by the following limit

$$\delta(K) = \lim_{n \rightarrow \infty} \frac{1}{n} |\{k \leq n : k \in K\}|$$

and denoted by $\delta(K)$, where $|\cdot|$ represents the cardinality of the set in vertical bars.

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In the event that for every $\epsilon > 0$ the set $K(\epsilon) = \{k \in \mathbb{N} : |x_k - l| \geq \epsilon\}$ has natural density zero, then sequence x is called statistically convergent to l . This type of convergence is denoted by $st - \lim x = l$ ([8]). According to [9], we recall the statistical limit superior and the statistical limit inferior. Let x be a sequence and S_x, R_x denote the following sets respectively

$$\begin{aligned} &\{t \in \mathbb{R} : \delta\{k : x_k > t\} \neq 0\}, \\ &\{v \in \mathbb{R} : \delta\{k : x_k < v\} \neq 0\}, \end{aligned}$$

$$st - \limsup x = \begin{cases} \sup R_x, & R_x \neq \emptyset \\ -\infty, & R_x = \emptyset \end{cases},$$

is defined as the statistical limit superior of x and

$$st - \liminf x = \begin{cases} \inf S_x, & S_x \neq \emptyset \\ \infty, & S_x = \emptyset \end{cases}$$

is defined as the statistical limit inferior of x . We now present some fundamental definitions and properties related to sublinear operators and modulars.

Let $I = [a, b]$ be a measurable set equipped with the Lebesgue measure μ . Define $X(I)$ as the space of all real-valued measurable functions on I , identified up to almost everywhere equality. Additionally, let $C(I)$ denote the space of all continuous real-valued functions on I and $C^\infty(I)$ represents the space of all infinitely differentiable functions on I .

Definition 1.1. A functional $\rho : X(I) \rightarrow [0, \infty]$ is said to be modular if the following are satisfied:

- (i) $\rho[g] = 0 \iff g = 0$ a.e. on I .
- (ii) $\rho[-g] = \rho[g]$, for all g in $X(I)$.
- (iii) $\rho[\alpha_1 g + \alpha_2 h] \leq \rho[g] + \rho[h]$ for all g, h in $X(I)$ and for every $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$ [5].

A modular ρ is called Q -quasi convex, if

$$\rho[\alpha_1 g + \alpha_2 h] \leq Q\alpha_1 \rho[Qg] + Q\alpha_2 \rho[Qh]$$

is satisfied for some constant $Q \geq 1$, for any g, h in $X(I)$ and $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$. In the case of $Q = 1$, ρ is called convex [5].

A modular ρ is called Q -quasi semiconvex whenever the inequality

$$\rho[\alpha g] \leq Q\alpha \rho[Qg]$$

holds at least a constant $Q \geq 1$ and for all g in $X(I)$, $g \geq 0$ and $\alpha \in (0, 1]$. Observe that Q -quasi convexity yields Q -quasi semiconvexity.

A modular ρ is called monotone if $\rho[g] \leq \rho[h]$ for all $g, h \in X(I)$ satisfying $|g| \leq |h|$ [5].

Now let us give some subspaces of $X(I)$ [6] (see also [7], [14])

$$L^\rho(I) := \left\{ f \in X(I) : \lim_{\lambda \rightarrow 0^+} \rho[\lambda f] = 0 \right\}$$

and

$$E^\rho(I) := \{f \in L^\rho(I) : \rho[\lambda f] < \infty \text{ for every } \lambda > 0\}.$$

Some definitions about modulars are as follows [6]:

- a) ρ is finite, if $1 \in L^\rho(I)$ denoting by 1 the function $1(t) = 1$ for every $t \in I$.
- b) ρ is strongly finite, if $1 \in E^\rho(I)$.
- c) ρ is absolutely continuous, if there exists $\alpha > 0$ such that for every $g \in X(I)$, with $\rho[g] < +\infty$, satisfying that for every $\epsilon > 0$ there is $\delta > 0$ such that $\rho[\alpha g \chi_B] < \epsilon$, for every measurable subset $B \subset I$ with $\mu(B) < \delta$.

d) A modular ρ is called absolutely finite, i.e. ρ is finite and for every $\epsilon > 0, r > 0$ there is $\delta > 0$ such that $\rho[r\chi_E] < \epsilon$ for all measurable subset $E \subset I$ with $\mu(E) < \delta$.

