Filomat 40:7 (2026), 2551–2567
https://doi.org/10.2298/FIL2607551P

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Some identities of the probabilistic degenerate Genocchi polynomials and numbers

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Abstract. At 1979, L. Carlitz introduced a degenerate version of the Stirling, Bernoulli, and Eulerian numbers. Recently, some researchers have studied various degenerate special polynomials and numbers. Let Y be a random variable whose moment generating function, $E\left[e^{Yt}\right] = \sum_{n=0}^{\infty} E\left[Y^n\right] \frac{t^n}{n!}$, $(|t| < r)$. The purpose of this paper is to introduce the probabilistic extension of degenerate Genocchi polynomials, namely the probabilistic degenerate Genocchi polynomials associated with random variable Y, $\frac{2t}{E\left[e_{\lambda}^{Y}(t)\right]+1}\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{x} = \sum_{n=0}^{\infty} G_{n,\lambda}^{Y}(x)\frac{t^n}{n!}$. We obtain some properties, explicit expressions, recurrence relations and certain identities for those polynomials and numbers. As the various special cases of random variable Y, we deal with the gamma random variable with parameters $\alpha, \beta > 0$, the Bernoulli random variable with probability of success p and the Poisson random variable with parameter $\alpha > 0$. Also, we investigate the properties of the $G_{n,\lambda}^{Y}(x)$ and illustrate our results with some examples.

1. Introduction

For a given nonzero real number λ, the *degenerate exponential function* is defined to be

$$e_{\lambda}^{x}(t) = (1+\lambda t)^{\frac{x}{\lambda}}, \text{ and } e_{\lambda}(t) = (1+\lambda t)^{\frac{1}{\lambda}}, \text{ (see [4, 12]).} \tag{1.1}$$

The study of degenerate versions of special functions was started by L. Carlitz ([4]), and since then many researchers have introduced degenerate versions of various special functions and found various properties of them. In [22], Kurk considered the degenerate Korobov polynomials and gave recurrence relations for poly-Korobov polynomials, the poly-Korobov-type Changhee polynomials and the degenerate Korobov polynomials. Kim-Kim gave several expressions for the generating function of the sum of the values of the generalized falling factorials at positive consecutive integers in [14]. In [26], authors gave

2020 *Mathematics Subject Classification*. Primary 05A10; Secondary 05A15, 11B73, 11B83, 60C05.
Keywords. Genocchi polynomials, degenerate exponential function, probabilistic degenerate Genocchi polynomials, probabilistic Bell polynomials, probabilistic Stirling numbers of the second kind.
Received: 08 September 2025; Revised: 26 December 2025; Accepted: 09 January 2026
Communicated by Dragan S. Djordjević
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the degenerate version of derangement polynomials and found some interesting identities. In [18], Kim-Kim-Lee-Kwon considered the degenerate Daehee polynomials and found the coefficients when any formal power series represented the linear combination of these polynomials using umbral calculus. Kim-Park-Kwon introduced the degenerate version of q-Changhee polynomials and gave some explicit formula of those polynomials (see [21]).

L. Carlitz defined defined the *degenerate Bernoulli numbers* by using generating function as follows:

$$\frac{t}{e_\lambda(t)-1}=\sum_{n=0}^{\infty}B_{n,\lambda}\frac{t^n}{n!},\ \text{(see [4, 7])},$$

and Kim-Kim defined the *degenerate Bell polynomials* as follows:

$$e^{x(e_\lambda-1)}=\sum_{n=0}^{\infty}Bel_{n,\lambda}(x)\frac{t^n}{n!},\ \text{(see [11, 13, 14, 16])}.$$

In [10, 20, 23], Kim et al. gave *degenerate Genocchi polynomials* which are defined as follows

$$\frac{2t}{e_\lambda(t)+1}e_\lambda^x(t)=\sum_{n=0}G_{n,\lambda}(x)\frac{t^n}{n!}.$$

When $x=0$, $G_{n,\lambda}=G_{n,\lambda}(0)$ are called the *degenerate Geoncchi numbers*.

In [9, 17], Kim et al. defined the *degenerate Stirling numbers of the first kind* and the *second kind* $S_{1,\lambda}(n,k)$ and $S_{2,\lambda}(n,k)$, respectively defined by the generating function to be

$$\frac{1}{k!}\left(\log_\lambda(1+t)\right)^k=\sum_{l=k}^{\infty}S_{1,\lambda}(n,k)\frac{t^n}{n!},\ \text{and}\ \frac{1}{k!}\left(e_\lambda(t)-1\right)^k=\sum_{l=k}^{\infty}S_{2,\lambda}(n,k)\frac{t^n}{n!},\tag{1.2}$$

where $\log_\lambda(t)$ is the compositional inverse function of e_λ such that $e_\lambda\left(\log_\lambda(t)\right)=\log\left(e_\lambda(t)\right)=t$.

The *Genocchi polynomials* are defined by the generating function to be

$$\frac{2t}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!},\ \text{(see [6, 8])}.$$

When $x=0$, $G_n=G_n(0)$ are called the *Genocchi numbers*.

For nonzero integers n and k with $n\geq k$, the *Stirling numbers of the first kind* and the *Stirling numbers of the second kind* $S_1(n,k)$, $S_2(n,k)$, respectively are defined by generating function to be

$$\frac{1}{k!}\left(\log(1+t)\right)^k=\sum_{l=k}^{\infty}S_1(n,k)\frac{t^l}{l!},\ \text{and}\ \frac{1}{k!}\left(e^t-1\right)^k=\sum_{l=k}^{\infty}S_2(l,k)\frac{t^l}{l!},\ \text{(see [5])}.\tag{1.3}$$

The *Bell polynomials* are defined as follows:

$$e^{x\left(e^t-1\right)}=\sum_{n=0}^{\infty}Bel_n(x)\frac{t^n}{n!},\ \text{(see [5, 11, 25])}.$$

In the special case $x=1$, $Bel_n=Bel_n(1)$ are called the *Bell numbers*.

Let Y be a random variable with the moment generating function of Y

$$E\left[e^{Yt}\right]=\sum_{n=0}^{\infty}E\left[Y^n\right]\frac{t^n}{n!},\ (|t|<r)\ \text{exists for some}\ r>0,$$

(see [1, 2]). Note that if $Y = 1$, then $E\left[e^{Yt}\right] = e^t$.

By above note, Kim-Kim defined the *probabilistic Bernoulli polynomials* associated with random variable Y as follows:

$$\frac{t}{E\left[e^{Yt}\right]-1}\left(E\left[e^{Yt}\right]\right)^x = \sum_{n=0}^{\infty} B_n^Y(x)\frac{t^n}{n!}, \text{ (see [15]).} \tag{1.4}$$

As the generalization of the Stirling numbers of the second kind, the *probabilistic Stirling numbers of the second kind* associated with random variable Y are defined as follows:

$$\frac{1}{k!}\left(E\left[e^{Yt}\right]-1\right)^k = \sum_{l=k}^{\infty} S_2^Y(n,k)\frac{t^n}{n!}, \text{ (see [1, 2, 13, 19]).} \tag{1.5}$$

In the special case $Y = 1$, $S_2^Y(n,k) = S_2(n,k)$.

