Filomat 40:8 (2026), 2779–2785
https://doi.org/10.2298/FIL2608779M

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Bernstein-type inequalities for $\mathcal{N}$-operator

Abdullah Mir[a,*], Wasim Ahmad Thoker[a]

[a] *Department of Mathematics, University of Kashmir, Srinagar-190006, Jammu and Kashmir, India*

Abstract. In this paper, we investigate various comparison inequalities for a certain class of linear differential operators acting on complex polynomials. We introduce a generalized operator, called the $\mathcal{N}$-operator, which encompasses extensions of several polynomial inequalities and specializes to the classical B-operator under suitable parameter selections. Assuming natural constraints on the zeros of the involved polynomials, we establish new Bernstein-type inequalities in the uniform norm that compare the action of $\mathcal{N}$ on two polynomials, potentially of different degrees. Our results generalize and sharpen classical inequalities such as those of Erdős–Lax and Ankeny–Rivlin, while also relaxing the assumptions made in earlier works, including those by Rather, Gulzar, Mir and others. Several known results are recovered as special cases, and our framework highlights the analytic behavior of polynomials under zero-preserving differential operators.

1. Introduction and Preliminaries

The investigation of comparison inequalities involving the norms of polynomials on a disc is a well-established area in analysis, largely due to its wide-ranging applications in geometric function theory. Among these, Bernstein-type inequalities providing norm estimates that extend classical results for polynomials are particularly prominent (see, e.g., [1, 2, 6, 7, 10, 13, 15, 16, 18, 21]). An effective strategy is to examine linear operators that maintain or strengthen these inequalities. In particular, the $\mathcal{B}_n$-operators introduced by Rahman [19] have served to unify various Bernstein-type inequalities. In this paper, we extend this framework by establishing new inequalities for a broader class of B_n-operators, relating their action on polynomials to the maximum norm on the unit disk.

To proceed with our results, we begin by recalling some basic notations and definitions. We consider the space $\mathcal{P}_n$ consisting of all complex polynomials with degree at most n. For a polynomial $P \in \mathcal{P}_n$, let P' denote its derivative and P^v its v-th derivative, with the convention $P^0 = P$.

A fundamental result due to Bernstein [4] asserts that if two polynomials $f,\ h \in \mathcal{P}_n$ with $\deg f \le \deg h$, satisfy $|f(z)| \le |h(z)|$ on the unit circle $|z| = 1$ and $h(z)$ does not vanish for $|z| > 1$, then the same inequality holds for their derivatives as well. That is

$$|f'(z)| \le |h'(z)|, \quad |z| = 1.$$

2020 *Mathematics Subject Classification*. Primary 30A10; Secondary 30C10, 30C15.
Keywords. Complex polynomials, B_n-operator, Zeros, Bernstein inequality.
Received: 25 June 2025; Accepted: 12 January 2026
Communicated by Dragan S. Djordjević
This research was supported by the NBHM (R.P), Department of Atomic Energy, GoI (No. 02011/19/2022/R&D-II/10212).
* Corresponding author: Abdullah Mir
Email addresses: drabmir@yahoo.com (Abdullah Mir), thoker.wasim.313@gmail.com (Wasim Ahmad Thoker)
ORCID iDs: https://orcid.org/0000-0003-0930-6391 (Abdullah Mir), https://orcid.org/0009-0000-2981-6896 (Wasim Ahmad Thoker)

From this, the classical Bernstein inequality [3] follows: if $P \in \mathcal{P}_n$, then

$$\max_{|z|=1} |P'(z)| \le n \max_{|z|=1} |P(z)|. \tag{1}$$

Equality holds in (1) for $P(z) = \alpha z^n$, where α is a complex number.
Beyond the unit circle, the Bernstein-Walsh lemma [20] provides an important estimate. It says that if $f, h \in \mathcal{P}_n$ be such that $\deg f \le \deg h$ and $h(z)$ does not vanish for $|z| > 1$ with $|f(z)| \le |h(z)|$ for $|z| = 1$, then $|f(z)| < |h(z)|$ for $|z| > 1$ unless $f(z) = e^{i\theta} g(z), \theta \in \mathbb{R}$. From this it can be easily deduced that for $P \in \mathcal{P}_n$ and $R \ge 1$,