Classical examples of modular spaces include Orlicz spaces, which are defined using a φ -function φ . More broadly, Musielak-Orlicz spaces, generated by a φ -function φ that varies with a parameter and satisfies specific growth conditions relative to that parameter, also fall under this category [4]. The modular functionals defining these spaces fulfill all the previously stated conditions.

A sequence $(f_i)_{i \in \mathbb{N}} \subset L^\rho(I)$ is said to be statistically modularly convergent to f in $L^\rho(I)$, whenever

$$st - \lim \rho[\lambda(f_i - f)] = 0$$

holds for some $\lambda > 0$. This expression is a generalization of the norm convergence in L^p spaces.

Also, a sequence $(f_i)_{i \in \mathbb{N}} \subset L^\rho(I)$ is said to be statistically strongly convergent to f in $L^\rho(I)$ whenever

$$st - \lim \rho[\lambda(f_i - f)] = 0$$

for all $\lambda > 0$.

Clearly, the concept of statistical modular convergence is weaker than that of statistical strong convergence in $L^\rho(I)$.

Now we give some definitions about sublinear operators in general form. Consider Hausdorff topological spaces X with vector sublattice E and Y with vector sublattice F , respectively belonging to $C(X)$ and $C(Y)$. Let V represent an operator mapping from E to F .

- If for every $g, h \in E$ and $a \geq 0$, the conditions

$$V(g + h) \leq V(g) + V(h)$$

and

$$V(ag) = aV(g) \quad (\text{positively homogeneous})$$

hold, then the operator is classified as sublinear.

- If for every $g, h \in E, g \leq h$ implies $V(g) \leq V(h)$ then the operator is said to be monotone.
- If for any pair of comonotone functions g, h in E , meaning

$$(g(x) - g(s)) \cdot (h(x) - h(s)) \geq 0 \text{ for every } x, s \in X,$$

the property

$$V(g + h) = V(g) + V(h)$$

holds, then the operator is said to be comonotone additive.

Let $T = (T_i)_{i \in \mathbb{N}}$ be a sequence of monotone and sublinear operators being $T_i : K \rightarrow X(I)$ where $C(I) \subset K \subset X(I)$. Also along this paper assume that $(T_i)_{i \in \mathbb{N}}$ satisfies the following property which will be called as (CS): there exists $X_T \subset K \cap L^\rho(I)$ that includes $C(I)$ such that for all functions $f \in X_T$ we get $T_i f \in L^\rho(I)$ and

$$st - \lim \sup \rho[\lambda(T_i f)] \leq P \cdot \rho[\lambda f] \tag{1}$$

for any $\lambda > 0$ and an absolute constant P .

2. Approximation Results for Sublinear Operators on Modular Spaces

Within the section, we introduce Korovkin-type results concerning sublinear operators on modular spaces. Initially we give the next lemma.

Lemma 2.1. Consider a modular ρ being absolutely finite, monotone and absolutely continuous on $X(I)$. Hence we get $\overline{C(I)} = L^\rho(I)$.

Proof. Because $C^\infty(I) \subset C(I) \subset L^\rho(I)$ and $\overline{C^\infty(I)} = L^\rho(I)$ [4], we have $\overline{C(I)} = L^\rho(I)$. \square

We will use the notations f_i for the test functions $x^i, i = 0, 1, 2$.

Theorem 2.2. Assume that modular ρ is strongly finite, monotone, absolutely continuous and Q -quasi semiconvex on $X(I)$. Consider $(T_i)_{i \in \mathbb{N}}$ being a sequence of sublinear and monotone operators conforming to the property (CS). If $h \in \{f_0, f_1, -f_1, f_2\}$

$$T_i h \rightarrow h$$

statistically strongly in $L^\rho(I)$, we get

$$T_i f \rightarrow f$$

statistically modularly in $L^\rho(I)$ for any nonnegative $f \in L^\rho(I) \cap K$ such that $f - C(I) \subset X_T$.