The *probabilistic Bell polynomials* associated with random variable Y are defined by the generating function to be

$$e^{x\left(E\left[e^{Yt}\right]-1\right)} = \sum_{n=0}^{\infty} Bel_n^Y(x)\frac{t^n}{n!}, \text{ (see [13, 16, 25])} \tag{1.6}$$

When $Y = 1$, $Bel_n(x) = Bel_n^Y(x)$ are called the *Bell polynomials* and $Bel_n = Bel_n(1)$ are called the *Bell numbers*.

Special functions are used extensively not only in pure and applied mathematics, but also in all fields that utilize mathematics, such as engineering, medicine, and economics (see [3]). In particular, research actively exploring the properties of special functions using probability theory has been progressing rapidly in recent years (see [1, 2, 13, 15, 16, 19, 24, 25]).

This aim of this paper is to discover various properties of degenerate Genocchi polynomials and numbers using probabilistic methods and derive some interesting identities of those polynomials and numbers. The applications of this paper are to establish a theoretical framework for a new probabilistic model and develop theories applicable across various fields. In particular, it seeks to provide practical solutions to real-world problems in areas such as finance, image processing, regression modeling in statistics, and plasma physics.

The outline of this paper is as follows. In Section 1, we recall the degenerate Bernoulli numbers, degenerate Bell polynomials, degenerate Genocchi polynomials, degenerate Stirling numbers of the first kind and the second kind. Assume that Y is a random variable such that the moment generating function of Y, $E\left[e^{Yt}\right] = \sum_{n=0}^{\infty} E\left[Y^n\right]\frac{t^n}{n!}$, $(|t| < r)$. In Section 2, let $(Y_j)_{j\geq 1}$ be a sequence of mutually independent copies of the random variable Y, and let $S_0 = 0$, $S_k = Y_1 + Y_2 + \cdots + Y_k$, $k \in \mathbb{N}$. Then we define the probabilistic degenerate Genocchi polynomials associated with Y, $G_{n,\lambda}^Y(x)$. Then we derive explicit expressions for the probabilistic degenerate Genocchi polynomials from Theorem 2.1 to 2.7. We determine $G_{n,\lambda}^Y(x)$ when Y is the gamma random variable with parameters α,$\beta > 0$ in Theorem 2.8 and the Bernoulli random variable with probability of success p in Theorem 2.9 and the Poisson random variable with parameter $\alpha > 0$ in Theorem 2.10. In Section 3, we investigate the properties of the $G_{n,\lambda}^Y(x)$ and illustrate our results with some examples. Finally, we conclude our study in Section 4.

2. Probabilistic Genocchi polynomials and numbers

Let $\left(Y_j\right)_{j\geq 1}$ be sequence of mutually independent copies of the random variable Y, and let

$$S_0 = 0,\ S_k = Y_1 + Y_2 + \cdots + Y_k,\ k \in \mathbb{N}.$$

In viewpoint of (1.4), we define the *probabilistic degenerate Genocchi polynomials associated with* Y as follows:

$$\frac{2t}{E\left[e_\lambda^Y(t)\right]+1}\left(E\left[e_\lambda^Y(t)\right]\right)^x = \sum_{n=0}^{\infty} G_{n,\lambda}^Y(x)\frac{t^n}{n!}. \tag{2.1}$$

In the special case $x = 0$, $G^Y_{n,\lambda} = G^Y_{n,\lambda}(0)$ are called the *probabilistic degenerate Genocchi numbers associated with Y*.

By (1.5) and (2.1), we see that

$$\begin{aligned}\sum_{n=0}^{\infty} G^Y_{n,\lambda}(x)\frac{t^n}{n!} &= \frac{2t}{E\left[e^Y_\lambda(t)\right]+1}\left(E\left[e^Y_\lambda(t)\right]-1+1\right)^x \\ &= \left(\sum_{n=0}^{\infty} G^Y_{n,\lambda}\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}(x)_n\frac{1}{n!}\left(E\left[e^Y_\lambda(t)\right]-1\right)^n\right) \\ &= \left(\sum_{n=0}^{\infty} G^Y_{n,\lambda}\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}(x)_n\sum_{l=n}^{\infty}S^Y_{2,\lambda}(l,n)\frac{t^l}{l!}\right) \\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}S^Y_{2,\lambda}(n,m)G^Y_{n-m,\lambda}(x)_m\right)\frac{t^n}{n!}.\end{aligned} \tag{2.2}$$

By (2.2), we obtain the following theorem.

Theorem 2.1. *For each nonnegative integer n, we have*

$$G^Y_{n,\lambda}(x) = \sum_{m=0}^{n}\binom{n}{m}G^Y_{n-m,\lambda}S^Y_{2,\lambda}(n,m)(x)_m.$$

In particular, if we put $Y = 1$, then

$$G_{n,\lambda}(x) = \sum_{m=0}^{n}\binom{n}{m}G_{n-m,\lambda}S_{2,\lambda}(n,m)(x)_m.$$

In addition, by (2.2), we get

$$\begin{aligned}\sum_{n=0}^{\infty} G^Y_{n,\lambda}(x)\frac{t^n}{n!} &= \left(\sum_{n=0}^{\infty} G^Y_{n,\lambda}\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}(x)_n\frac{1}{n!}\left(E\left[e^Y_\lambda(t)\right]-1\right)^n\right) \\ &= \left(\sum_{n=0}^{\infty} G^Y_{n,\lambda}\frac{t^n}{n!}\right)\left(1+\sum_{n=1}^{\infty}(x)_n\frac{1}{n!}\left(E\left[e^Y_\lambda(t)\right]-1\right)^n\right) \\ &= \sum_{n=0}^{\infty}\left(G^Y_{n,\lambda}\frac{t^n}{n!}+\sum_{n=1}^{\infty}\sum_{r=0}^{n}\sum_{m=0}^{r}\binom{n}{r}S^Y_{2,\lambda}(r,m+1)G^Y_{n-r,\lambda}(x)_{m+1}\right)\frac{t^n}{n!}.\end{aligned} \tag{2.3}$$

By (2.3), we obtain the following theorem.