$$\max_{|z|=R} |P(z)| \le R^n \max_{|z|=1} |P(z)|. \tag{2}$$

Govil, Qazi, and Rahman [9] established the equivalence of inequalities (1) and (2). Moreover, for polynomials that are zero-free within the unit disk, stronger bounds are available. Indeed, for such polynomials, Erdős conjectured and later Lax [11] confirmed that

$$\max_{|z|=1} |P'(z)| \le \frac{n}{2} \max_{|z|=1} |P(z)|, \tag{3}$$

whereas Ankeny and Rivlin [1] established

$$\max_{|z|=R} |P(z)| \le \frac{R^n + 1}{2} \max_{|z|=1} |P(z)|, \quad R \ge 1. \tag{4}$$

Over the past several decades, a wide range of generalizations of these inequalities have been developed by mathematicians, extending their applicability to various norms and classes of functions. This kind of research has given rise to a rich theoretical framework with significant applications in approximation theory, complex analysis, and operator theory. Prominent contributions in this area can be found in the comprehensive books of of Marden [12], Milovanović et al. [14], and Rahman and Schmeisser [20], among others. The classical inequalities (3) and (4) along with their extensions form the basis for the more general class of operator inequalities studied in this paper.
Rahman [19] introduced the class $\mathcal{B}_n$ of linear operators $B : \mathcal{P}_n \to \mathcal{P}_n$, which preserve specific polynomial inequalities, especially those concerning the distribution of zeros. More recently, Rather et al. [21] studied a generalized linear operator $\mathcal{N}$, defined by

$$\mathcal{N}[P](z) := \sum_{v=0}^{s} \lambda_v \left(\frac{nz}{2}\right)^v \frac{P^v(z)}{v!}, \tag{5}$$

where the coefficients λ_v, for $v = 0, 1, \ldots, s$, are chosen such that the polynomial

$$\phi(z) := \sum_{v=0}^{s} \binom{n}{v} \lambda_v z^v, \qquad s \le n \tag{6}$$

has all its zeros in the half-plane

$$|z| \le \left| z - \frac{n}{2} \right|. \tag{7}$$

It is important to observe that when $\lambda_v = 0$ for all $v \ge 3$, the $\mathcal{N}$-operator simplifies to the classical B-operator introduced by Rahman. They further established the following result:

Theorem 1.1. If $f(z)$ is a polynomial of degree n with all zeros in $|z| \le 1$, and $P \in \mathcal{P}_n$ such that $|P(z)| \le |f(z)|$ on $|z| = 1$, then

$$|\mathcal{N}[P](z)| \le |\mathcal{N}[f](z)| \quad \text{for } |z| \ge 1.$$

Equality holds for $P(z) = e^{i\gamma} f(z), \gamma \in \mathbb{R}$.

Recently, Mir [15] obtained the following generalization of Theorem 1.1:

Theorem 1.2. If $f(z)$ is a polynomial of degree n with all zeros in $|z| \leq 1$, and $P \in \mathcal{P}_n$ such that $|P(z)| \leq |f(z)|$ on $|z| = 1$, then for any complex number α with $|\alpha| \leq 1$, and $R \geq r \geq 1$, we have

$$|\mathcal{N}[P(Rz)] - \alpha\mathcal{N}[P(rz)]| \leq |\mathcal{N}[f(Rz)] - \alpha\mathcal{N}[f(rz)]| \quad \text{for } |z| \geq 1.$$

Equality holds for $P(z) = e^{i\gamma} f(z)$, $\gamma \in \mathbb{R}$.

In this paper, we further explore a range of comparison inequalities for the linear differential $\mathcal{N}$-operator acting on complex polynomials, extending and unifying several classical Bernstein-type inequalities. By imposing appropriate conditions on the zero distribution of the underlying polynomials, we establish new inequalities involving the supremum norms associated with this operator. These results not only include classical inequalities as special cases but also provide new insights into the behavior of polynomials under the action of such operators.