Proof. Consider any nonnegative function f in $C(I)$. Hence we write for every $\epsilon > 0$, there exists $\delta > 0$ such that

$$|f(x) - f(y)| \leq \frac{2\|f\|_\infty}{\delta^2} \cdot |x - y|^2$$

holds for any $x, y \in I$ with $|x - y| < \delta$. Therefore we obtain

$$\begin{aligned} |f(x) - f(y)| &\leq \epsilon + \frac{2\|f\|_\infty}{\delta^2} \cdot |x - y|^2 \\ &= \epsilon + \frac{2\|f\|_\infty}{\delta^2} [f_2(x) + 2f_1(x) \cdot (M - f_1(y)) + 2M \cdot (-f_1(x)) + f_2(y)] \end{aligned}$$

for any $x, y \in I$, where $M = \max\{f_1(y), f_2(y), 0\}$. Since T_i is positively homogeneous, for $f \geq 0$, we can write

$$\begin{aligned} |T_i(f)(y) - f(y)| &\leq |T_i(f)(y) - T_i(f(y) \cdot f_0)| + |f(y) \cdot (T_i(f_0) - f_0)| \\ &\leq \epsilon \cdot T_i(f_0) + \frac{2\|f\|_\infty}{\delta^2} \left[\begin{array}{l} T_i(f_2) + 2(M - f_1(y)) \cdot T_i(f_1) + 2M \cdot T_i(-f_1) \\ + f_2(y) \cdot T_i(f_0) \end{array} \right] \\ &\quad + f(y) \cdot |T_i(f_0) - f_0|. \end{aligned}$$

For all $\lambda > 0$, we obtain

$$\begin{aligned} \rho[\lambda \cdot (T_i(f) - f)] &\leq \rho[3\lambda \epsilon T_i(f_0)] + \rho \left[\frac{6\lambda \|f\|_\infty}{\delta^2} \left\{ \begin{array}{l} T_i(f_2) + 2(M - f_1(y)) \cdot T_i(f_1) \\ + 2M \cdot T_i(-f_1) + f_2(y) \cdot T_i(f_0) \end{array} \right\} \right] \\ &\quad + \rho[3\lambda \cdot f(y) |T_i(f_0) - f_0|]. \end{aligned}$$

Let us denote

$$B := T_i(f_2) - f_2 + 2M(T_i(f_1) - f_1) - 2f_1(T_i(f_1) - f_1) + 2M \cdot (T_i(-f_1) + f_1) + f_2(y) \cdot (T_i(f_0) - f_0).$$

Hence we have

$$\begin{aligned} \rho \left[\frac{6\lambda \cdot \|f\|_\infty}{\delta^2} \cdot B \right] &\leq \rho \left[\frac{30\lambda \cdot \|f\|_\infty}{\delta^2} \cdot |T_i(f_2) - f_2| \right] \\ &+ \rho \left[\frac{60.M.\lambda \cdot \|f\|_\infty}{\delta^2} |T_i(f_1) - f_1| \right] + \rho \left[\frac{60.M.\lambda \cdot \|f\|_\infty}{\delta^2} |T_i(-f_1) + f_1| \right] \\ &+ \rho \left[\frac{60.M.\lambda \cdot \|f\|_\infty}{\delta^2} |T_i(f_1) - f_1| \right] + \rho \left[\frac{30.M.\lambda \cdot \|f_2\|_\infty \cdot \|f\|_\infty}{\delta^2} |T_i(f_0) - f_0| \right]. \end{aligned}$$

By using our assumptions, we have for any constant $\eta > 0$,

$$st - \limsup \rho [\eta(T_i(f) - f)] \leq P.\rho [3\eta\epsilon f_0].$$

Then by using Q -quasi semiconvexity and strongly finiteness of ρ , we obtain by assuming $\epsilon < 1$ that

$$\rho [3\eta\epsilon f_0] \leq Q\epsilon\rho [3Q\eta f_0].$$

Hence we get the statistical strong convergence to nonnegative $f \in C(I)$. Take into consideration nonnegative $f \in K \cap L_\rho(I)$. From Lemma 2.1, there exist a nonnegative sequence $(g_k)_{k \in \mathbb{N}} \subset C(I)$ and a $\lambda > 0$ such that $\rho [3\lambda f] < \infty$ and

$$\lim_{k \rightarrow \infty} \rho [3\lambda (g_k - f)] = 0. \tag{2}$$

Since T_i is subadditive for $i \in \mathbb{N}$, we find

$$\begin{aligned} |T_i(f) - f| &\leq |T_i(f) - T_i(g_k)| + |T_i(g_k) - g_k| + |g_k - f| \\ &\leq T_i(|f - g_k|) + |T_i(g_k) - g_k| + |g_k - f|. \end{aligned}$$