Theorem 2.2. *For each positive integer n, we have*

$$G^Y_{n,\lambda}(x) - G^Y_{n,\lambda} = \sum_{r=0}^{n}\sum_{m=0}^{r}\binom{n}{r}S^Y_{2,\lambda}(r,m+1)G^Y_{n-r,\lambda}(x)_{m+1}.$$

In particular, if we put $Y = 1$, then

$$G^Y_{n,\lambda}(x) - G^Y_{n,\lambda} = \sum_{r=0}^{n}\sum_{m=0}^{r}\binom{n}{r}S_{2,\lambda}(r,m+1)G_{n-r,\lambda}(x)_{m+1}.$$

Note that by (1.2)

$$\begin{aligned}\sum_{m=0}^{n}\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{2m} &=\frac{\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{2(n+1)}-1}{\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{x}-1}\\ &=\frac{1}{2t^2}\frac{t}{E\left[e_{\lambda}^{Y}(t)\right]-1}\frac{2t}{E\left[e_{\lambda}^{Y}(t)\right]+1}\left(\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{2n+2}-1\right)\\ &=\frac{1}{2t^2}\left(\sum_{a=0}^{\infty}B_a^Y\frac{t^a}{a!}\right)\left(\sum_{r=0}^{\infty}\left(G_{r,\lambda}^{Y}(2n+2)=G_{r,\lambda}^{Y}\right)\frac{t^r}{r!}\right)\\ &=\frac{1}{2t^2}\sum_{a=0}^{\infty}\sum_{r=0}^{a}\binom{a}{r}\left(G_{r,\lambda}^{Y}(2n+2)-G_{r,\lambda}^{Y}\right)B_{a-r}^{Y}\frac{t^n}{n!}\\ &=\frac{1}{2}\sum_{a=0}^{\infty}\sum_{r=0}^{a+2}\binom{a+2}{r}\frac{\left(G_{r,\lambda}^{Y}(2n+2)-G_{r,\lambda}^{Y}\right)B_{a-r+2}^{Y}}{(a+2)(a+1)}\frac{t^n}{n!},\end{aligned} \tag{2.4}$$

and

$$\begin{aligned}\sum_{m=0}^{n}\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{2m} &=\sum_{m=0}^{n}E\left[e_{\lambda}^{Y_1+Y_2+\cdots+Y_{2m}}(t)\right]=\sum_{m=0}^{n}E\left[e_{\lambda}^{S_{2m}}(t)\right]\\ &=\sum_{a=0}^{\infty}\sum_{m=0}^{n}E\left[(S_{2m})_{a,\lambda}\right]\frac{t^a}{a!}.\end{aligned} \tag{2.5}$$

Hence, by (2.4) and (2.5), we obtain the following theorem.

Theorem 2.3. *For each nonnegative integer a, n, we have*

$$\sum_{m=0}^{n}E\left[(Y_1+Y_2+\cdots+Y_{2m})_{a,\lambda}\right]=\sum_{r=0}^{a+2}\binom{a+2}{r}\frac{\left(G_{r,\lambda}^{Y}(2n+2)-G_{r,\lambda}^{Y}\right)}{(a+2)(a+1)}B_{a-r+2}^{Y}.$$

If we put $Y=1$, then

$$\sum_{m=0}^{n}(2m)_{a,\lambda}=\sum_{r=0}^{a+2}\binom{a+2}{r}\frac{(G_{r,\lambda}(2n+2)-G_{r,\lambda})}{(a+2)(a+1)}B_{a-r+2}.$$

Since

$$\begin{aligned}
\frac{2t}{E\left[e_\lambda^Y(t)\right]+1}\left(E\left[e_\lambda^Y(t)\right]\right)^x &= \frac{2t}{1-\left(-E\left[e_\lambda^Y(t)\right]\right)}\left(E\left[e_\lambda^Y(t)\right]\right)^x \\
&= 2t\sum_{m=0}^{\infty}(-1)^m\left(E\left[e_\lambda^Y(t)\right]\right)^{x+m} \\
&= 2t\sum_{m=0}^{\infty}(-1)^m\left(E\left[e_\lambda^Y(t)\right]-1+1\right)^{x+m} \\
&= 2t\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\binom{x+m}{l}(-1)^m\left(E\left[e_\lambda^Y(t)\right]-1\right)^l \\
&= 2t\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}(x+m)_l\sum_{r=l}^{\infty}S_{2,\lambda}^Y(r,l)\frac{t^r}{r!} \\
&= 2t\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{l=0}^{n}(x+m)_l(-1)^m S_{2,\lambda}^Y(r,l)\frac{t^n}{n!} \\
&= \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{n-1}n(-1)^m(x+m)_l S_{2,\lambda}^Y(r,l)\frac{t^n}{n!}.
\end{aligned} \tag{2.6}$$

By (2.6), we obtain the following theorem.

Theorem 2.4.

$$G_{0,\lambda}^Y(x)=0 \text{ and } G_{n,\lambda}^Y(x)=\sum_{m=0}^{\infty}\sum_{l=0}^{n-1}n(-1)^m(x+m)_l S_{2,\lambda}^Y(r,l),\ (n\geq 1).$$

In particular, if we put $Y=1$, then

$$G_{0,\lambda}(x)=0 \text{ and } G_{n,\lambda}n(-1)^m(x+m)_l S_{2,\lambda}(r,l),\ (n\geq 1).$$

From Theorem 2.4, we see that $G_{1,\lambda}^Y(x)$ is also constant.
By (2.1), we get

$$\begin{aligned}
2t &= \left(\sum_{n=0}^{\infty}G_{n,\lambda}^Y\frac{t^n}{n!}\right)\left(E\left[e_\lambda^Y(t)\right]+1\right) \\
&= \left(\sum_{n=0}^{\infty}G_{n,\lambda}^Y\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}E\left[(Y)_{n,\lambda}\right]\frac{t^n}{n!}+1\right) \\
&= \sum_{n=0}^{\infty}\left(\sum_{r=0}^{n}\binom{n}{r}E\left[(Y)_{r,\lambda}\right]G_{n-r,\lambda}^Y+G_{n,\lambda}^Y\right)\frac{t^n}{n!},
\end{aligned} \tag{2.7}$$

and so by (2.7), we obtain the following theorem.

Theorem 2.5. *For each nonnegative integer n, we have*

$$2\delta_{1,n}=\sum_{r=0}^{n}\binom{n}{r}E\left[(Y)_{r,\lambda}\right]G_{n-r,\lambda}^Y+G_{n,\lambda}^Y.$$

In particular, if we put $Y=1$, then

$$2\delta_{1,n}=\sum_{r=0}^{n}\binom{n}{r}(1)_{r,\lambda}G_{n-r,\lambda}+G_{n,\lambda}.$$

Note that

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda}^{Y}\frac{t^n}{n!} &= \frac{2t}{E\left[e_{\lambda}^{Y}(t)\right]+1} = 2t\sum_{m=0}^{\infty}(-1)^m\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^m\\ &=2t\sum_{m=0}^{\infty}(-1)^m E\left[e_{\lambda}^{Y_1+Y_2+\cdots+Y+m}(t)\right]\\ &=2t\sum_{m=0}^{\infty}(-1)^m\sum_{n=0}^{\infty}E\left[(Y_1+Y_2+\cdots+Y_m)_{n,\lambda}\right]\frac{t^n}{n!}\\ &=2\sum_{m=0}^{\infty}(-1)^m\sum_{n=1}^{\infty}nE\left[(Y_1+Y_2+\cdots+Y_m)_{n-1,\lambda}\right]\\ &=\sum_{n=1}^{\infty}\left(2n\sum_{m=0}^{\infty}(-1)^m E\left[(Y_1+Y_2+\cdots+Y_m)_{n-1,\lambda}\right]\right)\frac{t^n}{n!},\end{aligned} \tag{2.8}$$

and so by (2.8), we obtain the following theorem.