2. Main Results

We begin by introducing the operator $\mathcal{T} : \mathcal{P}_n \to \mathcal{P}_n$ that depends on several parameters and extends $\mathcal{N}$-operator introduced in previous section.

Definition 2.1. *Let $P \in \mathcal{P}_n$, $R \geq r \geq 1$ and the complex numbers α, β are such that $|\alpha| \leq 1$ and $|\beta| \leq 1$ and the operator $\mathcal{N}$ is defined by* (5). *We introduce the operator $\mathcal{T} \equiv \mathcal{T}(\alpha, \beta, R, r) : \mathcal{P}_n \to \mathcal{P}_n$ by*

$$\mathcal{T}[P](z) = \mathcal{N}[P(Rz)] + \psi(\alpha, \beta, R, r)\mathcal{N}[P(rz)] \tag{8}$$

where

$$\psi(\alpha, \beta, R, r) = \beta\left\{\left(\frac{1+R}{1+r}\right)^n - |\alpha|\right\} - \alpha. \tag{9}$$

The first result generalizes Theorems 1.1 and 1.2 and includes (2) as a special case.

Theorem 2.2. If $f(z)$ is a polynomial of degree n having all its zeros in $|z| \leq 1$ and $P \in \mathcal{P}_m, m \leq n$ such that $|P(z)| \leq |f(z)|$ for $|z| = 1$, then for every $|\alpha| \leq 1$, $|\beta| \leq 1$, and $R \geq r \geq 1$, we have

$$\left|\mathcal{T}[P](z) + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha, \beta, R, r)P^v(rz)}{v!} \frac{(n^v - m^v)}{2^v} z^v\right| \leq \left|\mathcal{T}[f](z)\right| \quad \text{for } |z| \geq 1, \tag{10}$$

Equality in (10) holds for $P(z) = e^{i\gamma} f(z)$, $\gamma \in \mathbb{R}$.

Setting $r = 1$ in (10) and using (8) and (9), the following result follows immediately from Theorem 2.2.

Corollary 2.3. If $f(z)$ is a polynomial of degree n having all its zeros in $|z| \leq 1$ and $P \in \mathcal{P}_m, m \leq n$ such that $|P(z)| \leq |f(z)|$ for $|z| = 1$, then for every $|\alpha| \leq 1$, $|\beta| \leq 1$, and $R \geq 1$, we have

$$\left|\mathcal{T}[P](z) + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha, \beta, R, 1)P^v(z)}{v!} \frac{(n^v - m^v)}{2^v} z^v\right| \leq \left|\mathcal{T}[f](z)\right| \quad \text{for } |z| \geq 1. \tag{11}$$

where

$$\mathcal{T}[P](z) = \mathcal{N}[P(Rz)] + \psi(\alpha, \beta, R, 1)\mathcal{N}[P(z)] = \mathcal{N}[P(Rz)] + \left[\beta\left\{\left(\frac{1+R}{2}\right)^n - |\alpha|\right\} - \alpha\right]\mathcal{N}[P(z)].$$

Equality in (11) holds for $P(z) = e^{i\gamma} f(z)$, $\gamma \in \mathbb{R}$.

By setting $m = n$ in Theorem 2.2, we obtain the following result (see also [17], Theorem 3).

Corollary 2.4. If $f(z)$ is a polynomial of degree n with all zeros in $|z| \leq 1$, and $P \in \mathcal{P}_n$ such that $|P(z)| \leq |f(z)|$ on $|z| = 1$, then for every $|\alpha| \leq 1, |\beta| \leq 1$, and $R \geq r \geq 1$, we have

$$|\mathcal{T}[P](z)| \leq |\mathcal{T}[f](z)| \quad \text{for } |z| \geq 1. \tag{12}$$

Equality holds in (12) for $P(z) = e^{i\gamma} f(z), \gamma \in \mathbb{R}$.

Remark 2.5. For $\beta = 0$, Corollary 2.4 reduces to Theorem 1.2. Further if we take $\alpha = \beta = 0$ in (12), we get Theorem 1.1.