For a $\lambda > 0$, $f - g_k \in X_T$, we obtain

$$\rho [\lambda |T_i f - f|] \leq \rho [3\lambda.T_i(|f - g_k|)] + \rho [3\lambda \cdot |T_i(g_k) - g_k|] + \rho [3.\lambda |g_k - f|]. \tag{3}$$

If we take advantage of property (CS), we write

$$\rho [3\lambda.T_i(|f - g_k|)] \leq P.\rho [4\lambda \cdot |f - g_k|].$$

If we apply statistical lim sup operator to each side of the above inequality, we get

$$st - \limsup \rho [\lambda \cdot |T_i f - f|] \leq (P + 1) \cdot \rho [3\lambda \cdot (f - g_k)].$$

By using equality (2), for some $\lambda > 0$ we observe

$$st - \limsup \rho [\lambda |T_i f - f|] = 0.$$

The proof is completed. \square

Theorem 2.3. Building on the assumptions of Theorem 2.2, also assume that T_i is comonotone additive for each $i \in \mathbb{N}$. Then

$$T_i f \rightarrow f$$

holds statistically modularly in $L^\rho(I)$ for any f in $L^\rho(I) \cap K$ such that $f - C(I) \subset X_T$.

Proof. Let each T_i be monotone, sublinear and comonotone additive. Because of Theorem 2.2, we get

$$T_i(f + \|f\|_\infty) \rightarrow f + \|f\|_\infty \tag{4}$$

statistically modularly in $L^p(I)$ for each f in $L^p(I) \cap K$ such that $f - C(I) \subset X_T$. Each constant function consists of a comonotone pair with any function f . Since T_i is positively homogeneous and comonotone additive for any $i \in \mathbb{N}$, we get

$$T_i(f + \|f\|_\infty \cdot f_0) = T_i(f) + \|f\|_\infty \cdot T_i(f_0). \tag{5}$$

By using (4), (5) and our assumption

$$T_i(f_0) \rightarrow f_0, \text{ statistically strongly in } L^p(I),$$

we obtain

$$T_i(f) \rightarrow f$$

statistically modularly in $L^p(I)$ for each $f \in L^p(I) \cap K$ such that $f - C(I) \subset X_T$. \square

Now let us determine the class X_T in a special case. Initially we recall the Orlicz space [16]. Let φ be a convex function from \mathbb{R}_0^+ into itself such that $\varphi(0) = 0$, $\varphi(v) > 0$ for $v > 0$ and $\lim_{v \rightarrow \infty} \varphi(v) = \infty$. Let us denote the class formed by these functions with Φ . Let us define the

$$\rho_\varphi[f] = \int_I \varphi(|f(s)|) ds$$

functional for $\varphi \in \Phi$ and $f \in X(I)$. The functional ρ_φ is convex on $X(I)$ [4].

$$L_\varphi(I) = \{f \in X(I) : \exists \lambda > 0 \ni \rho_\varphi[\lambda f] < \infty\}.$$

The subspace $L_\varphi(I)$ generated by φ is called Orlicz space [16]. The ρ_φ modular is finite, strongly finite and absolutely continuous. Here, if $\varphi(v) = v^p$ is taken for $v \geq 0$, $p \geq 1$, we have $L_\varphi(I) = L^p(I)$ and

$$\|f\|_\varphi = \|f\|_p$$

is obtained. On the other hand, let F_φ be the set of functions in $L_\varphi(I)$ satisfying the inequality (1) for a constant P which is independent for λ and f and for every λ . Also let us consider the following classical Bernstein operator $B_i : C[0, 1] \rightarrow C[0, 1]$

$$B_i(f)(x) = \sum_{k=0}^i p_{i,k}(x) f\left(\frac{k}{i}\right)$$

where $p_{i,k}(x) = \binom{i}{k} x^k (1-x)^{i-k}$. By using this operator we will define sublinear operator $T_i : C[0, 1] \rightarrow C[0, 1]$ as follows:

$$T_i(f) = u_i \max\{B_i(f), B_{i+1}(f)\}$$

where (u_i) is defined by

$$u_i = \begin{cases} 0, & i \text{ is prime number} \\ 1, & \text{otherwise.} \end{cases}$$