Theorem 2.6. *For each positive integer n, we have*

$$G_{n,\lambda}^{Y} = 2n\sum_{m=0}^{\infty}(-1)^m E\left[(S_m)_{n-1,\lambda}\right].$$

In particular, if we put $Y = 1$, then

$$G_{n,\lambda} = 2n\sum_{m=0}^{\infty}(-1)^m\,(m)_{n-1,\lambda}\,.$$

Let n be a positive odd integer. Then

$$\begin{aligned}2t\sum_{k=0}^{n-1}(-1)^k\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^k &= \frac{2t}{E\left[e_{\lambda}^{Y}(t)\right]+1}\left(1+\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^n\right)\\ &=\sum_{m=0}^{\infty}\left(G_{m,\lambda}^{Y}+G_{m,\lambda}^{Y}(n)\right)\frac{t^m}{m!},\end{aligned} \tag{2.9}$$

and

$$\begin{aligned}2t\sum_{k=0}^{n}(-1)^k\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^k &=2t\sum_{k=0}^{n}(-1)^k E\left[e_{\lambda}^{dY_1+Y_2+\cdots+Y_k}(t)\right]\\ &=2t\sum_{k=0}^{n}(-1)^k\sum_{m=0}^{\infty}E\left[(Y_1+Y_2+\cdots+Y_k)_{m,\lambda}\right]\frac{t^m}{m!}\\ &=\sum_{m=1}^{\infty}2m\sum_{k=0}^{n}(-1)^k E\left[(S_k)_{m-1,\lambda}\right]\frac{t^m}{m!}.\end{aligned} \tag{2.10}$$

By (2.9) and (2.10), we obtain the following theorem.

Theorem 2.7. *Let n be a positive odd integer. Then $G_{0,\lambda}^{Y} = -G_{0,\lambda}^{Y}(n)$ and for each positive integer m, we have*

$$\sum_{k=0}^{n}(-1)^k E\left[(S_k)_{m-1,\lambda}\right] = \frac{G_{m,\lambda}^{Y}+G_{m,\lambda}^{Y}(n)}{2m}.$$

In particular, if we put $Y = 1$, then

$$\sum_{k=0}^{n}(-1)^k(k)_{m-1,\lambda} = \frac{G_{m,\lambda} + G_{m,\lambda}}{2m}.$$

A continuous random variable Y whose density function is given by

$$f(y) = \begin{cases} \beta e^{-\beta y}\frac{(\beta y)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } y \geq 0, \\ 0 & \text{if } y < 0, \end{cases}$$

for some $\alpha, \beta > 0$ is called the *gamma random variable* with parameters α, β, which is denoted by $Y \sim \Gamma(\alpha, \beta)$.

Let $Y \sim \Gamma(1, 1)$. Then

$$\begin{aligned} E\left[e_\lambda^Y(t)\right] &= \int_0^\infty e^{-y}e_\lambda^y(t)dy = \int_0^\infty e^{-y\left(1-\frac{1}{\lambda}\log(1+\lambda t)\right)}dy \\ &= \frac{1}{1-\frac{1}{\lambda}\log(1+\lambda t)}, \end{aligned} \tag{2.11}$$

and so by (1.3), we see that

$$\begin{aligned} \sum_{n=0}^{\infty} G_{n,\lambda}^Y \frac{t^n}{n!} &= \frac{2t}{E\left[e_\lambda^Y(t)\right]+1} = \frac{2t\left(1-\frac{1}{\lambda}\log(1+\lambda t)\right)}{2-\frac{1}{\lambda}\log(1+\lambda t)} \\ &= t\left(\frac{1}{1-\frac{1}{2\lambda}\log(1+\lambda t)} - 2\frac{\frac{1}{2\lambda}\log(1+\lambda t)}{1-\frac{1}{2\lambda}\log(1+\lambda t)}\right) \\ &= t\left(\sum_{n=0}^{\infty}\left(\frac{1}{2\lambda}\log(1+\lambda t)\right)^n - 2\sum_{n=0}^{\infty}\left(\frac{1}{2\lambda}\log(1+\lambda t)\right)^{n+1}\right) \\ &= t\left(2 - \sum_{n=0}^{\infty}\left(\frac{1}{2\lambda}\right)^n(\log(1+\lambda t))^n\right) \\ &= 2t - t\sum_{n=0}^{\infty}\frac{n!}{(2\lambda)^n}\frac{1}{n!}(\log(1+\lambda t))^n \\ &= 2t - \sum_{n=1}^{\infty}\sum_{m=0}^{n-1}\frac{n\lambda^{n-1}m!}{(2\lambda)^m}S_1(n-1,m)\frac{t^n}{n!} \end{aligned} \tag{2.12}$$

By (2.12), we obtain the following theorem.

Theorem 2.8. *Let $Y \sim \Gamma(1, 1)$. For each nonnegative integer n, we have*

$$G_{n,\lambda}^Y = 2\delta_{1,n} - \sum_{m=0}^{n-1}\frac{n\lambda^{n-1}m!}{(2\lambda)^m}S_1(n-1,m).$$

Let Y be the Bernoulli random variable with probability success p. Note that

$$E\left[e_\lambda^Y(t)\right] = p(e_\lambda(t)-1)+1. \tag{2.13}$$

By (2.13), we get

$$
\begin{aligned}
\sum_{n=0}^{\infty} G_{n,\lambda}^{Y} \frac{t^n}{n!} =& \frac{2t}{E\left[e_{\lambda}^{Y}(t)\right]+1} \\
=& \frac{2t}{p\left(e_{\lambda}(t)-1\right)+2} = \frac{t}{1-\left(-\frac{p}{2}\left(e_{\lambda}(t)-1\right)\right)} \\
=& t\sum_{n=0}^{\infty}\left(-\frac{p}{2}\right)^{n} n!\frac{1}{n!}\left(e_{\lambda}(t)-1\right)^{n} \\
=& t\sum_{n=0}^{\infty}\left(-\frac{p}{2}\right)^{n} n!\sum_{l=n}^{\infty} S_{2,\lambda}(l,n)\frac{t^l}{l!} \\
=& t\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(-\frac{p}{2}\right)^{m} m! S_{2,\lambda}(n,m)\frac{t^n}{n!} \\
=& \sum_{n=1}^{\infty}\sum_{m=0}^{n-1} n\left(-\frac{p}{2}\right)^{m} m! S_{2,\lambda}(n-1,m)\frac{t^n}{n!},
\end{aligned}
\tag{2.14}
$$

and by (2.14), we obtain the following theorem.