If in Theorem 2.2, we take $f(z) = Mz^n$, where $M = \max_{|z|=1} |P(z)|$, then we get the following result.

Corollary 2.6. If $P \in P_n$, then for every $|\alpha| \leq 1, |\beta| \leq 1$, and $R \geq r \geq 1$, we have

$$\begin{aligned}\left|\mathcal{T}[P](z) + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha,\beta,R,r)P^v(rz)}{v!} \frac{(n^v - m^v)}{2^v} z^v\right| \\ \leq \left|R^n + r^n \psi(\alpha,\beta,R,r)\right| \left|\mathcal{N}[\varphi_n(z)]\right| \max_{|z|=1} |P(z)| \quad \text{for } |z| \geq 1,\end{aligned} \tag{13}$$

where $\varphi_n(z) = z^n$. Equality in (13) holds for $P(z) = \gamma z^n, \gamma \neq 0$.

Substituting the value of $\mathcal{N}[\varphi_n(z)]$ in (13), we get for every $|\alpha| \leq 1, |\beta| \leq 1$, and $R \geq r \geq 1$,

$$\begin{aligned}\left|\mathcal{T}[P](z) + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha,\beta,R,r)P^v(rz)}{v!} \frac{(n^v - m^v)}{2^v} z^v\right| \\ \leq \left|R^n + r^n \psi(\alpha,\beta,R,r)\right| |z|^n \left|\sum_{v=0}^{s} \lambda_v \binom{n}{v} \left(\frac{n}{2}\right)^v\right| \max_{|z|=1} |P(z)|,\end{aligned} \tag{14}$$

for $|z| \geq 1$, where $\lambda_v; 0 \leq v \leq s$, are such that all zeros of $\phi(z)$ defined in (6) lie in the half plane (7). Taking $\lambda_v = 0, v = 1, 2, 3, \ldots, s$ in (14) and noting that $\mathcal{N}[P](z) = \lambda_0 P(z)$, we get the following result (see also [17], Corollary 7).

Corollary 2.7. If $P \in P_n$, then for every $|\alpha| \leq 1, |\beta| \leq 1$, and $R \geq r \geq 1$, we have

$$|P(Rz) + \psi(\alpha,\beta,R,r)P(rz)| \leq \left|R^n + r^n \psi(\alpha,\beta,R,r)\right| |z|^n \max_{|z|=1} |P(z)| \quad \text{for } |z| \geq 1. \tag{15}$$

Equality in (15) holds for $P(z) = \gamma z^n, \gamma \neq 0$.

If in (15), we take $\alpha = r = 1, \beta = 0$ and divide both sides of it by $R - 1$ and make $R \to 1$, we get

$$|P'(z)| \leq n|z|^{n-1} \max_{|z|=1} |P(z)| \quad \text{for } |z| \geq 1,$$

which in particular yields (1),whereas (2) is a special case of (15), if we take $\alpha = \beta = 0$.
Next, we establish the following estimate for the lower bound of $|\mathcal{T}[P](z)|$ on $|z| \geq 1$.

Theorem 2.8. If $P(z)$ is a polynomial of degree n with all zeros in $|z| \leq 1$, then for every $|\alpha| \leq 1, |\beta| \leq 1, m \leq n$ and $R \geq r \geq 1$, we have for $|z| \geq 1$,

$$|\mathcal{T}[P](z)| \geq \left|(R^m + r^m \psi(\alpha,\beta,R,r)) \left(\mathcal{N}[\varphi_n(z)] + \sum_{v=1}^{s} \lambda_v \binom{m}{v} \frac{(n^v - m^v)}{2^v} z^m\right)\right| \min_{|z|=1} |P(z)|, \tag{16}$$

where $\varphi_n(z) = z^n$.

If we take $m = n$ in (16), we get the following result.