These operators are monotone and sublinear. (These operators are obviously comonotone additive). It is well known that

$$\begin{aligned} B_i(f_0)(x) &= f_0, \\ B_i(-f_1)(x) &= -x, \end{aligned}$$

$$B_i(f_2)(x) = x^2 + \frac{x \cdot (1-x)}{i}.$$

By the help of above equalities, we get $T_i(f_0)(x) = u_i f_0(x)$, $T_i(-f_1)(x) = -u_i x$ and $T_i(f_2)(x) = u_i \max\{B_i(f_2)(x), B_{i+1}(f_2)(x)\} = u_i \left(x^2 + \frac{x(1-x)}{i}\right)$. Since φ is convex, for every $\lambda > 0$ we have

$$\begin{aligned} \rho_\varphi[\lambda B_i f] &= \int_0^1 \varphi\left(\left|\lambda B_i(f)(s)\right|\right) ds \\ &= \int_0^1 \varphi\left(\lambda \left|\sum_{k=0}^i p_{i,k}(s) f\left(\frac{k}{i}\right)\right|\right) ds \\ &\leq \int_0^1 \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) p_{i,k}(s) ds \\ &= \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) \int_0^1 p_{i,k}(s) ds \\ &= \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) \binom{i}{k} \frac{k!(i-k)!}{(i+1)!} \\ &= \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) \frac{1}{i+1} \\ &= \frac{1}{i+1} \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) \\ &= \frac{1}{i+1} \rho_i^\varphi[\lambda f]. \end{aligned}$$

By using the last inequality, we can write for every integrable function f ,

$$\begin{aligned} \limsup_{i \rightarrow \infty} \frac{1}{i+1} \rho_i^\varphi[\lambda f] &= \limsup_i \frac{1}{i+1} \cdot \frac{i}{i} \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) \\ &= \limsup_i \frac{i}{i+1} \cdot \sum_{k=0}^i \varphi\left(\left|\lambda f\left(\frac{k}{i}\right)\right|\right) \cdot \left(\frac{k+1}{i} - \frac{k}{i}\right) \\ &= 1 \cdot \rho_\varphi[\lambda f]. \end{aligned}$$

From the above equation, $f \in F_\varphi$, that is every integrable function in the Riemann sense, belongs to the class F_φ . So there exists a subset $X_T \subset K \cap L^p(I)$ including the space $C(I)$ for the operator (T_i) with

$$\begin{aligned} st - \limsup_i \rho_\varphi[\lambda(T_i f)] &= st - \limsup_i \rho_\varphi[\lambda u_i \max\{B_i f, B_{i+1} f\}] \\ &= st - \limsup_i u_i \rho_\varphi[\lambda \max\{B_i f, B_{i+1} f\}] \\ &\leq P \cdot \rho_\varphi[\lambda f]. \end{aligned}$$

That is, the property (CS) is satisfied at least for the class of Riemann integrable functions. This shows that $F_\varphi \subset X_T$. Then we get

$$\begin{aligned}\rho_\varphi [\lambda (T_i(f_0) - f_0)] &= \int_0^1 \varphi(\lambda |u_i - 1|) dx \\ &= \begin{cases} \varphi(\lambda), & i \text{ is prime number} \\ 0, & \text{otherwise} \end{cases}, \text{ for all } \lambda > 0 \\ \rho_\varphi [\lambda (T_i(f_1) - f_1)] &= \int_0^1 \varphi(\lambda |u_i x - x|) dx \\ &= \begin{cases} \int_0^1 \varphi(\lambda x) dx, & i \text{ is prime number} \\ 0, & \text{otherwise} \end{cases}, \text{ for all } \lambda > 0 \\ \rho_\varphi [\lambda (T_i(-f_1) + f_1)] &= \int_0^1 \varphi(\lambda |-u_i x + x|) dx \\ &= \begin{cases} \int_0^1 \varphi(\lambda x) dx, & i \text{ is prime number} \\ 0, & \text{otherwise} \end{cases}, \text{ for all } \lambda > 0\end{aligned}$$

and

$$\begin{aligned}\rho_\varphi [\lambda (T_i(f_2) - f_2)] &= \int_0^1 \varphi\left(\lambda \left|u_i x^2 + \frac{u_i x(1-x)}{i} - x^2\right|\right) dx \\ &= \begin{cases} \int_0^1 \varphi(\lambda x^2) dx, & i \text{ is prime number} \\ \int_0^1 \varphi\left(\lambda \frac{x(1-x)}{i}\right) dx, & \text{otherwise} \end{cases}, \text{ for all } \lambda > 0.\end{aligned}$$

As one can see, for $h \in \{f_0, f_1, -f_1, f_2\}$ we have

$$T_i h \rightarrow h$$

statistically strongly in $L^p(I)$. This sequence of operators T_i fulfills all the requirements of our theorem.