Theorem 2.9. *Let Y be the Bernoulli random variable with probability success p. For each positive integer n, we have*

$$
G_{n,\lambda}^{Y} = n\sum_{m=0}^{n-1}\left(-\frac{p}{2}\right)^{m} m! S_{2,\lambda}(n-1,m).
$$

Let Y be the Poisson random variable with parameter $\alpha > 0$. Note that

$$
E\left[e_{\lambda}^{Y}(t)\right] = \sum_{y} e_{\lambda}^{y}(t) e^{-\alpha}\frac{\alpha^y}{y!} = e^{-\alpha}\sum_{y}\frac{(\alpha e_{\lambda}(t))^y}{y!} = e^{\alpha(e_{\lambda}(t)-1)}. \tag{2.15}
$$

By (2.15), we get

$$
\begin{aligned}
\sum_{n=0}^{\infty} G_{n,\lambda}^{Y}(x)\frac{t^n}{n!} =& \frac{2t}{E\left[e_{\lambda}^{Y}(t)\right]+1}\left(E\left[e_{\lambda}^{Y}(t)\right]\right)^{x} \\
=& \frac{2t}{e^{\alpha(e_{\lambda}(t)-1)}+1} e^{\alpha x(e_{\lambda}(t)-1)} \\
=& \frac{t}{\alpha\left(e_{\lambda}(t)-1\right)}\frac{2\alpha\left(e_{\lambda}(t)-1\right)}{e^{\alpha(e_{\lambda}(t)-1)}+1} e^{\alpha x(e_{\lambda}(t)-1)} \\
=& \frac{1}{\alpha}\left(\sum_{n=0}^{\infty} B_{n,\lambda}\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty} G_n\frac{\alpha^n}{n!}\left(e_{\lambda}(t)-1\right)^n\right)\left(\sum_{n=0}^{\infty} Bel_{n,\lambda}(\alpha x)\frac{t^n}{n!}\right) \\
=& \frac{1}{\alpha}\left(\sum_{n=0}^{\infty}\sum_{m=0}^{n}\binom{n}{m} B_{n-m,\lambda} Bel_{m,\lambda}(\alpha x)\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}\alpha^n G_n\sum_{l=n}^{\infty} S_{2,\lambda}(l,n)\frac{t^l}{l!}\right) \\
=& \frac{1}{\alpha}\sum_{n=0}^{\infty}\left(\sum_{a=0}^{n}\sum_{m=0}^{a}\sum_{r=0}^{n-a}\binom{n}{a}\binom{a}{m}\alpha^r G_r S_{2,\lambda}(n-a,r) B_{a-m,\lambda} Bel_{m,\lambda}(\alpha x)\right)\frac{t^n}{n!}.
\end{aligned}
\tag{2.16}
$$

Moreover, by (2.16)

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda}^{Y}(x)\frac{t^n}{n!} =&\frac{2t}{e^{\alpha(e_\lambda(t)-1)}+1}e^{\alpha x(e_\lambda(t)-1)}\\ =&\frac{1}{\alpha}\left(\sum_{n=0}^{\infty} B_{n,\lambda}\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}\alpha^n G_n(x)\frac{1}{n!}\left((e_\lambda-1)\right)^n\right)\\ =&\frac{1}{\alpha}\left(\sum_{n=0}^{\infty} B_{n,\lambda}\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}\sum_{m=0}^{n} G_m(x)\alpha^m S_{2,\lambda}(n,m)\frac{t^n}{n!}\right)\\ =&\frac{1}{\alpha}\sum_{n=0}^{\infty}\left(\sum_{r=0}^{n}\sum_{m=0}^{r}\binom{n}{r}\alpha^m S_{2,\lambda}(r,m)B_{n-r,\lambda}G_m(x)\right)\frac{t^n}{n!}.\end{aligned} \tag{2.17}$$

By (2.16) and (2.17), we obtain the following theorem.

Theorem 2.10. *Let Y be the Poisson random variable with parameter $\alpha > 0$. For each nonnegative integer n, we have*

$$\begin{aligned}\alpha G_{n,\lambda}^{Y}(x) =&\sum_{a=0}^{n}\sum_{m=0}^{a}\sum_{r=0}^{n-a}\binom{n}{a}\binom{a}{m}\alpha^r G_r S_{2,\lambda}(n-a,r)B_{a-m,\lambda}Bel_{m,\lambda}(\alpha x)\\ =&\sum_{r=0}^{n}\sum_{m=0}^{r}\binom{n}{r}\alpha^m S_{2,\lambda}(r,m)B_{n-r,\lambda}G_m(x).\end{aligned}$$

Let Y be the binomial distribution with parameters m and p. Then

$$E\left[e_\lambda^Y(t)\right] = \left(p\left(e_\lambda(t)-1\right)+1\right)^m. \tag{2.18}$$

By (2.18), we get

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda}^{Y}\frac{t^n}{n!} =&\frac{2t}{E\left[e_\lambda^Y(t)\right]+1} = \frac{2t}{(p(e_\lambda(t)-1)+1)^m+1}\\ =&2t\sum_{l=0}^{\infty}(-1)^l\left((e_\lambda(t)-1)+1\right)^{ml}\\ =&2t\sum_{l=0}^{\infty}(-1)^l\sum_{a=0}^{ml}\binom{ml}{a}p^a(e_\lambda(t)-1)^a\\ =&2t\sum_{l=0}^{\infty}(-1)^l\sum_{a=0}^{\infty}(ml)_a p^a\frac{1}{a!}(e_\lambda(t)-1)^a\\ =&2t\sum_{l=0}^{\infty}(-1)^l\sum_{a=0}^{\infty}(ml)_a p^a\sum_{r=a}^{\infty}S_{2,\lambda}(r,a)\frac{t^r}{r!}\\ =&2t\sum_{l=0}^{\infty}(-1)^l\sum_{a=0}^{\infty}\sum_{b=0}^{a}(ml)_b p^b S_{2,\lambda}(a,b)\frac{t^a}{a!}\\ =&\sum_{n=1}^{\infty}2\sum_{l=0}^{\infty}\sum_{b=0}^{n-1}(-1)^l(ml)_b p^b n S_{2,\lambda}(n-1,b)\frac{t^n}{n!}.\end{aligned} \tag{2.19}$$

By (2.19), we obtain the following theorem.