Corollary 2.9. If $P(z)$ is a polynomial of degree n with all zeros in $|z| \leq 1$, then for every $|\alpha| \leq 1$, $|\beta| \leq 1$ and $R \geq r \geq 1$, we have for $|z| \geq 1$,

$$|\mathcal{T}[P](z)| \geq \left|R^n + r^n\psi(\alpha,\beta,R,r)\right| \left|\mathcal{N}[\varphi_n(z)]\right| \min_{|z|=1} |P(z)|, \tag{17}$$

where $\varphi_n(z) = z^n$.

If in (17), we take $\beta = 0$, we get for every $|\alpha| \leq 1$ and $R \geq r \geq 1$,

$$|\mathcal{N}[P(Rz)] - \alpha\mathcal{N}[P(rz)]| \geq |R^n - r^n\alpha| \left|\mathcal{N}[\varphi_n(z)]\right| \min_{|z|=1} |P(z)| \quad \text{for } |z| \geq 1.$$

3. Auxiliary Results

To establish our main results, we need the following auxiliary results.

Lemma 3.1. If $P \in P_n$, and $P(z)$ has all its zeros in $|z| \leq 1$, then for every $R \geq r \geq 1$, and $|z| = 1$,

$$|P(Rz)| \geq \left(\frac{1+R}{1+r}\right)^n |P(rz)|.$$

The proof of this lemma is similar to the proof of Lemma 2.1 of Govil et al. [8], and hence we omit the details.
If we take $r = s = 1$ and $\sigma = \frac{n}{2}$ in Theorem 1.1 of Rather et al. [21], we get the following:

Lemma 3.2. If all the zeros of polynomial $P \in P_n$ lie in $|z| \leq 1$, then all the zeros of $\mathcal{N}[P(z)]$ also lie in $|z| \leq 1$.

4. Proofs of the Main Results

Proof of Theorem 2.2: For $R = r \geq 1$, the result follows from (4.3) in [5]. Thus, we assume $R > r \geq 1$. If λ is any complex number such that $|\lambda| > 1$, then by Rouché's theorem, the polynomial $T(z) = P(z) - \lambda f(z)$ has all its zeros in $|z| \leq 1$. On applying Lemma 3.1 to the polynomial $T(z)$, we get for $R > r \geq 1$ and for each $0 \leq \theta < 2\pi$,

$$|T(Re^{i\theta})| \geq \left(\frac{1+R}{1+r}\right)^n |T(re^{i\theta})|. \tag{18}$$

Since $T(Re^{i\theta}) \neq 0$ and $\frac{1+R}{1+r} > 1$, for every $R > r \geq 1$, it follows from (18) that

$$|T(Re^{i\theta})| > \left(\frac{1+r}{1+R}\right)^n |T(Re^{i\theta})| \geq |T(re^{i\theta})|,$$

which is equivalent to

$$|T(Rz)| > |T(rz)| \quad \text{for } |z| = 1 \text{ and } R > r \geq 1. \tag{19}$$

If α is any complex number with $|\alpha| \leq 1$, we have

$$\begin{aligned} |T(Rz) - \alpha T(rz)| &\geq |T(Rz)| - |\alpha||T(rz)| \\ &\geq \left\{\left(\frac{1+R}{1+r}\right)^n - |\alpha|\right\} |T(rz)| \quad \text{for } |z| = 1. \end{aligned} \tag{20}$$

Since $T(Rz)$ has all its zeros in $|z| \leq \frac{1}{R} < 1$, it follows by a direct application of Rouché's theorem and inequality (19) that the polynomial $T(Rz) - \alpha T(rz)$ has all its zeros in $|z| < 1$. Again from inequality(20) , by direct application of Rouché's theorem, it follows that all the zeros of the polynomial

$$\begin{aligned} H(z) :&= T(Rz) + \psi(\alpha,\beta,R,r)T(rz) = T(Rz) - \alpha T(rz) + \beta\left\{\left(\frac{1+R}{1+r}\right)^n - |\alpha|\right\} T(rz) \\ &= P(Rz) - \alpha P(rz) + \beta\left\{\left(\frac{1+R}{1+r}\right)^n - |\alpha|\right\} P(rz) \\ &- \lambda\left[f(Rz) - \alpha f(rz) + \beta\left\{\left(\frac{1+R}{1+r}\right)^n - |\alpha|\right\} f(rz)\right] \end{aligned}$$