3. Concluding Remarks

In this study, we establish a comprehensive Korovkin-type approximation theorem for a class of sublinear operators defined within the framework of modular function spaces. The main objective of this work is to extend the classical Korovkin approximation theory to a more general setting that accommodates sublinear structures under statistical convergence. Furthermore, we construct and analyze a concrete example to demonstrate the applicability and validity of the proposed theorem, thereby illustrating how the theoretical results can be effectively utilized in practical approximation processes. As a future work, it would be interesting to study abstract Korovkin type theorem.

References

- [1] A. Arif, T. Yurdakadim, *Approximation results on nonlinear operators by P_p -statistical convergence*, Adv. Stud. Euro-Tbil. Math. J. **15** (2022), 1–10.
- [2] I. Aslan, *Convergence in phi-variation and rate of approximation for nonlinear integral operators using summability process*, Mediterr. J. Math. **18** (2021), paper no 5.
- [3] I. Aslan, O. Duman, *Nonlinear approximation in N-dimension with the help of summability methods*, RACSAM **115** (2021), paper no 105.
- [4] C. Bardaro, I. Mantellini, *Korovkin theorem in modular spaces*, Comment Math **47** (2007), 239–253.
- [5] C. Bardaro, I. Mantellini, *Multivariate moment type operators: approximation properties in Orlicz spaces*, J. Math. Inequal. **2** (2008), 247–259.

- [6] C. Bardaro, I. Mantellini, *A Korovkin theorem in multivariate modular function spaces*, J. Funct. Spaces. Appl. 2 (2009), 105–120.
- [7] C. Bardaro, A. Boccuto, X. Dimitriou, I. Mantellini, *Abstract Korovkin-type theorems in modular spaces and applications* Cent.Eur. J. Math. 11 (2013), 1774-1784.
- [8] J. A. Fridy, *On statistical convergence*, Analysis 5 (1985) 301–313.
- [9] J. A. Fridy, C. Orhan, *Statistical limit superior and limit inferior*, Proc. Amer. Math. Soc. 125 (1997) 3625–3631.
- [10] S. G. Gal, *Uniform and pointwise quantitative approximation by Kantorovich-Choquet type integral operators with respect to monotone and submodular set functions*, Mediterr. J. Math. 14 (2017), 205–216.
- [11] S. G. Gal, C. P. Niculescu, *A nonlinear extension of Korovkin's theorem*, Mediterr. J. Math. 17 (2020), paper no 145.
- [12] S. G. Gal, C. P. Niculescu, *Korovkin-type theorems for weakly nonlinear and monotone operators*, Mediterr. J. Math. 20 (2023), 56.
- [13] T. Y. Gökçer, O. Duman, *Approximation by max-min operators: A general theory and its applications*. Fuzzy Sets and Systems 394 (2020), 146–161.
- [14] S. Karakus, K. Demirci, O. Duman, *Statistical approximation by positive linear operators on modular spaces*, Positivity 14 (2010), 321–334.
- [15] P. P. Korovkin, *Linear Operators and Approximation Theory*, Hindustan Publ. Corp, Delhi (1960).
- [16] J. Musielak, *Orlicz spaces and modular spaces*, Berlin: Springer-Verlag, 1983.
- [17] C. P. Niculescu, *An overview of absolute continuity and its applications*, Int. Ser. Numer. Math 157 (2009), 201–214.
- [18] E. Taş, S. Aykanat, *Some Approximation Results for Nonlinear Operators in Modular Space*, Adv. Stud. Euro-Tbil. Math. J. Accepted.
- [19] E. Taş, T. Yurdakadim, *Approximation results by statistical convergence based on a power series in modular spaces*, Lobachevskii J. Math. 44 (2023), 4913–4919.
- [20] E. Taş, T. Yurdakadim, *Approximation by positive linear operators in modular spaces by power series method*, Positivity 21 (2017), 1293–1306.
- [21] T. Yurdakadim, *Abstract Korovkin theory in modular spaces in the sense of power series method*, Hacet. J. Math. Stat. 47 (2018), 1467–1477.