Theorem 2.11. *Let Y be the binomial distribution with parameter m and p. For each positive integer n, we have*

$$G_{n,\lambda} = 2\sum_{l=0}^{\infty}\sum_{b=0}^{n-1}(-1)^l (ml)_b p^b n S_{2,\lambda}(n-1,b)\frac{t^n}{n!}.$$

Let Y be the geometric distribution with probability success p. Then

$$E\left[e_\lambda^Y(t)\right] = \frac{pe_\lambda(t)}{1-qe_\lambda(t)},\ p+q=1. \tag{2.20}$$

By (2.20), we get

$$\begin{aligned}\sum_{n=0}^{\infty}G_{n,\lambda}\frac{t^n}{n!} &= \frac{2t(1-qe_\lambda(t))}{(2p-1)e_\lambda(t)+1}\\ &= \frac{2t}{2p-1}(1-qe_\lambda(t))\sum_{l=0}^{\infty}(-1)^l e_\lambda^l(t)\\ &= \frac{2t}{2p-1}\left(\sum_{l=0}^{\infty}(-1)^l e_\lambda^l(t) - q\sum_{l=0}^{\infty}(-1)^l e_\lambda^{l+1}(t)\right)\\ &= \sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{2(-1)^l}{2p-1}\left((l)_{n,\lambda}-q(l+1)_{n,\lambda}\right)\frac{t^{n+1}}{n!}\\ &= \sum_{n=1}^{\infty}\sum_{l=0}^{\infty}\frac{2n(-1)^l}{2p-1}\left((l)_{n-1,\lambda}-q(l+1)_{n-1,\lambda}\right)\frac{t^n}{n!}.\end{aligned} \tag{2.21}$$

By (2.21), we obtain the following theorem.

Theorem 2.12. *Let Y be the geometric distribution with probability success p. For each positive integer n, we have*

$$G_{n,\lambda} = \sum_{l=0}^{\infty}\frac{2n(-1)^l}{2p-1}\left((l)_{n-1,\lambda}-q(l+1)_{n-1,\lambda}\right).$$

3. The properties of $G_{n,\lambda}^{Y_s}(x)$

The polynomial $G_{n,\lambda}^{Y_s}(x)$, $s=g,b,p$ can be explicitly expressed by using Mathematica. For example, the polynomials with $n=1,2,3,4$ are listed as following

$$\begin{aligned}
&G_{1,\lambda}^{Y_g}(x) = 1,\\
&G_{2,\lambda}^{Y_g}(x) = 2x+1,\\
&G_{3,\lambda}^{Y_g}(x) = 3x^2-3\lambda x+\frac{3}{2}(\lambda-1),\\
&G_{4,\lambda}^{Y_g}(x) = 4x^3+(6-12\lambda)x^2+\left(-4+8\lambda^2\right)x+\left(-3+6\lambda-4\lambda^2\right).\\
&G_{1,\lambda}^{Y_b}(x) = 1,\\
&G_{2,\lambda}^{Y_b}(x) = p(2x-1),\\
&G_{3,\lambda}^{Y_b}(x) = 3p^2x^2+3p(1-2p-\lambda)x+\frac{3p}{2}(\lambda+p-1),\\
&G_{4,\lambda}^{Y_b}(x) = 4p^3x^3+6p^2(2-3p-2\lambda)x^2
\end{aligned}$$

$$+4p\left(1-6p+5p^2-3\lambda+6p\lambda+2\lambda^2\right)x+p\left(-2+6p-3p^2+6\lambda-6p\lambda-4\lambda^2\right).$$
$$G_{1,\lambda}^{Y_p}(x)=1,$$
$$G_{2,\lambda}^{Y_p}(x)=\alpha(2x-1),$$
$$G_{3,\lambda}^{Y_p}(x)=3\alpha^2x^2+3\alpha\left(1-\alpha-\lambda\right)x+\frac{3\alpha}{2}\left(\lambda-1\right),$$
$$G_{4,\lambda}^{Y_p}(x)=4\alpha^3x^3+6\alpha^2\left(2-\alpha-2\lambda\right)x^2$$
$$+4\alpha\left(1-3\alpha-3\lambda+3\alpha\lambda+2\lambda^2\right)x+\alpha\left(-2+\alpha^2+6\lambda-4\lambda^2\right).$$

where $Y_g \sim \Gamma(1,1)$, Y_b is the Bernoulli random variable with probability success p and Y_p is the Poisson random variable with parameter α.

To investigate the numerical patten of the root of the polynomials $G_{n,\lambda}^{Y_s}(x)$, $s=g,b,p$, we compute there roots by using the Mathematica with 100 working precision, for $n=20,40,60$ and $\lambda=0.2,0.4,1.0$. Here, we take $\alpha=0.2$ and $p=0.5$. The numerical results are plotted in Fig.1-3. Studying how many real roots a polynomial has is also important in algebra. The number of real roots of the polynomials $G_{n,\lambda}^{Y_s}(x)$, $s=g,b,p$ are counted for $n=20,40,60$ and $\lambda=0.2,0.4,1.0$. The results are shown in Table 1-Table 3. Observing the results in Table 1-Table 3 and Fig. 1-3, the polynomials $G_{n,\lambda}^{Y_s}(x)$ have the following properties:

- The degree of polynomials $G_{n,\lambda}^{Y_s}(x)$ is $n-1$.

- The distributions of the roots of the polynomials $G_{n,\lambda}^{Y_s}(x)$, $s=g,b,p$ are symmetrical with respect to the $x-$ axis.

- Both polynomials $G_{n,\lambda}^{Y_g}(x)$ and $G_{n,\lambda}^{Y_p}(x)$ have a similar property, i.e. polynomials with smaller λ have more real roots.

- $G_{n,\lambda}^{Y_b}(x)$ with larger λ has more real roots.

Table 1: The number of real roots of polynomial $G_{n,\lambda}^{Y_g}(x)$.