lie in $|z| < 1$, for any complex number β with $|\beta| \leq 1$, and $R > r \geq 1$. Applying Lemma 3.2 and noting that $\mathcal{N}$ is a linear operator, we conclude that all the zeros of the polynomial

$$\begin{aligned} \mathcal{N}[H(z)] &= \sum_{v=0}^{s} \lambda_v \frac{H^V(z)}{v!}\left(\frac{nz}{2}\right)^v \\ &= \sum_{v=0}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha,\beta,R,r)P^v(rz)}{v!}\left\{\left(\frac{mz}{2}\right)^v + \left(\frac{nz}{2}\right)^v - \left(\frac{mz}{2}\right)^v\right\} \\ &- \lambda \sum_{v=0}^{s} \lambda_v \frac{f^v(Rz) + \psi(\alpha,\beta,R,r)f^v(rz)}{v!}\left(\frac{nz}{2}\right)^v \\ &= \mathcal{N}[P(Rz)] + \psi(\alpha,\beta,R,r)\mathcal{N}[P(rz)] + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha,\beta,R,r)P^v(rz)}{v!}\frac{(n^v - m^v)}{2^v} z^v \\ &- \lambda\left\{\mathcal{N}[f(Rz)] + \psi(\alpha,\beta,R,r)\mathcal{N}[f(rz)]\right\} \\ &= \mathcal{T}[P](z) + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha,\beta,R,r)P^v(rz)}{v!}\frac{(n^v - m^v)}{2^v} z^v - \lambda\mathcal{T}[f](z) \end{aligned}$$

lie in $|z| < 1$, for every $|\alpha| \leq 1$, $|\beta| \leq 1$, and $R > r \geq 1$.
This implies

$$\left|\mathcal{T}[P](z) + \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz) + \psi(\alpha,\beta,R,r)P^v(rz)}{v!}\frac{(n^v - m^v)}{2^v} z^v\right| \leq \left|\mathcal{T}[f](z)\right| \quad \text{for } |z| \geq 1.$$

For if this is not true, there exist a point $z = z_0$ with $|z_0| \geq 1$ such that

$$|\mathcal{T}[P](z_0) + A| > \left|\mathcal{T}[f](z_0)\right|,$$

where

$$A = \sum_{v=1}^{s} \lambda_v \frac{P^v(Rz_0) + \psi(\alpha,\beta,R,r)P^v(rz_0)}{v!}\frac{(n^v - m^v)}{2^v} z_0^v$$

Taking

$$\lambda = \frac{\mathcal{T}[P](z_0) + A}{\mathcal{T}[f](z_0)},$$

so that $|\lambda| > 1$ and $\mathcal{N}[H(z_0)] = 0$ for $|z_0| \geq 1$, which is a contradiction as all zeros of $\mathcal{N}[H(z)]$ lie in $|z| < 1$. This completes the proof of Theorem 2.2.

Proof of Theorem 2.8: Let $m_1 = \min_{|z|=1} |P(z)|$. If $m_1 = 0$ then there is nothing to prove. Assume that $m_1 > 0$, so that all the zeros of $P(z)$ lie in $|z| < 1$ and we have $m_1|z|^m \leq |P(z)|$ for $|z| = 1$. Applying Theorem 2.2 to $m_1 z^m$ and $P(z)$, we get for every $|\alpha| \leq 1$, $|\beta| \leq 1$, $m \leq n$ and $R \geq r \geq 1$,

$$|\mathcal{T}[P](z)| \geq \left|(R^m + r^m\psi(\alpha,\beta,R,r))\left(\mathcal{N}[\varphi_n(z)] + \sum_{v=1}^{s} \lambda_v \binom{m}{v} \frac{(n^v - m^v)}{2^v} z^m\right)\right| \min_{|z|=1} |P(z)|,$$

for $|z| \geq 1$, which is precisely inequality (16). This completes the proof of Theorem 2.8.