	$\lambda=0.2$	$\lambda=0.4$	$\lambda=1.0$
$n=20$	7	3	1
$n=40$	5	5	1
$n=60$	7	3	3

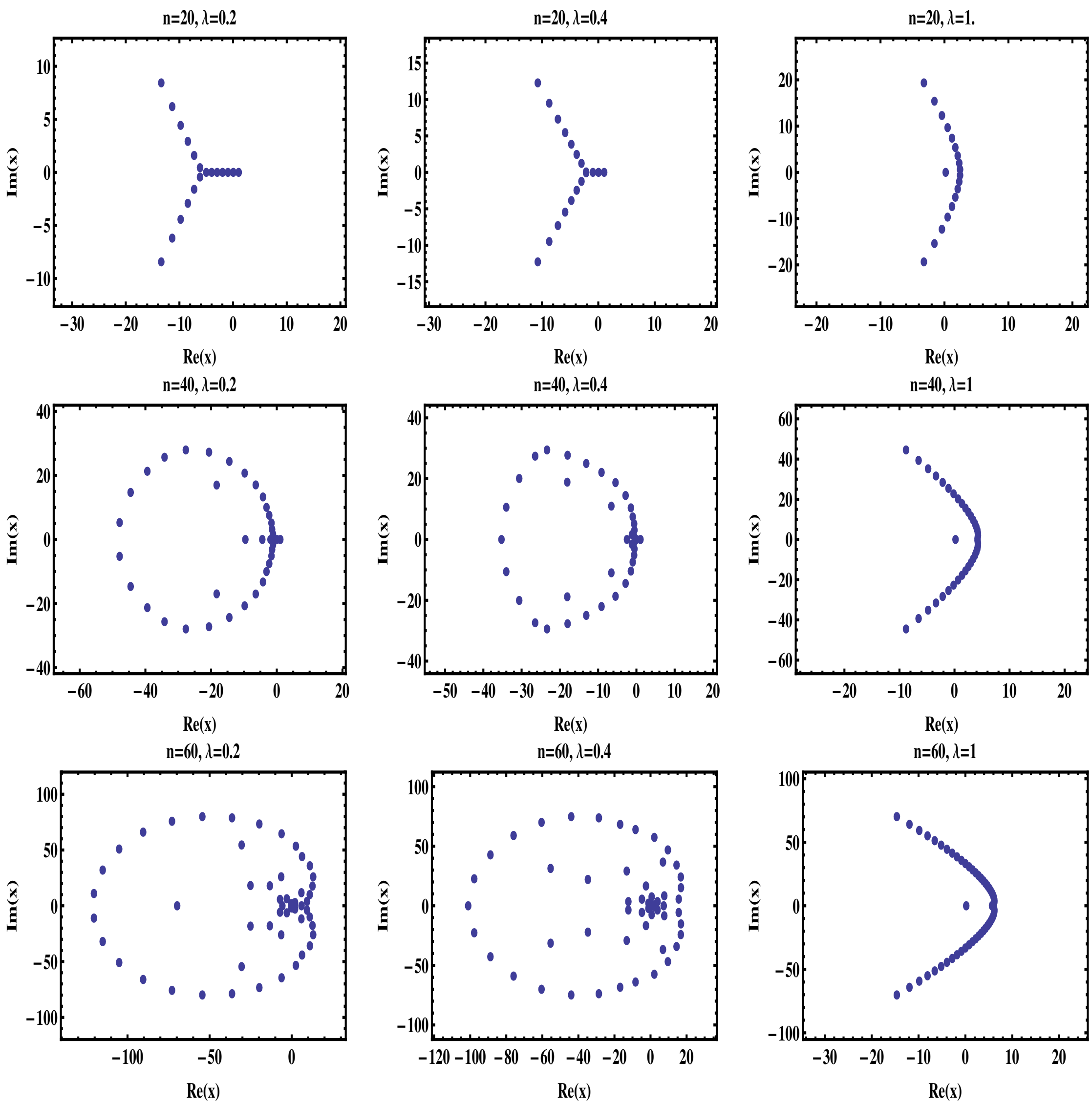

Figure 1: The zeros of the polynomials $G_{n,\lambda}^{Y_g}(x)$ varying $n = 20, 40, 60$ and $\lambda = 0.2, 0.4, 1.0$.

Table 2: The number of real roots of polynomial $G_{n,\lambda}^{Y_b}(x)$.

	$\lambda = 0.2$	$\lambda = 0.4$	$\lambda = 1.0$
$n = 20$	13	15	19
$n = 40$	19	17	39
$n = 60$	7	11	59

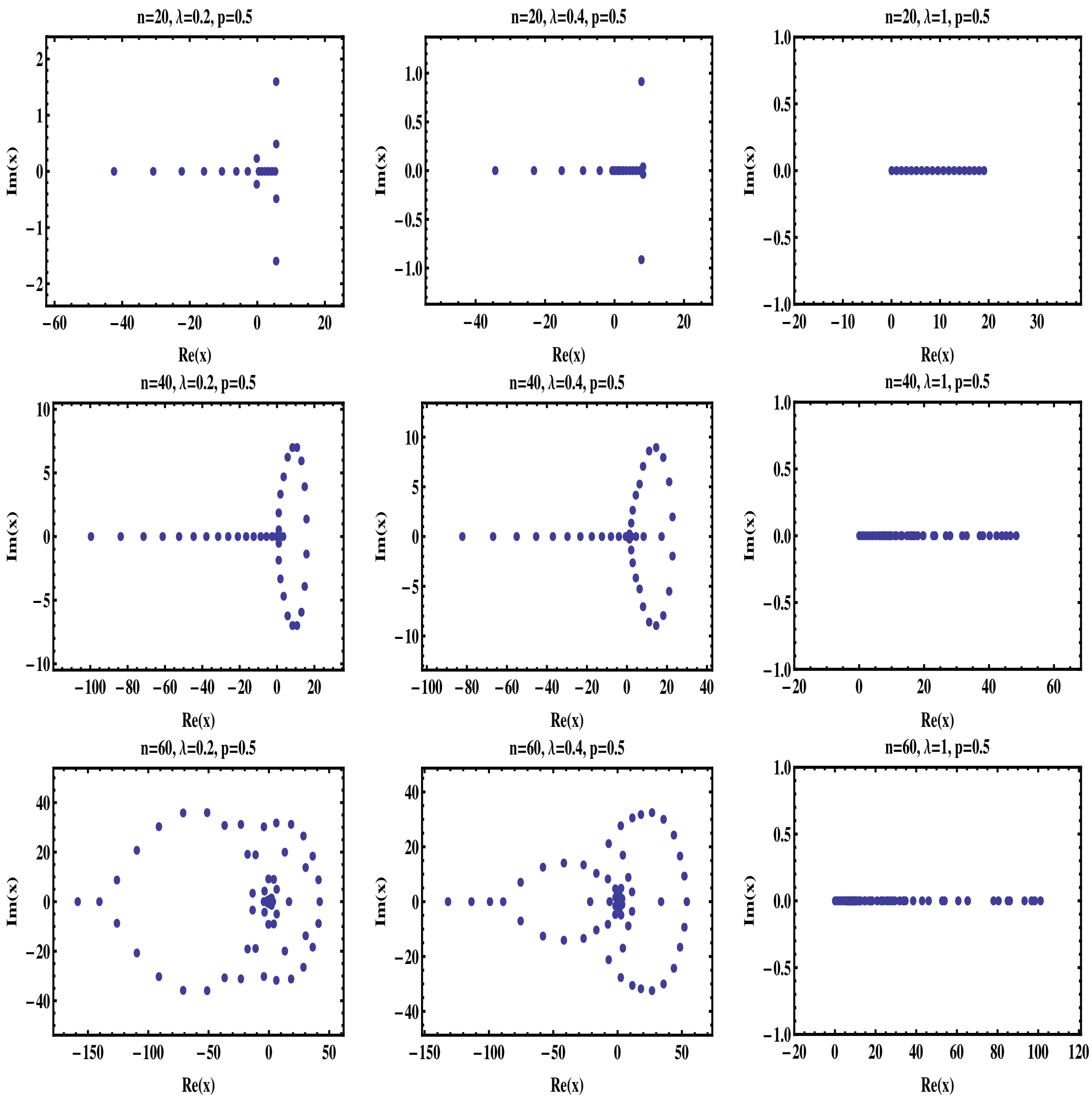

Figure 2: The zeros of the polynomials $G_{n,\lambda}^{Y_b}(x)$ varying $n = 20, 40, 60$ and $\lambda = 0.2, 0.4, 1.0$.