5. Conclusion

Polynomial inequalities form a foundational aspect of both approximation theory and geometric function theory. In this work, we extend a class of such inequalities to a generalized linear operator $\mathcal{N}$, showcasing how operator-theoretic techniques can unify and strengthen classical results. While not all known inequalities naturally admit generalization, many exhibit inherent structural properties that make them suitable for extension via $\mathcal{N}$. The results presented here point towards a promising direction for further research. For an extensive overview of related inequalities that may serve as a basis for future developments, refer to [20].

References

[1] N. C. Ankeny and T. J. Rivlin, *On a theorem of S. Bernstein*, Pacific J. Math., **5** (1955), 849-852.
[2] A. Aziz and Q. M. Dawood, *Inequalities for a polynomial and its derivative*, J. Approx. Theory, **54** (1998), 306-313.
[3] S. Bernstein, *Sur l'ordre de la meilleure approximation des fonctions continues par des polynômes de degré donné*, Mem. Acad. R. Belg., **4** (1912), 1-103.
[4] S. Bernstein, *Sur la limitation des dérivées des polynômes*, C. R. Acad. Sci. (Paris), **190** (1930), 338-340.
[5] F. A. Bhat and H. A. Dar, *Generalization of polynomial inequalities involving N-operator*, Publ. De L'Inst. Math. (accepted, to appear)
[6] M. Bidkham and K. K. Dewan, *Inequalities for a polynomial and its derivative*, J. Math. Anal. Appl., **166** (1992), 319-324.
[7] R. B. Gardner, N. K. Govil and G. V. Milovanović, *Extremal Problems and Inequalities of Markov-Bernstein Type for Algebraic Polynomials*, Mathematical Analysis and Its Applications, London: Elsevier/Academic Press, 2022.
[8] N. K. Govil, A. Liman and W. M. Shah, *Some inequalities concerning derivative of polynomials and polynomials*, Austr. J. Math. Anal. Appl., **8** (2011), 1-8.
[9] N. K. Govil and Q. I. Rahman, *Inequalities describing the growth of polynomials not vanishing in a disk of prescribed radius*, Math. Inequal. Appl., **6** (2003), 453-466.
[10] P. Kumar and G. V. Milovanović, *On sharpening and generalization of Rivlin's inequality*, Turk. J. Math., **46** (2022), 1436-1445.
[11] P. D. Lax, *Proof of a conjecture of P. Erdös on the derivative of a polynomial*, Bull. Amer. Math. Soc., **50** (1944), 509-513.
[12] M. Marden, *Geometry of Polynomials*, Math. Surveys, No. 3, Amer. Math. Soc., Providence, RI, (1966).
[13] G. V. Milovanović, A. Mir and A. Hussain, *Extremal problems of Bernstein-type and an operator preserving inequalities between polynomials*, Sib. Math. J., **63** (2022), 138-148.
[14] G. V. Milovanović, D. S. Mitrinović and Th. M. Rassias, *Topics in Polynomials, Extremal problems, Inequalities, Zeros*, World Scientific, Singapore, (1994).
[15] A. Mir, *Comparison inequalities of Bernstein-type between polynomials with restricted zeros*, Appl. Anal. Discrete Math., **16** (2022), 55-65.
[16] A. Mir and A. Hussain, *Operator preserving Bernstein-type inequalities between polynomials*, J. Contemp. Math. Anal., **58** (2023), 347-356.
[17] A. Mir and A. Hussain, *Comparison inequalities between polynomials with constraints on their zeros*, Appl. Anal. Discrete Math., **18** (2024), 28-43.
[18] A. Mir and B. Dar, *Inequalities concerning the rate of growth of Polynomials*, Afr. Mat., **27** (2016), 279-290.
[19] Q. I. Rahman, *Functions of exponential type*, Trans. Amer. Math. Soc., **135** (1969), 295-309.
[20] Q. I. Rahman and G. Schmeisser, *Analytic Theory of Polynomials*, Oxford University Press, (2000).
[21] N. A. Rather, I. Dar and S. Gulzar, *On the zeros of certain composite polynomials and an operator preserving inequalities*, Ramanujan J., **54** (2021), 605-612.