Table 3: The number of real roots of polynomial $G_{n,\lambda}^{Y_p}(x)$.

	$\lambda = 0.2$	$\lambda = 0.4$	$\lambda = 1.0$
$n = 20$	17	11	7
$n = 40$	31	19	5
$n = 60$	11	7	5

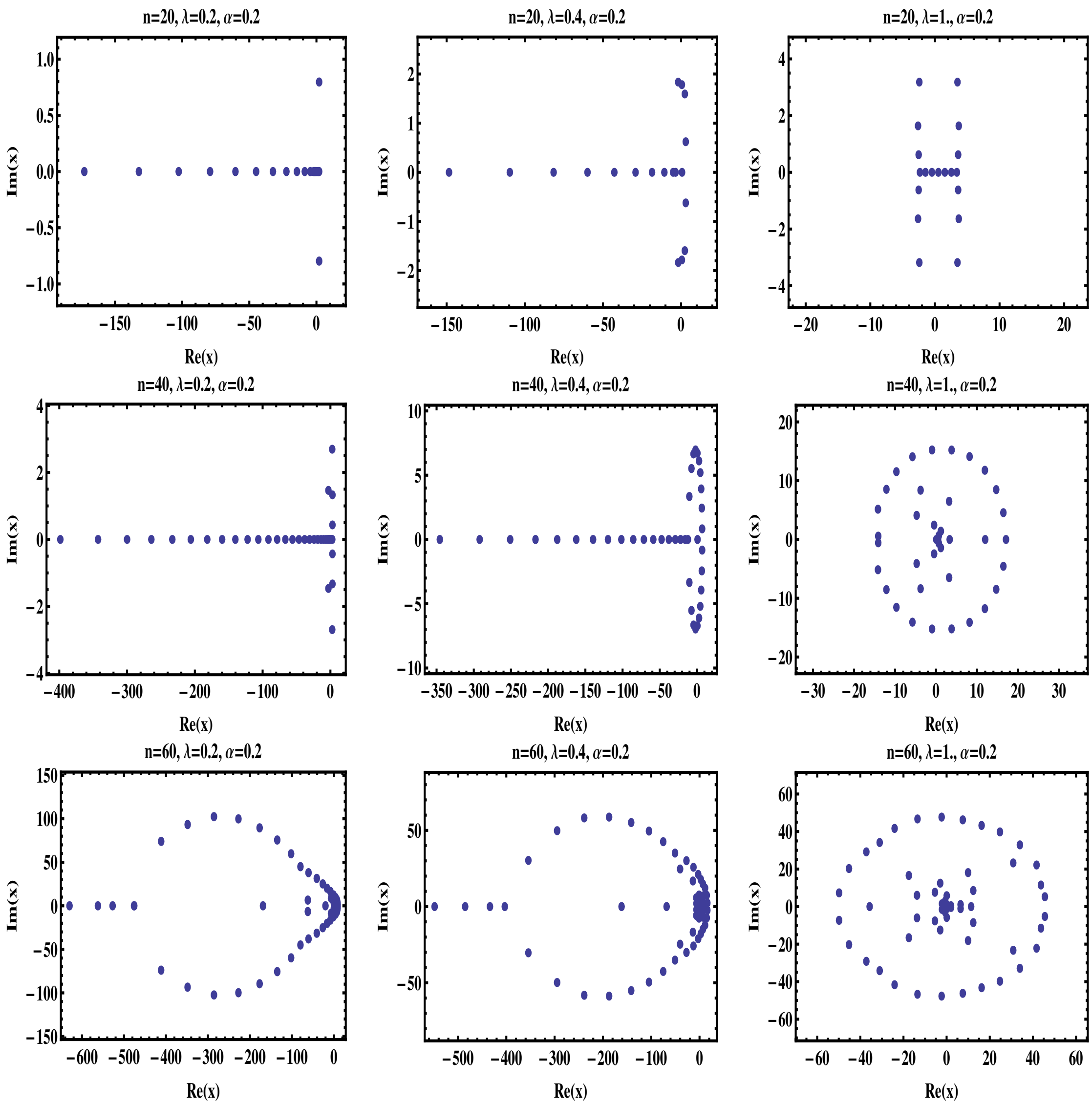

Figure 3: The zeros of the polynomials $G^{Y_p}_{n,\lambda}(x)$ varying $n = 20, 40, 60$ and $\lambda = 0.2, 0.4, 1.0$.

4. Conclusion

Let Y be a random variable whose moment generating function, $E\left[e^{Yt}\right] = \sum_{n=0}^{\infty} E\left[Y^n\right] \frac{t^n}{n!}$, $(|t| < r)$. The aim of this paper is to introduce the probabilistic extension of degenerate Genocchi polynomials, namely the probabilistic degenerate Genocchi polynomials associated with random variable Y. We obtained several properties, explicit expressions, recurrence relations and certain identities for those polynomials and numbers. As the some special cases of random variable Y, we dealt with the gamma random variable with parameters $\alpha, \beta > 0$, the Bernoulli random variable with probability of success p and the Poisson random variable with parameter $\alpha > 0$. In a concrete form, we obtained several explicit expressions for $G^{Y}_{n,\lambda}(x)$ related to the probabilistic Bernoulli polynomials associated with Y(see Theorem 2.3), the probabilistic Stirling numbers of the second kind of the second kind associated with Y(see Theorem 2.1, 2.2, 2.4) and the probabilistic Bell polynomials associated with Y(see Theorem 2.10). Further more, we derived three identities about probabilistic degenerate extensions of recurrence relations and $S_k = Y_1 + Y_2 + \cdots + Y_k$, $k \in \mathbb{N}$ (see Theorems 2.5- 2.7). We represented explicit expressions $G^Y_{n,\lambda}(x)$ when $Y \sim \Gamma(1, 1)$ in Theorem 2.8 and Y is the Bernoulli random variable with probability of success p in Theorem 2.9 and Y is the Poisson random variable with parameter $\alpha > 0$ in Theorem 2.10. We investigated an explicit expression and the numerical pattern of the roots for the polynomials $G^{Y_s}_{n,\lambda}(x)$, $s = g, b, p$. Also, we compute their roots by using the Mathematica with 100 working precision, for $n = 20, 40, 60$, $\lambda = 0.2, 0.4, 1.0$, $\alpha = 0.2$ and $p = 0.5$ and showed the results in Table 1,2,3. As one of our future projects, we would like to continue to study probabilistic versions of Riemann-Zeta functions and L functions.

Acknowledgements

The authors would like to thank the referees for their valuable and detailed comments which have significantly improved the presentation of this paper.

Author Contributions

All authors have same contributions.

Conflicts of Interest

There is no conflict of interest as regard this work.

Data availability

Data sharing is not applicable to this article as no data sets were generated or analyzed during this study.

Funding

We received no funding for this work.

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